Solving the Rubik Cube Using Only Four-Turn Formulas

(1) Complete the First Layer Cross (White Cross on Face 6)

This is easy! Try it yourself.

Or refer to steps L1.1 and L1.2 on page 2 of the other note for SLL method.

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(2) Complete the Second-Layer Edge Pieces

Standard Procedure: Repeat the following steps (until no target edge piece remains):

1. Find a target edge piece in the third layer whose two colors are **not yellow**.
2. Rotate the third face (.3) so the **top color (u)** of the target edge matches the **center color (c)** of its side, then turn that side to the front (face 1) (u1c).
3. If the side color of the target edge matches the center color of the **right side (face 2)**: perform -2+3+2 to insert the edge into the second layer.
   -2+3+2 (RU'R') is the first three moves of the **forward four-turn formula**.
   The whole sequence is .3 -2+3+2.
4. If the side color of the target edge matches the center color of the **left side (face 5)**: perform +5-3-5 to insert the edge into the second layer.
   +5-3-5 (L'U L) is the first three moves of the **symmetric four-turn formula**.
   The whole sequence is .3 +5-3-5.

If an edge piece is already in the second layer but flipped incorrectly, move it to between the **front (1)** and **right (2)** faces, then perform: -2+3+2 -3-z +5-3-5. This lifts the edge to the third layer, and then reinserts it into the second layer using the standard second-layer method.

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(3) Complete the Third-Layer Cross (Yellow Cross on Face 3)

Standard Procedure: Alternate between the following two actions (until no more target edge pieces remain):

1. Find a target edge in the third layer with **yellow (y)** on its side (s). Rotate the third face (*3) to place the edge above the **front face (s1y)**, then perform -2+3+2.
   (First three moves of the forward four-turn formula)
2. Find a target edge in the third layer with **yellow (y)** on its side (s). Rotate the third face (*3) to place the edge above the **right face (s2y)**, then perform +1-3-1.
   (First three moves of the symmetric four-turn formula)

After this, one of the second-layer edge pieces will be displaced to the third layer; reinsert it using the standard second-layer method.

The full sequence is:
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s1y -2+3+2 s2y +1-3-1 {s1y -2+3+2 s2y +1-3-1} u1c -2+3+2
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(4) Complete the Six-Face Cross

After completing the yellow cross, check whether the side colors of the four edge pieces match their corresponding center pieces.
(At least two sides must match; if not, rotate the third face and check again.)

- If all four sides match: the six-face cross is complete.
- If only two adjacent sides match: place them on the **right (2)** and **back (4)** faces, then perform:
  U1 = -2+3+2#3 -2+3+2 to complete the six-face cross.
- If only two opposite sides match: place them on the **front (1)** and **back (4)** faces, then perform:
  U1 -3 U1 to complete the six-face cross.

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(5) Position All 8 Corner Pieces

Pivot Face / Pivot Corner vs Target Face / Target Corner:

- Pivot face is placed on the right. Pivot corner is the bottom-right-front corner.
- Target face is placed on the left. Target corner is the top-left-front corner.

Phase 1: Use the white face as the target face and the yellow face as the pivot face. Position the three corners of the target face, leaving one incorrect corner for Phase 2.

Phase 2: Use the yellow face as the target face and the white face as the pivot face. Position the four corners of the target face to complete the cube.

This step uses only the following two short, mutually inverse **four-turn formulas:**

- Forward Four-Turn Formula: -2+3+2-3 (+ sequence: up → right → down → left)
- Reverse Four-Turn Formula: +3-2-3+2 (- sequence: right → up → left → down)

These formulas swap two diagonally opposite corners, rotate three edges, and change the orientation of seven pieces.
Try them and observe which corners and edges are affected.

1. One pair of corners is the target and pivot corners.
2. The other pair is the two top corners on the pivot face.
3. The three edges are those between the four corners.

A. **3 times (+3 or -3)** restores the three edges with no orientation change.
B. **6 times (+6 or -6)** restores all seven pieces with no orientation change.
C. **2 times (+2 or -2)** restores the two pairs of corners but with a twist.

The target face can be rotated freely (-5,+5,#5)(L,L',L2) to bring the target corner piece to the target corner position at top-left-front.
The formula swaps the pivot and target corners and also twists them.

1. If the piece in the pivot corner belongs to the target face, move the correct target position of this piece to the target corner, and apply the four-turn formula once to insert it. If 3 four-turn formulas are needed, it is better to use the next method.
2. If the piece in the pivot corner does **not** belong to the target face, or item 1 needs 3 four-turn formulas, move any incorrect target corner piece to the target corner, and apply the formula once to bring the target corner piece to the pivot corner.

A. When the **yellow face is the pivot face**: after each **+3 or -3** four-turn formula, you may change the pivot corner by rotating the right face to bring a white corner piece into the pivot corner.
B. When the **white face is the pivot face**: after each **+6 or -6** four-turn formula, you may change the pivot corner, finish the cube, or continue the following steps.
C. If you encounter the case where both yellow and white faces each have only one misoriented corner (their twists are opposite):
   Use the face containing both corners as the new target face, and its opposite face as the new pivot face. Move one of the misoriented corner pieces to the target corner, perform the four-turn formula twice to correct it, then move the other misoriented corner piece to the target corner and perform the reverse formula twice to correct it.

(3a) Advanced To Make face 3 yellow Cross

Symbols used: (*NG -NG -^NG ... v-E4 |E4 ^-E4 ... A2 B2 ...)
E1 or / means the adjacent two edges of the corner 1 need to be swapped.
OK means 4 edges are in the correct sequence.
E4 or \ means the adjacent two edges of the corner 4 need to be swapped.
NG means 4 edges are in the reverse sequence.

```
   *NG      -NG      -^NG      *OK       *E4       *E1
     \        \      3 \
    3       3 3 3    3 3         3         3         3
  \        \        \                    \            /


   v-E4      |E4      ^-E4      -vE1      |E1      v-E1
              3        3                   3
    3 3       3        3 3      3 3         3       3 3
  \ 3       \ 3       \           3 /      3 /      3 /
```

A2 = Noted as (-2+3+2 *) = -2+3+2 *3 +1-3-1 .3 -2+3+2
   or = -2+3+2 *3 +1-3-1 *3 -2+3+2 *3 +1-3-1 .3 -2+3+2
B2 = Noted as (-2#3+2 *) = -2#3+2 *3 +1-3-1 .3 -2+3+2
B5 = Noted as (+5#3-5 *) = +5#3-5 *3 +1-3-1 .3 -2+3+2
C2 = Noted as (-2-3+2 *) = -2-3+2 *3 +1-3-1 .3 -2+3+2
U1 = -2+3+2 #3 -2+3+2

(3a1) Fastest method: rotates used
  *NG -NG -^NG use A2 (-2+3+2 *) (19) (11) (11)
  *OK *En   use A2+(A2 B*,C2) (19) (19)
  v-OK |OK   use B2 + U1   (19)
 v-E4 |E4 ^-E4 use B2 (-2#3+2 *) (11)
 -vE1 |E1   use B5 (+5#3-5 *) (11)
 v-E1   use C2 (-2-3+2 *) (11)

(3a2) Simple method: At worst, add U1 to the fastest method.
  *NG -NG -^NG use A2
  *OK *En   use A2+(A2 U1)
 v-EnOK |EnOK use B2 + U1

(3a3) Standard easy method: All use A2. At worst, add U1+U1 to the
fastest method.

(6) Optional: Following steps get the test cases above from +OK.

-2+3+2 +1-3-1 .3 -2+3+2   +OK --> v-NG --> *OK   -^NG --> +OK
-2#3+2 +1-3-1 .3 -2+3+2   +OK --> -vE1 --> |E4 --> +OK
-2-3+2 +1-3-1 .3 -2+3+2   +OK --> |E4

Recommended method: CFEC method

CFEC method = First Layer **White Cross** + First **2** Layers +
                      Third Layer **Yellow Edges** + Third Layer **Yellow Corners**
The **F2L** of the **CFOP** method is used to replace the steps: **(2) Complete
the Second-Layer Edge Pieces,** and the **Phase 1** of **(5) Position All 8 Corner Pieces.**
The last pair of edge and corner is replaced by inserting the edge
into layer 2 by using the standard second-layer method.
And no formula is needed for F2L, and so for CFEC.

Compare CFEC vs CFOP (EC vs OP):

Since EC uses one free corner at the first layer compared to OP, this
makes EC easier than OP, and in many cases, EC may be faster than OP.
The cons of OP are that it uses too many formulas (OLL:57, PLL:21),
and most with long steps.
The cons of EC are that it uses too many steps in some cases. In that
case, if 3 corner pieces need to be relocated, we can use L3.23 (also
twist the corner pieces) or L3.3 (no twist) in Fig.5 of the other
note for SLL method. Although we may use other formulas to relocate
corner pieces, it is not so necessary.

The F2L does not require remembering any formulas. But need to know
how to position the pieces. A brief note is given as follows. Or see
the other detail note.

1. **E3C3**: 2 cases as Lucky pairs (E|C, E/C). Need to do first.
Otherwise, it will be broken.(E3C3 = Edge and Corner pieces in L3)
It is easy to put them in the position. Try it yourself.
To put the pieces as stated follows: For E3, C3 pieces rotate layer 3
(L3); for E2, C1 or slot rotate layer 1 and 2 together (L1+L2).

2. **E3C1**: 3 cases. Maybe do it early.(Edge piece in L3; Corner in L1)
When white is on the side of the corner piece, put the edge side to
the right face, and if the corner side color is the same as that on
the upside of the edge, put this side of the corner to the right
face; otherwise, put the white side of the corner to the right face.
When white is on the down face of the corner piece, put the edge side

to the left face; and put the corner side with the same color as the one on the upside of the edge to the right face.

3. **E3C3**: 7 cases.
If EC connects, and if the end-face and the upside of the edge are the same color, put the end-face to the right (06); else, it is (23). Else if the upside of the corner piece is white, put the edge to the right (12). Else put the white side of the corner to the right (345).
For (03), put the slot position under the corner piece.
For (456), put the slot position on the other side.
For (12), put the cube face with the same color as that on the side of the edge to the right; this also determines the slot position.
(Note: The upside of the corner piece is white for (012))

4. **C3E2**: 3 cases.
When the white color is on the side of the corner piece, put this white side to the right, and put the edge to the other side of the right face. Maybe do it early.
When the white color is on the upper side of the corner piece, connect the corner and the edge, then turn to layer 3 to become E3C3.

5. **C1E2**: Turn to layer 3 to become E3C3.
Do it at the last. It may not appear.

A. The first step is always to turn the right face, which turns the corner piece (2.), or slot (3.), or edge piece (4.) upward.
B. The third step is a reverse turn to the first step.
C. The second step is to turn the upper face, such that after the third step, the edge and the corner form a lucky pair.
   Try it yourself.

3.(0) is a special case, use formula: -2-3+2#3 -2-3+2+3 -2-3+2 or sign reversed: +2+3-2#3 +2+3-2-3 +2+3-2. It is easy to remember as: -2-3+2 or +2+3-2 repeat 3 times with #3, +3 or -3 in between.

Advanced to choose the rotating sequence of the third layer yellow corners

The table below lists the rotation of the pivot face and the number of 4-turn steps for the different combinations of the corner pieces. (Note that: exchange the +- sign can obtain the other combination, or the same, then neglect it.)
If there are many choices (resulting from the different rotation directions and the sequence of the corner pieces), you can choose the best one to minimize the total number of the 4-turn steps and keep the number of pivot face turns to be 0, +6, or -6.

nC = Number of corner pieces
O = Oritation (+)(-)
R = Rotation of the pivot face (+)(-)
S = Number of 4-turn steps

nClO	R:S			Rotating sequence of the corner pieces			
1++	++:2			++			
2++	+++:3			++ ++			
2+-	+:3	-:3		+- -+	-+ +-		
1++ --	+:5	-:5		-- ++	++ --		
1++ +-	+++:5	+:3		+- ++	++ +-		
3++	++++:4			++ ++ ++			
3+-	ok:4			+- -+ +-			
2+- ++	ok:4	++:4		-+ ++ +-	++ +- -+		
2++ --	ok:6	++:6		++ ++ --	-- ++ ++		
2++ +-	ok:8	++:4	++++:6	+- ++ ++	++ ++ +-	+- ++ ++	
1++ +- --	ok:4	++:6	++++:6 ----:6	++ +- --	+- -- ++	-+ -- ++	-- ++ -+