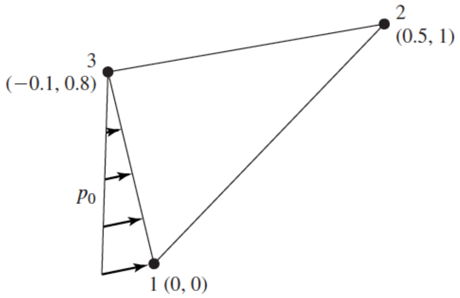
1. The T3 element shown below is subjected to the linearly varying pressure as shown below. Determine the equivalent nodal forces.



●

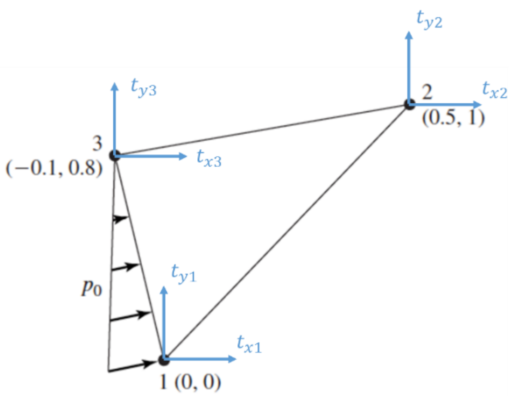
$$N_1(x,y) = \frac{A_1}{A_e} = 1 + 0.4x - 1.2y$$

$$N_2(x,y) = \frac{A_2}{A_e} = 1.6x + 0.2y$$

$$N_3(x,y) = \frac{A_3}{A_e} = -2x + y$$

●

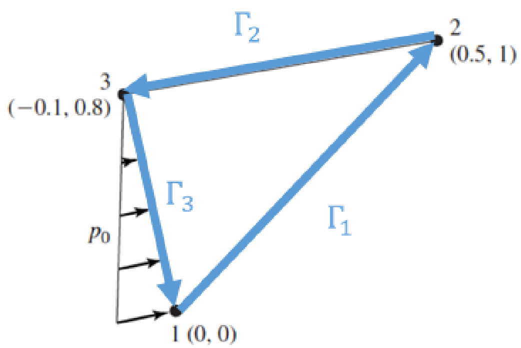
$$\bar{\boldsymbol{t}} = \boldsymbol{N}^e \boldsymbol{t} = \boldsymbol{N}^e \begin{bmatrix} t_{x1} \\ t_{y1} \\ t_{x2} \\ t_{y2} \\ t_{x3} \\ t_{y3} \end{bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{bmatrix} p_0 * \frac{8}{\sqrt{65}} \\ p_0 * \frac{1}{\sqrt{65}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



●

$$\boldsymbol{f}_\Gamma^e = t \oint \boldsymbol{N}^{eT} \bar{\boldsymbol{t}} d\Gamma = 0 + 0 + t \int_{\Gamma_3} \boldsymbol{N}^{eT} \boldsymbol{N}^e \boldsymbol{t} d\Gamma$$

$$= \frac{t}{\sqrt{65}} \begin{bmatrix} \int_{\Gamma_3} N_1^2 \cdot 8 \cdot p_0 d\Gamma \\ \int_{\Gamma_3} N_1^2 \cdot 1 \cdot p_0 d\Gamma \\ 0 \\ 0 \\ \int_{\Gamma_3} N_1 N_3 \cdot 8 \cdot p_0 d\Gamma \\ \int_{\Gamma_3} N_1 N_3 \cdot 1 \cdot p_0 d\Gamma \end{bmatrix}$$



There are no traction on Γ_1 & Γ_2, so equivalent nodal force from Γ_1 & Γ_2 are zero.

- $$y=-8x,\quad \begin{cases}N_1(x)=1+10x\\ N_3(x)=-10x\end{cases}$$

$$\Gamma^2=x^2+y^2=65x^2\longrightarrow \Gamma=\sqrt{65}x\longrightarrow \frac{d\Gamma}{dx}=\sqrt{65}\ (Jacobian)$$

$$\boldsymbol{f}_\Gamma^e=\frac{t}{\sqrt{65}}\begin{bmatrix}\int_{\Gamma_3}N_1^2\cdot 8\cdot p_0 d\Gamma\\ \int_{\Gamma_3}N_1^2\cdot 1\cdot p_0 d\Gamma\\ 0\\ 0\\ \int_{\Gamma_3}N_1N_3\cdot 8\cdot p_0 d\Gamma\\ \int_{\Gamma_3}N_1N_3\cdot 1\cdot p_0 d\Gamma\end{bmatrix}=\frac{t}{\sqrt{65}}\begin{bmatrix}\int_{-0.1}^{0}N_1^2(x)\cdot 8\cdot p_0\cdot \sqrt{65}dx\\ \int_{-0.1}^{0}N_1^2(x)\cdot 1\cdot p_0\cdot \sqrt{65}dx\\ 0\\ 0\\ \int_{-0.1}^{0}N_1(x)N_3(x)\cdot 8\cdot p_0\cdot \sqrt{65}dx\\ \int_{-0.1}^{0}N_1(x)N_3(x)\cdot 1\cdot p_0\cdot \sqrt{65}dx\end{bmatrix}=p_0t\begin{bmatrix}\frac{4}{15}\\ \frac{1}{30}\\ 0\\ 0\\ \frac{2}{15}\\ \frac{1}{60}\end{bmatrix}\approx p_0t\begin{bmatrix}0.2667\\ 0.0333\\ 0\\ 0\\ 0.1333\\ 0.0167\end{bmatrix}$$

- $$x=-\frac{1}{8}y,\quad \begin{cases}N_1(y)=1-\frac{5}{4}y\\ N_3(y)=\frac{5}{4}y\end{cases}$$

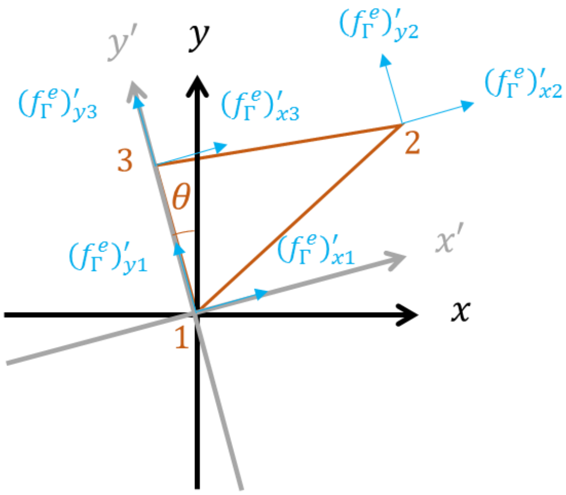
$$\Gamma^2=x^2+y^2=\frac{65}{64}y^2\longrightarrow \Gamma=\sqrt{\frac{65}{64}}y\longrightarrow \frac{d\Gamma}{dy}=-\frac{\sqrt{65}}{8}\ (Jacobian)$$

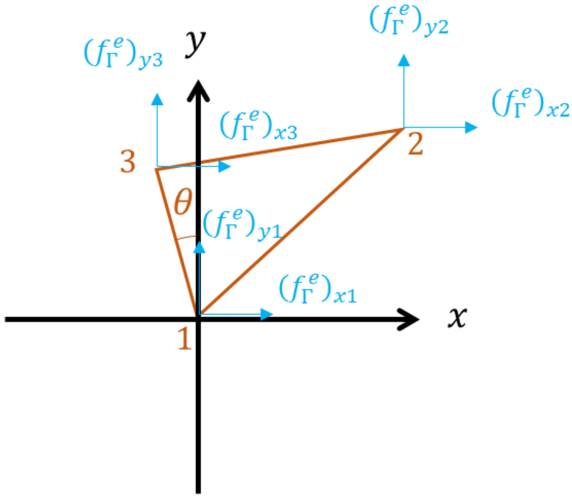
$$\boldsymbol{f}_\Gamma^e=\frac{t}{\sqrt{65}}\begin{bmatrix}\int_{\Gamma_3}N_1^2\cdot 8\cdot p_0 d\Gamma\\ \int_{\Gamma_3}N_1^2\cdot 1\cdot p_0 d\Gamma\\ 0\\ 0\\ \int_{\Gamma_3}N_1N_3\cdot 8\cdot p_0 d\Gamma\\ \int_{\Gamma_3}N_1N_3\cdot 1\cdot p_0 d\Gamma\end{bmatrix}=\frac{t}{\sqrt{65}}\begin{bmatrix}\int_{0.8}^{0}N_1^2(y)\cdot 8\cdot p_0\cdot -\frac{\sqrt{65}}{8}dy\\ \int_{0.8}^{0}N_1^2(y)\cdot 1\cdot p_0\cdot -\frac{\sqrt{65}}{8}dy\\ 0\\ 0\\ \int_{0.8}^{0}N_1(y)N_3(y)\cdot 8\cdot p_0\cdot -\frac{\sqrt{65}}{8}dy\\ \int_{0.8}^{0}N_1(y)N_3(y)\cdot 1\cdot p_0\cdot -\frac{\sqrt{65}}{8}dy\end{bmatrix}=p_0t\begin{bmatrix}\frac{4}{15}\\ \frac{1}{30}\\ 0\\ 0\\ \frac{2}{15}\\ \frac{1}{60}\end{bmatrix}\approx p_0t\begin{bmatrix}0.2667\\ 0.0333\\ 0\\ 0\\ 0.1333\\ 0.0167\end{bmatrix}$$

- $$\begin{bmatrix}x'\\ y'\end{bmatrix}=\begin{bmatrix}cos\theta & sin\theta\\ -sin\theta & cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\boldsymbol{T}\begin{bmatrix}x\\ y\end{bmatrix}$$

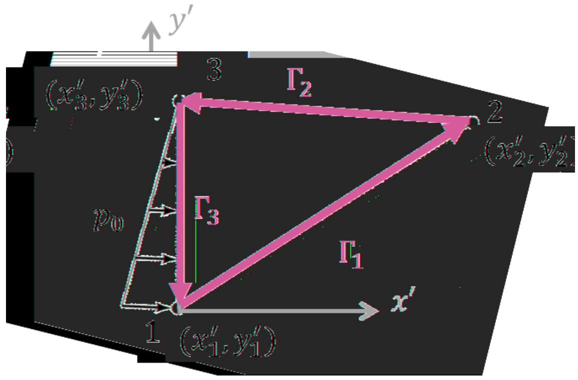
- $$A_e=\frac{1}{2}\begin{vmatrix}1 & x_1' & y_1'\\ 1 & x_2' & y_2'\\ 1 & x_3' & y_3'\end{vmatrix}=\frac{1}{4}\quad A_1'=\frac{1}{2}\begin{vmatrix}1 & x' & y'\\ 1 & x_2 & y_2\\ 1 & x_3 & y_3\end{vmatrix}\quad A_2'=\frac{1}{2}\begin{vmatrix}1 & x' & y'\\ 1 & x_3 & y_3\\ 1 & x_1 & y_1\end{vmatrix}\quad A_3'=\frac{1}{2}\begin{vmatrix}1 & x' & y'\\ 1 & x_1 & y_1\\ 1 & x_2 & y_2\end{vmatrix}$$

$$N_1(x',y')=\frac{A_1'}{A_e}\quad N_2(x',y')=\frac{A_2'}{A_e}\quad N_3(x',y')=\frac{A_3'}{A_e}$$

- 



- $$\bar{\boldsymbol{t}} = \boldsymbol{N}^e \boldsymbol{t} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{bmatrix} p_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



- $$\boldsymbol{f}_\Gamma^e = t \oint \boldsymbol{N}^{eT} \bar{\boldsymbol{t}} d\Gamma = 0 + 0 + t \int_{\Gamma_3} \boldsymbol{N}^{eT} \boldsymbol{N}^e \boldsymbol{t} d\Gamma = t \begin{bmatrix} \int_{\Gamma_3} N_1^2(x', y') p_0 dy' \\ 0 \\ 0 \\ 0 \\ \int_{\Gamma_3} N_1(x', y') N_3(x', y') p_0 dy' \\ 0 \end{bmatrix} = \begin{bmatrix} (f_\Gamma^e)'_{x1} \\ (f_\Gamma^e)'_{y1} \\ (f_\Gamma^e)'_{x2} \\ (f_\Gamma^e)'_{y2} \\ (f_\Gamma^e)'_{x3} \\ (f_\Gamma^e)'_{y3} \end{bmatrix}$$

- $$\because \boldsymbol{F}' = \boldsymbol{T}\boldsymbol{F}$$

$$\Rightarrow \boldsymbol{F} = \boldsymbol{T}^{-1} \boldsymbol{F}'$$

$$\Rightarrow \begin{bmatrix} (f_\Gamma^e)_{x1} & (f_\Gamma^e)_{x2} & (f_\Gamma^e)_{x3} \\ (f_\Gamma^e)_{y1} & (f_\Gamma^e)_{y2} & (f_\Gamma^e)_{y3} \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} (f_\Gamma^e)'_{x1} & (f_\Gamma^e)'_{x2} & (f_\Gamma^e)'_{x3} \\ (f_\Gamma^e)'_{y1} & (f_\Gamma^e)'_{y2} & (f_\Gamma^e)'_{y3} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (f_\Gamma^e)_{x1} & (f_\Gamma^e)_{x2} & (f_\Gamma^e)_{x3} \\ (f_\Gamma^e)_{y1} & (f_\Gamma^e)_{y2} & (f_\Gamma^e)_{y3} \end{bmatrix} = t\, p_0 \begin{bmatrix} 0.2667 & 0 & 0.1333 \\ 0.0333 & 0 & 0.0167 \end{bmatrix}$$

$$\boldsymbol{f}_\Gamma^e = p_0 t \begin{bmatrix} 0.2667 \\ 0.0333 \\ 0 \\ 0 \\ 0.1333 \\ 0.0167 \end{bmatrix}$$

ξ

- $N_1(\xi$