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A SPACE MARCHING CALCULATION METHOD FOR THREE-DIMENSIONAL FLOW PROBLEMS

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以空間發展方法解析三維流場問題

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摘 要

本文發展以空間發展方法的程式（STMAR)來解析在主要流向上没有曲率的內流場問題，以SIMPLEC的疊代程序計算非壓縮流在經過有限體積方法離散後的拋物線型化方程組。藉由此方程式求解內渠道發展流的結果並和理論解比較。位置一致性亦被測試。由渠道邊界上的熱負載所引起在截面上的迴轉流場亦在此三維計算問題中測試。

關鍵詞：空間發展，三維

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Abstract

A space marching code(SMART)for computing internal duct flow problem without curvature in the primary flow direction is developed. The resulting parabolized laminar Navier−Stokes equations for incompressible fluid are discretized using a finite volume method and solved numerically by the SIMPLEC algorithm. The computed results are compared with analytic solutions in the primary flow direction of developing laminar duct flow problem. The space consistency is also investigated. The effect of external thermal loading on the boundaries, which can induce a recirculating flow pattern on the cross−section plane,is also investigated in the present three−dimensional calculation.

Keywords: Space marching,three dimensional

1. Introduction

Many steady flow problems of practical interest,such as duct flow,jet and boundary layer flows,are classified as parabolic class. The flow belonging to this type is distinguished itself from elliptic ones by the following situations;(i)there exists a primary flow direction without reversed flow in that direction;(ii)the diffusive effect are negligible in the primary direction compared with others;(iii)there exists negligible downstream influence on the upstream flow conditions.

Under certain circumstances,however,the fluid flows may flow over a shape of abrupt change in curvature. Such flow is in fact elliptic but the diffusion of transport properties still can be neglected in flow direction. This partially parabolized flow problem [1]is influenced by the downstream pressure field and often found in the cases of strong−curved duct flow ,turbine and compressor cascades.

The inherent physical properties of parabolic or partially parabolic flow problems offer a space marching algorithm. The solutions at the downstream locations for parabolic flow problem are purely determined from the upstream data. Similar procedures are used for partially−parabolic problems except more than one relaxation sweep in the primary flow direction are required for marching to the specified downstream pressure condition. The computation, therefore, proceeds downstream plane−by−plane such that numerical advantage in terms of computer memory and cost can be saved a lot [2]

The investigated incompressible laminar flow equations are first parabolized by neglecting the streamwise diffusion terms. The resulting differental equations are discretized by a finite volume method. The dependent variables are stored at the staggered points that pressure oscillations can be avoided. The discretized equations are then solved by an semi—implicit SIMPLE—family algorithm.

The present analysis deals with the internal parabolized flow problem which has no curvature variation in the primary direction. The additional recirculating flowfield on the cross—section plane due to the applied thermal loading on the external surfaces is also investigated. The basic formulation and numerical algorithm, which is similar to the marching direction in time instead of marching direction in space, are included in the following sections.

2. Basic Formulation

The physics of steady incompressible Navier—Stokes fluid in a rectangular channel can be expressed by the following equation:

$$\frac{\partial}{\partial z}(\rho w \phi)+\frac{\partial}{\partial x}(\rho u \phi)+\frac{\partial}{\partial y}(\rho v \phi)+P^{\phi}$$

$$=\frac{\partial}{\partial x}(\Gamma\phi\frac{\partial \phi}{\partial x})+\frac{\partial}{\partial x}(\Gamma\phi\frac{\partial \phi}{\partial y})+S\phi \qquad (1)$$

where no reversed flow in the primary flow direction z is allowed. A Boussinesq approximation [3] is assumed in the energy equation for considering buoyancy effect. The resulting set of equations (1) is classified as a parabolic type since the diffusive transport in z direction is neglected. The specific meanings of ϕ, $p\phi$,$\Gamma\phi$, $S\phi$ in (1) for continuity, momentum equations, and energy equation are summarized in the following table:

	ϕ	$\Gamma\phi$	$P\phi$	$S\phi$
continuity	1	0	0	0
x – momentum	u	μ	$\partial p/\partial x$	0
y – momentum	v	μ	$\partial p/\partial y$	$g\beta\rho ref\theta$
z – momentum	w	μ	$\partial p/\partial z$	0
energy	θ	k	0	0

where $p=\bar{p}+\rho_{ref}\,g\,y$,and $\theta=\frac{T-Ti}{Th-Ti}$

3. Numerical Formulation

The above set of equations (1) is discretized by a finite volume method. The dependent variables in figure 1 are stored in a staggered manner such that the possible pressure oscillations for the incompressible flowfield can be avoided [4]. The velocity components w in the primary flow direction are stored at the same locations as scalar quantities p and θ in figure 1.

The resulting discretized equations, using power—law scheme for flux terms, are almost the same as those in [5,6] except some modifications on the time marching problem are made. The finite volume discretizing equations for

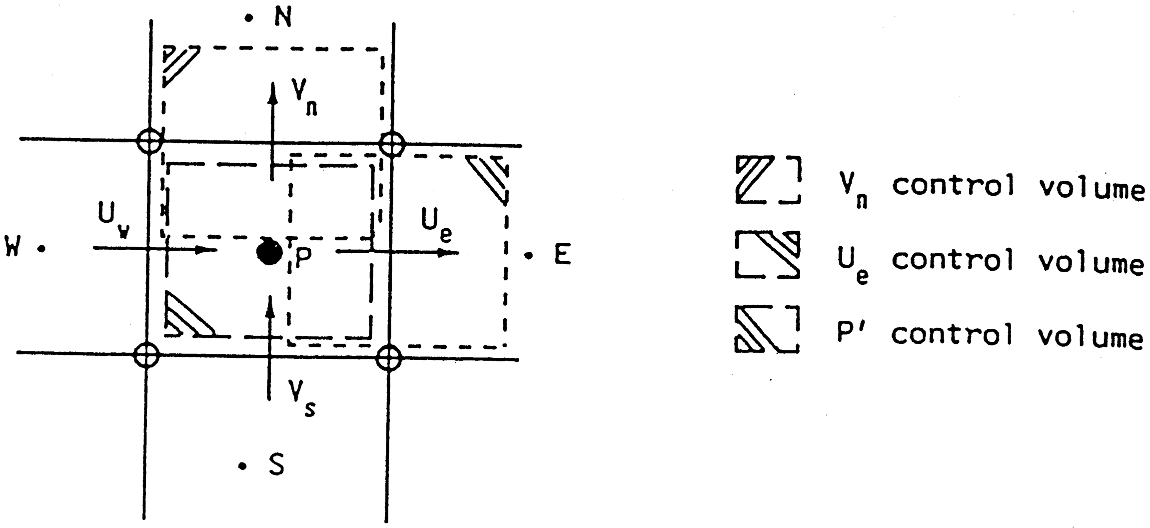

Figure 1. Locations of dependent variables at staggered mesh system.

(1) can be written by the following general form:

$$a_p \phi^{K+1} = a_E \phi_E^{k+1} + a_W \phi^{K+1}{}_W + a_N \phi^{K+1}{}_N + a_S \phi^{K+1}{}_S + b \qquad (2)$$

where

$$a_E = DeA(\mid Pe \mid) + [-Fe, o]$$

$$a_W = DwA(\mid Pw \mid) + [o, Fw]$$

$$a_N = DwA(\mid Pn \mid) + [-Fn, o]$$

$$a_S = DsA(\mid Ps \mid) + [o, Fs] \qquad (3)$$

$$a_P = a_E + a_W + a_S + a_N + a_P^k - Sp \Delta y \Delta z$$
$$a_P^k = (\rho w)_p^k = \Delta x \Delta y / \Delta z$$
$$b = S_c^\phi \Delta x \Delta y + a_P^k \phi_P^k$$

[A,B] in equation (3) denotes the greater of A and B. A(| Pe |) is a function of absolute Peclet number for different schemes.

The superscripts k, k+1 in (2) and (3) represent the locations of cross—section planes. Other notations in (3) are defined exactly the sameas those in [4.6]. The discretized continuity equation is

$$\frac{\Delta y \Delta x}{\Delta z}[(\rho w)_p^* - (\rho w)_p^k] + (F_n - F_s) + (F_e - F_w) = \text{residue} \qquad (4)$$

The residue in (4)is not equal to zero unless the final convergence solutions are obtained. The subscripts p in (2) stand for w, s, p respectively. for x, y momentum equations and energy equation. The differences in expression between space marching and time marching are as follows [5,6]:

$$a_w^k = \frac{\Delta x \Delta y}{\Delta z}(\rho w)_w^k \qquad \text{in x—momentum equation}$$

$$a_s^k = \frac{\triangle x \triangle y}{\triangle z}(\rho w)_s^k \quad \text{in y—momentum equation}$$

$$a_p^k = \frac{\triangle x \triangle y}{\triangle z}(\rho w)_p^k \quad \text{in energy equation}$$

The discretized momentum equation in the primary flow direction for solving W at the solution plane is written as:

$$a_P w_p^{k+1} = a_E W_E^{k+1} + a_W W_W^{k+1} + a_N W_N^{k+1} + a_S W_S^{k+1} + b \tag{5}$$

where

$$a_P = a_E + a_W + a_N + a_S + a_P^K$$

$$b = a_p^k w_p^k + S_c^w \triangle x \triangle y$$

The rest of coefficients are the same as those in energy equation [5,6].

The SIMPLEC iterative algorithm [4] is used to solve the discretized equations. The major procedures are as follows:

1. The available values for $u^k, v^k, w^k, p^k, \theta^k$ are specified at the cross—section plane xy. The solutions at the next downstream plane k+1 are obtained iteratively.
2. Evaluate all the coefficients in discretized x momentum equation and then u * at the grid points of solution plane.
3. Evaluate all the coefficients in discretized y momentum equation and then v * at the grid points of solution plane.
4. Guess a value of (dp/dz)* and evaluate all the coefficients in discretized z momentum equation. The corresponding solution for w * at the solution plane is then calculated.
5. Check if the global mass continuity is conserved. The correction of pressure gradient(dp/dz)* is made as the conservation of mass is not satified. The corrected w is then computed accordingly.
6. Substitute u, v, w into the mass continuity equation and check if the residue in (4) is smaller than the specified value. If not, the equation for pressure correction p' is solved.
7. Correct pressure p, and velocity u, v, using the computed pressure correction p'. The pressure correction is made using a global mass conservation on cross—plane which will be described later.
8. Solve energy equation for θ.
9. The iterative procedures from step 2 are continued until the residual in step 6 and velocity variations reach the allowable tolerances.

The converged solutions at the present solution plane will be used as the given conditions for obtaining solutions at the next downstream plane. Similar space marching procedures continue until the final downstream plane is reached.

The pressure gradient in the primary flow direction behaves elliptically that a boundary condition is required at the downstream plane. Since the equations have been parabolized and no information can be offered from the downstream plane that one has to impose a continuity of global mass as a constraint to correct streamwise pressure gradient for maintaining the elliptic pressure behavior over the cross—section.

The expression for pressure correction $(dp/dz)^*$, defined to be $(dp/dz)' = (dp/dz) - (dp/dz)^*$, can be derived similarly as[2] by substituting the correct pressure gradient term dp/dz into the discretized z momentum equation as

$$(dp/dz)' = \left[\iint_{A_k} (\rho w)^k dxdy - \iint_{A_k} (\rho w)^* dxdy\right] / \iint_{A_k} (\rho d_p)^* dxdy$$

$$\text{where } dp = A_p \Delta z / (a_p^k - \sum anb)$$

The velocity in z direction then can be corrected to be

$$w_p^* = w_p + d_p (dp/dz)'$$

4.Numerical Results

A developing laminar flowfield in a square duct is used as a test problem. The first case considers an adiabatic condition. The second study allows the temperature difference between walls. The upper and lower

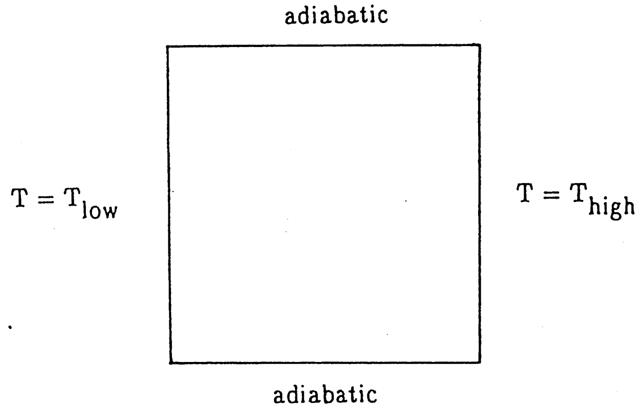

Figure 2. Configuration of cross section for second test problem.

boundaries of the enclosure are sdiabatic, and lower and higher emperatures are specified at the left and right vertical wall respectively in figure 2. A uniform velocity profile of Re=200 is given at the inlet plane z=0,

The velocity plot in the streamwise direction is shown in figure 3 which is on a cross section plane containing 12×12 uniform grid distribution. The decrease of velocity near the no—slip wall region, due to viscosity, accelerates the fluids at core flow region for preserving the global continuity of mass at each plane. The velocity keeps developing until the fully—developed length is swept where the shear and inertia forces are balanced. The corresponding velocity vector plots at cross sections are illustrated in figure 4. The fluid flows to the centroid of the cross—section in the beginning of few planes and keeps decreasing downstream. The velocity components on the xy plane

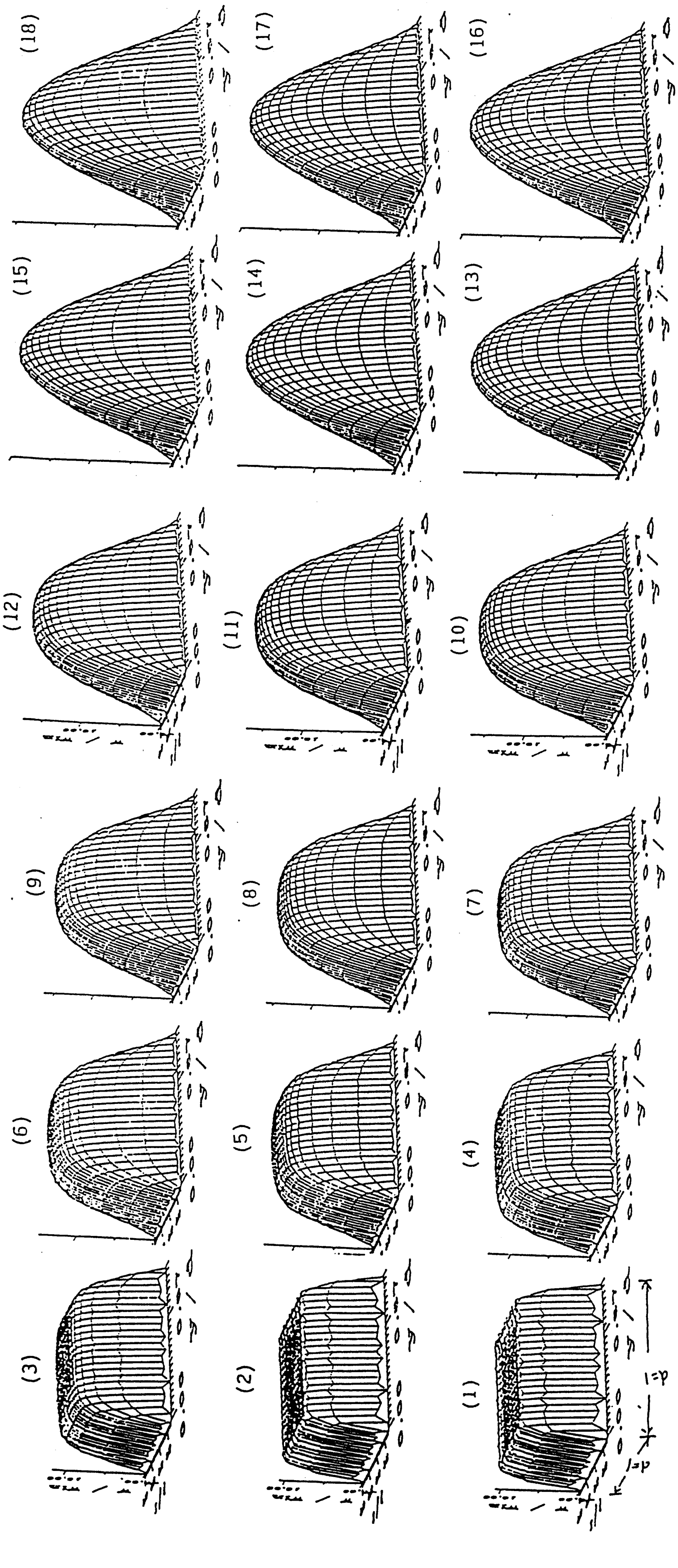

Figure 3. Streamwise velocity distribution along the duct at (1)z=0.1 (2)z= 0.2 (3)z=1.0 (4)z=2.0 (5)z=3.0 (6)z=4.0 (7)z=5.0 (8)z=6.0 (9)z=7.0 (10) z=8.0 (11)z=9.0 (12)z=10.0(13)z=20.0(14)z=30.0(15)z=40.0 (16)z=50.0(17)z=**fully developedu (18) analytic solution.**

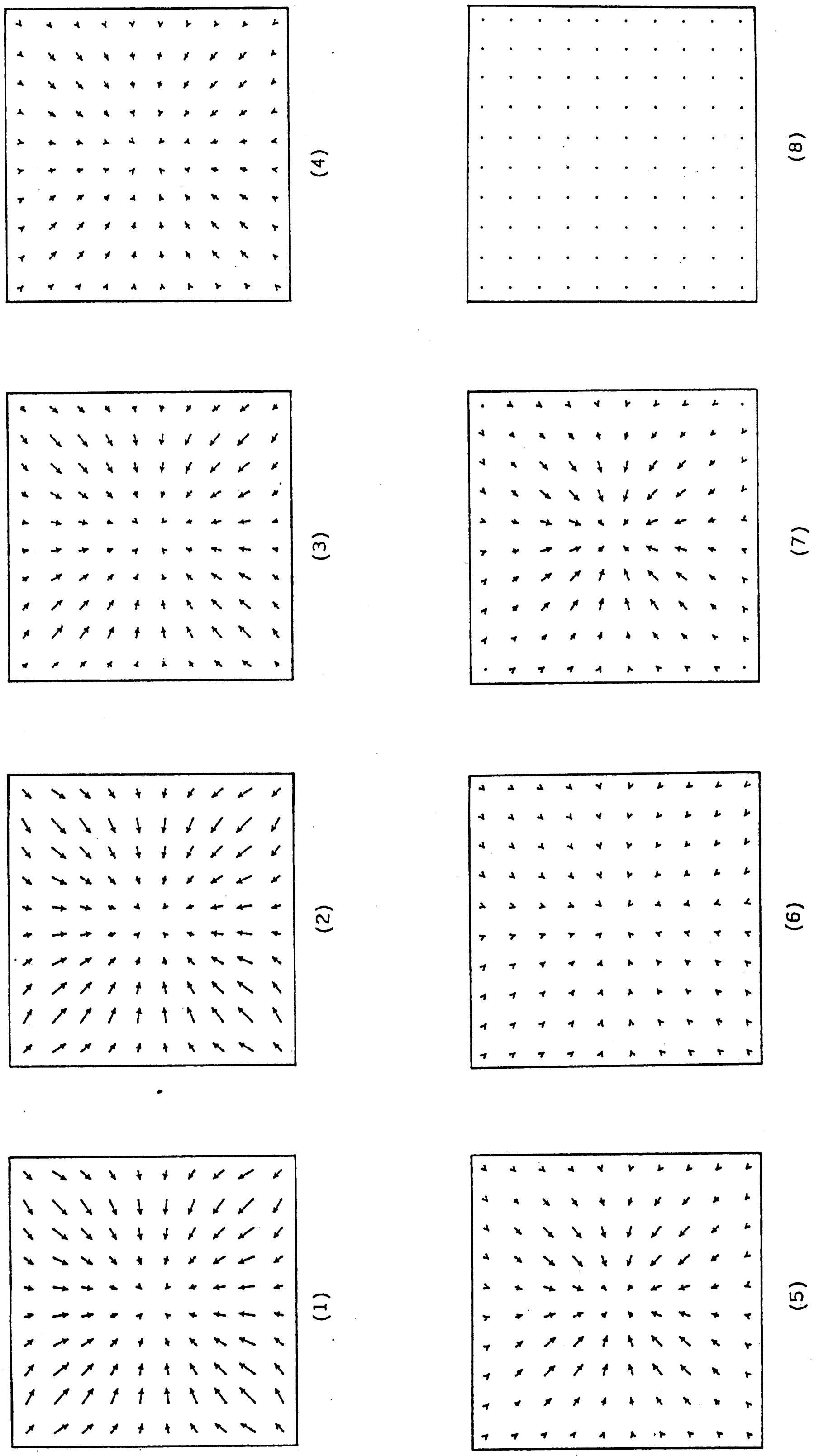

Figure 4. Velocity vector plots on the cross—section plane at(1)z=0.1(2)z=0.2(3)z=1.0(4)z=2.0(5)z=10.0(6)z=20.0
(7)z=50.0(8)z=100.0 **The magnitudes of vectors in figure** (1,2,3,4):(5,6):(7):(8)=10^{-5}:10^{-4}:10^{-2}:1

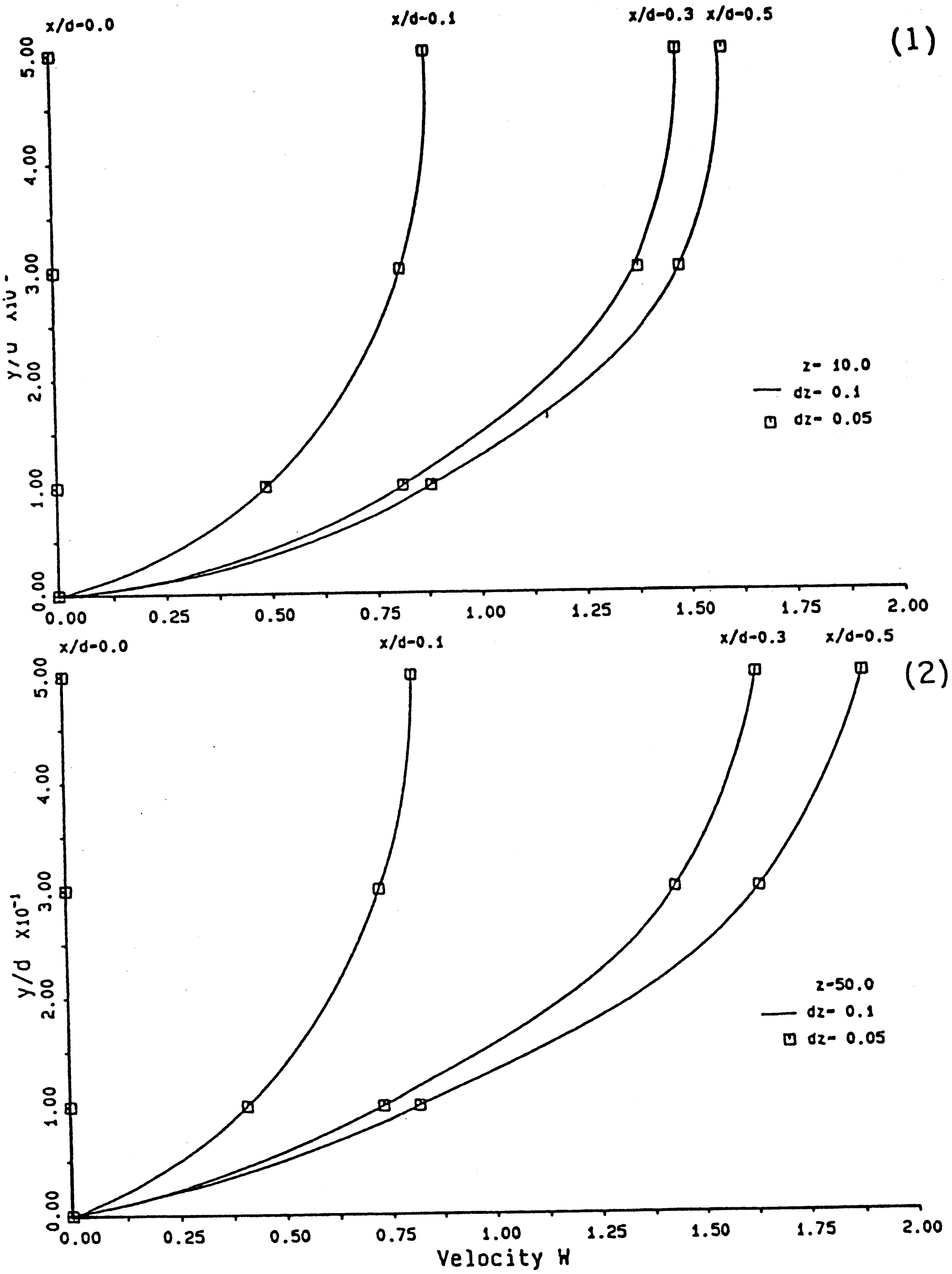

Figure 5. **Space consistence test using :$\triangle z=0.1$, :$\triangle z=0.05$ at streamwise locations (1)z=10 (2)z=50. d represents the length of square cross—section on x—y plane. The present computed results are plotted on the points□and linked by solid line.**

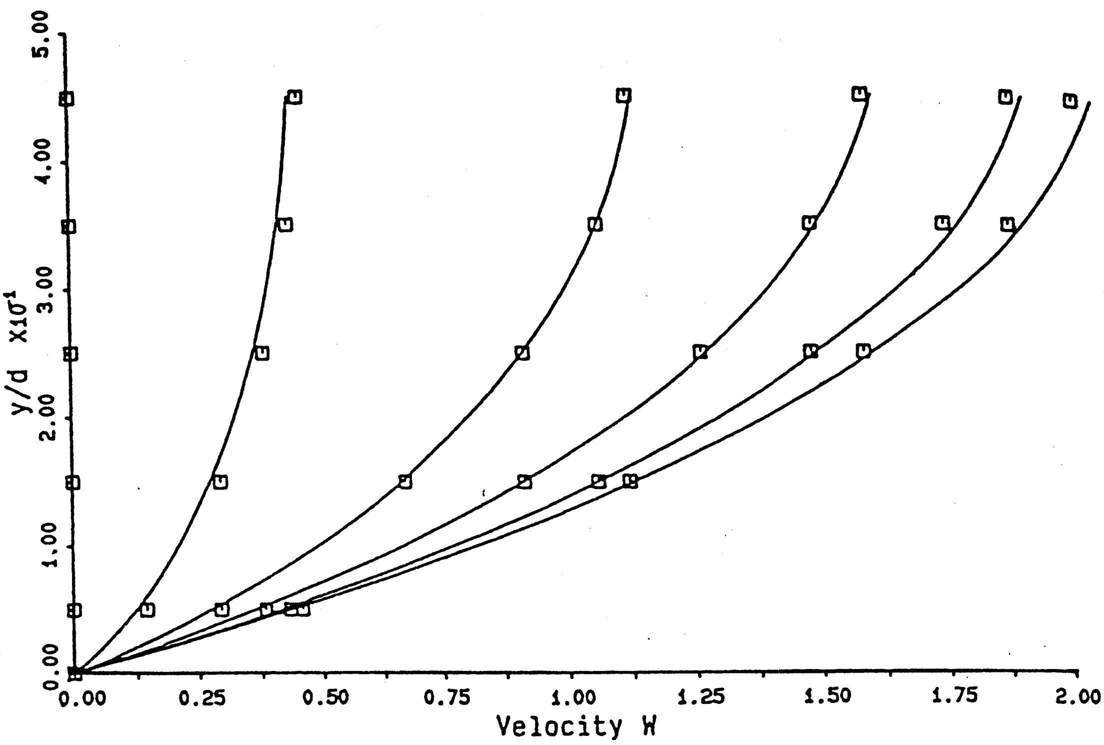

Figure 6. **Comparison of the computed streamwise result with □ by B. Lakshminarayana in [7]. d is defined as that in figure 5. The present computed results are represented by solid lines.**

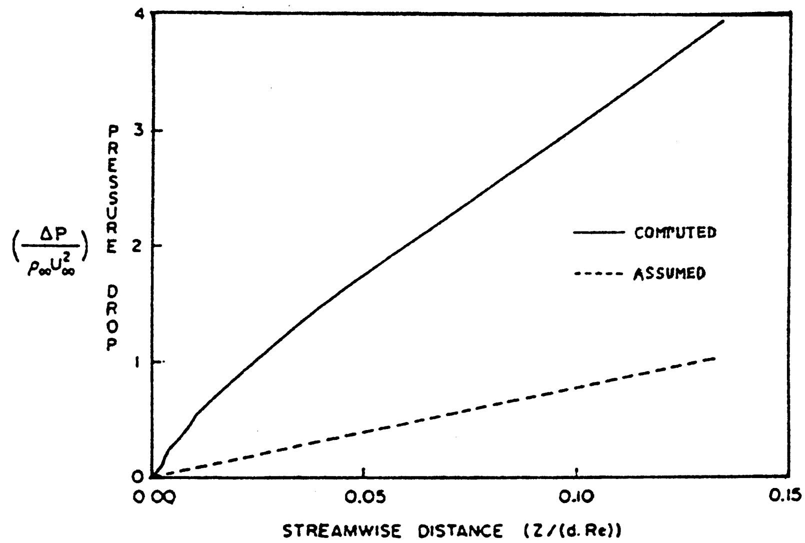

Figure 7. **The evolution of pressure drop along the streamwise direction**

vanish as the fully developed velocity profile is reached.

The consistence in the flow direction is verified in figure 5 by comparing the computed results from $\triangle z=0.1$ and 0.2. The present computed result at $z=100$ is further compared with the theoretical fully developed profile [7,8] where $\triangle z=0.1$ is used. The fairly good agreement is shown in figure 6. The evolution of pressure drop in the streamwise direction is shown in figure 7 which agrees with that from [7]. The slope in figure 7 decreases from the inlet section and approaches to a constant as the downstream station near the fully developing region is reached.

The velocity plots at cross–section of the second test problem are shown in figure 8. for Re=200, Pr=0.7. The flow directions are no longer pointed to the centers but instead more similar to those of thermal driven cavity flows. The upward flow motion,generated by the buoyancy effects, is observed on the region adjacent to the right hotter wall. The circulation pattern at the fully developed cross–section, as expected, looks similar to that of steady flow pattern with the same values of Pr and Ra. The streamwise velocity contour is almost identical to that of the first case.

5.Conclusion

A three–dimensional incompressible laminar Navier–Stokes flowfield within a straight duct is analyzed by a parabolic space marching mehtod. The solutions at the downstream cross–section are obtained by SIMPLEC iterative algorithm using the upstream information. The computed results without considering buoyancy effect agree with the available analytic data. The inclusion of thermal driven effect is also investigated and the computed solutions are physically correst.

Acknowledgement

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NOMENCLATURE

Standard

A	area which the pressure difference acts on
a,b	constants in finite difference equations (5)
F	convective terms in finite difference equation (4)
g	gravitational acceleration
H	height of square cavity
k	thermal conductivity
O(1)	order of 1.
p	pressure
P	Peclet number
Pr	Prandtl number, $2/\alpha$
$p\phi$	pressure gradient terms, equation (1)
p	$p=p+\rho ref\ g\ y$
Ra	Rayleigh number, $g\beta H^3 \triangle T/\alpha v$
$S\phi$	source terms, equation (1)
Th	heated wall temperature
Ti	initial temperature
$\triangle T$	initial temperature difference, Th−Ti
u,v,w	velocity components in x,y,z directions respectively
x,y,z	Cartesian coordinates

Greek

α	thermal diffusivity
β	coefficient of volumetric thermal expansion
ϕ	dependent variables, equation (1)
ρ	density
θ	dimensionless temperature
$\Gamma\phi$	diffusion coefficients, equation (1)
μ	dynamic viscosity
v	kinematic viscosity

Superscript

ϕ	dependent variable
*	temporary value at the present time step
′	correction
k	previous plane

Subscript

p	center of a control volume
E,W,N,S	eastern,western,northern,southern grid points of p
e,w,n,s	eastern,western,northern,southern middle points between p and points E,W,N,S.

References

[1] Pratap, V.S., and Spalding, D.B., Fluid Flow and Heat Transfer in Three−Dimensional Duct Flows, Int. J. Heat Mass Transfer, Vol. 19, 1976, pp.1183−1188.

[2] Patankar, S.V., and Spalding D.B., A Calculation Procedure for Three−Dimensional Parabolic Flows, Int. J. Heat Mass Transger, Vol. 15, 1972, pp.1787−1806.

[3] Eckert, E.R.G., Drake, R.M., Analysis of Heat and Mass Transfer, McGraw−Hill, New York, 1972.

[4] Patankar, S.V., Numerical Heat Transfer and Fluid Flow, McGraw−Hill, New York, 1980.

[5] Sheu, W.H., Liau, Y.C., Numerical Somulation of Three−Dimensional Viscous Flow Field by Parabolic Marching, NTU−INA, Report 317, 1989.

[6] Sheu, W.H., Liau, Y.C., Numerical Prediction of Free Convection Effects in an Arbitrary Cavity Using SIMPLE Family Algorithms, Proc. of the Sixth International Conference, Vol. 6, Part 2, 1989, pp 1145－1155.
[7] Govindan, T.R., Lakshminarayana, B.,A Space Marching Method for the Computation of Viscous Internal Flows, Computer & Fluids, Vol. 16, No. 1, 1988, pp.21－39
[8] White, F.M., Viscous Fluid Flow, McGraw－Hill, New York, 1974.

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