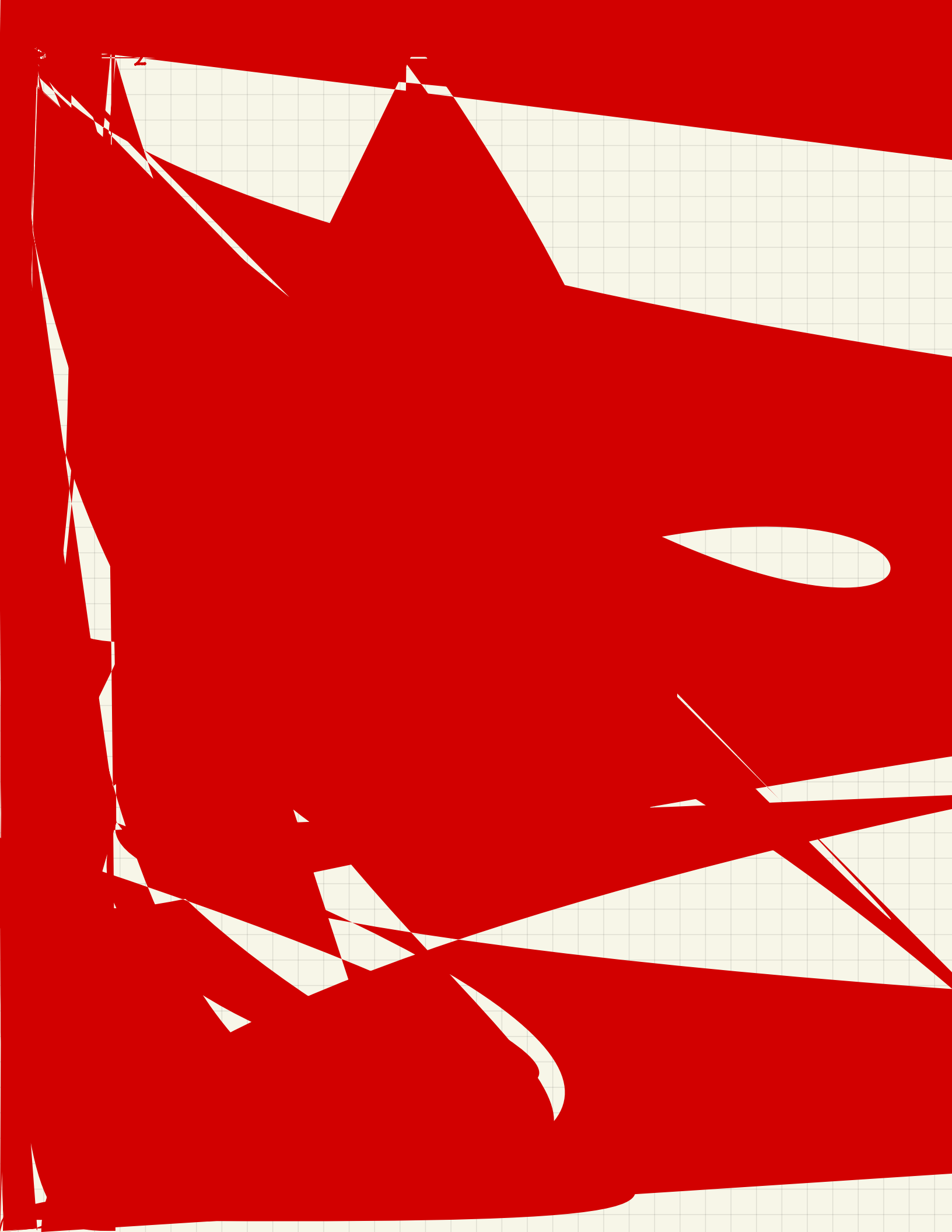




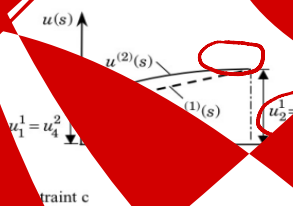


is used to solve the problem. The function f is usually assumed to be convex and the function g is usually assumed to be concave.

For two sets S and T , it is essential that $S \cap T \neq \emptyset$.

For a given u , the following is the case:

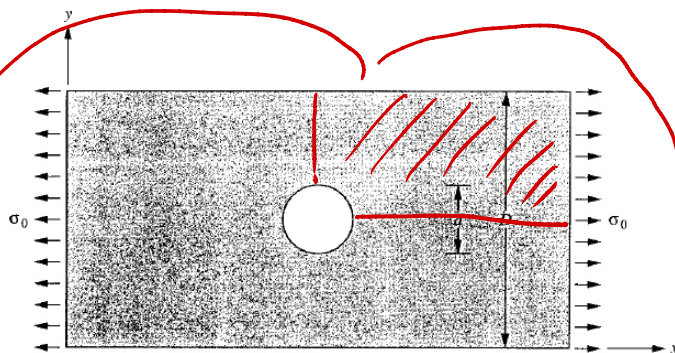
For a given u , the following is the case:



In the next section, we will see how to construct continuous elements for finite element analysis (FEM) (T, tetrahedron element 2).

3.7 -N E (3)

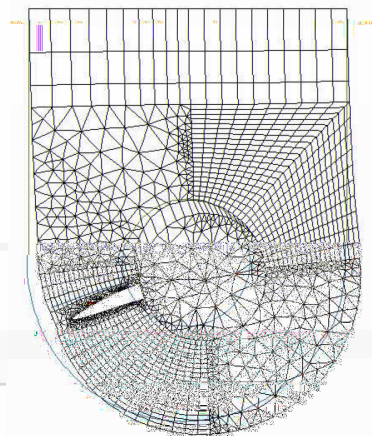
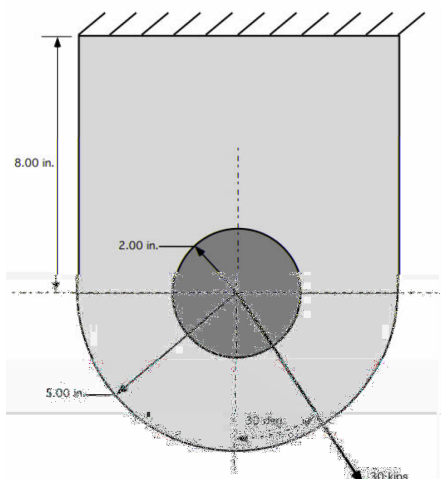
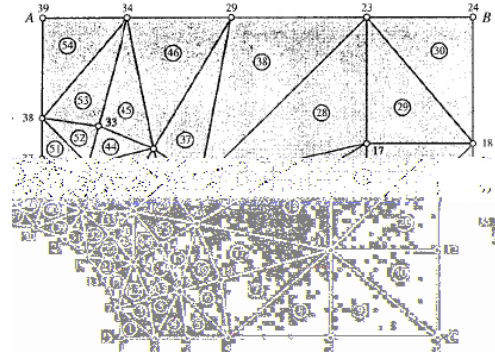
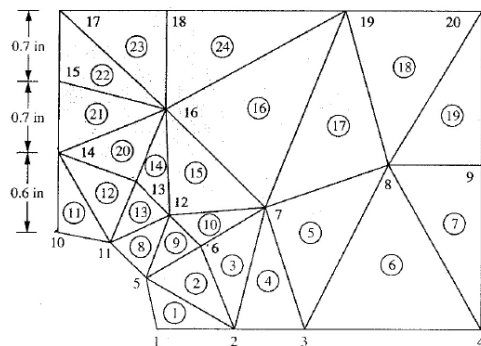
The element is a type of the continuous element is two-dimensional. One of the elements is a triangle, without too much trouble, construct the element is a stress element, so that the element is a type of the element.



Triangle package

Meshing	Triangle	Quadrilateral
2D	✓	△
3D	△	Hex

Tetrahedron





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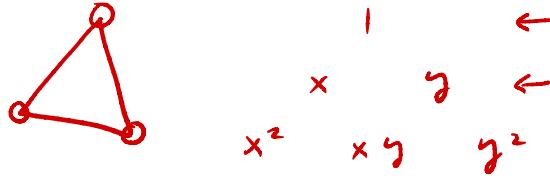
e ts,
e e t

The solution in the triangular element is a linear function of the nodal coordinates x and y :

$$(2) \quad u_x^e(x, y) = \alpha_0^e + \alpha_1^e x + \alpha_2^e y$$

O

$$(2) \quad u_x^e(x, y) = [1 \quad x \quad y] \begin{bmatrix} \alpha_0^e \\ \alpha_1^e \\ \alpha_2^e \end{bmatrix} = \mathbf{p}(x, y) \mathbf{a}^e$$



Noted that

is equal to

the triangular element field is a linear function of the nodal coordinates, so we should be able to uniquely express the

α_i^e in terms of the nodal values.

We will now obtain the expressions for the element field following the procedure we use in the 1D case. There are two ways to do it: the way that is less likely. The way is to derive the shape functions. The shape function is to utilize the Kronecker delta property of the shape functions. We shall do this below.

The Way to Obtain the Shape Functions

Let us first express the

$\alpha_0^e, \alpha_1^e, \alpha_2^e$ in terms of the

:

$$(1) \quad u_{x1}^e(x, y) = \alpha_0^e + \alpha_1^e x_1 + \alpha_2^e y_1$$

$$(2) \quad u_{x2}^e(x, y) = \alpha_0^e + \alpha_1^e x_2 + \alpha_2^e y_2$$

$$(3) \quad u_{x3}^e(x, y) = \alpha_0^e + \alpha_1^e x_3 + \alpha_2^e y_3$$

O

$$(4) \quad \begin{bmatrix} u_{x1}^e \\ u_{x2}^e \\ u_{x3}^e \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} \alpha_0^e \\ \alpha_1^e \\ \alpha_2^e \end{bmatrix} \quad \& \quad \mathbf{d}_x^e = \mathbf{M}^e \mathbf{a}^e$$

$$(\cdot) \quad \mathbf{a}^e = (\mathbf{M}^e)^{-1} \mathbf{d}_x^e$$

$$(\cdot) \quad u_x^e(x, y) = \mathbf{N}^e(x, y) \mathbf{d}_x^e \quad \text{where} \quad \mathbf{N}^e(x, y) = \mathbf{p}(x, y) (\mathbf{M}^e)^{-1}$$

where the row vector $\mathbf{N}^e(x, y) = [N_1^e(x, y) \quad N_2^e(x, y) \quad N_3^e(x, y)]$ is the

To develop these formulations, it is necessary to invert the matrix $\mathbf{M}^e$. This is done by using MATLAB's Symbolic Toolbox, which gives:

$$(\cdot) \quad (\mathbf{M}^e)^{-1} = \frac{1}{2A^e} \begin{bmatrix} x_2^e y_3^e - x_3^e y_2^e & x_3^e y_1^e - x_1^e y_3^e & x_1^e y_2^e - x_2^e y_1^e \\ y_2^e - y_3^e & y_3^e - y_1^e & y_1^e - y_2^e \\ x_3^e - x_2^e & x_2^e - x_1^e & x_1^e - x_3^e \end{bmatrix}$$

where A^e is the area of the element and is equal to half of the determinant of the matrix $\mathbf{M}^e$:

$$(\cdot) \quad 2A^e = \det(\mathbf{M}^e) = (x_2^e y_3^e - x_3^e y_2^e) + (x_3^e y_1^e - x_1^e y_3^e) + (x_1^e y_2^e - x_2^e y_1^e)$$

Thus the formulations are:

$$(\cdot) \quad N_1^e(x, y) = \frac{1}{2A^e} ((x_2^e y_3^e - x_3^e y_2^e) + (y_2^e - y_3^e)x + (x_3^e - x_2^e)y)$$

$$(\cdot) \quad N_2^e(x, y) = \frac{1}{2A^e} ((x_3^e y_1^e - x_1^e y_3^e) + (y_3^e - y_1^e)x + (x_1^e - x_3^e)y)$$

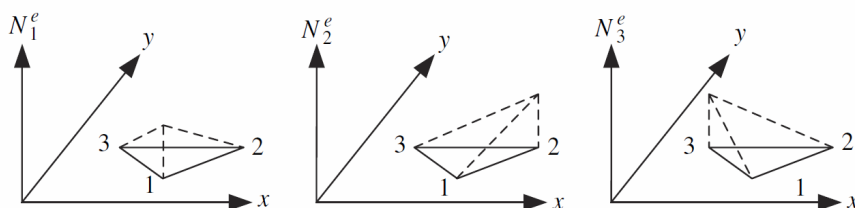
$$(\cdot) \quad N_3^e(x, y) = \frac{1}{2A^e} ((x_1^e y_2^e - x_2^e y_1^e) + (y_1^e - y_2^e)x + (x_2^e - x_1^e)y)$$

cyclic repetition

CST

constant strain triangle

These formulations are very useful to evaluate the stress field. It is seen that the stress field is constant over the element.



I ot e wo s, t ese s e fu tio s v e t e K o e e e l t o e ty:

$$(60) \quad N^e(x_j^e, y_j^e) = \delta_{j0}$$

T e Si le W y to o st u t S e Fu tio s

We use t e K o e e e l t o e ty to e t e i e t e s e fu tio s. Let us fi st o si e t e s e fu tio fo o e 1:

$$N^e x y = \beta^e + \beta^e x + \beta^e y$$

$$\text{We ve 3} \quad : \quad N_1^e(x_1^e, y_1^e) = 1, \quad N_1^e(x_2^e, y_2^e) = N_1^e(x_3^e, y_3^e) = 0$$

$$\Rightarrow \begin{bmatrix} 1 & x_1^e & y_1^e \\ 1 & x_2^e & y_2^e \\ 1 & x_3^e & y_3^e \end{bmatrix} \begin{bmatrix} \beta_0^e \\ \beta_1^e \\ \beta_2^e \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \beta_0^e = \frac{1}{2A^e} a_1, \quad \beta_1^e = \frac{1}{2A^e} b_1, \quad \beta_2^e = \frac{1}{2A^e} c_1$$

$$\text{w e e} \quad a_1 = x_2^e y_3^e - x_3^e y_2^e$$

$$b_1 = y_2^e - y_3^e$$

$$c_1 = x_3^e - x_2^e$$

$$A^e = \text{e of t e t i le} = \frac{1}{2} \begin{vmatrix} 1 & x_1^e & y_1^e \\ 1 & x_2^e & y_2^e \\ 1 & x_3^e & y_3^e \end{vmatrix} = \frac{1}{2} \det(\mathbf{M}^e)$$

Si il ly,

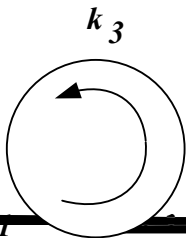
$$N_2^e(x, y) = \frac{1}{2A^e} (a_2 + b_2 x + c_2 y), \quad N_3^e(x, y) = \frac{1}{2A^e} (a_3 + b_3 x + c_3 y)$$

$$\text{w e e} \quad \begin{cases} a_2 = x_3^e y_1^e - x_1^e y_3^e \\ b_2 = y_3^e - y_1^e \\ c_2 = x_1^e - x_3^e \end{cases}, \quad \begin{cases} a_3 = x_1^e y_2^e - x_2^e y_1^e \\ b_3 = y_1^e - y_2^e \\ c_3 = x_2^e - x_1^e \end{cases}$$

Noti e t e y li e etitio e ul ity

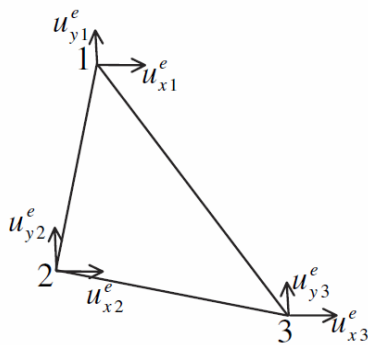
$$N_i^e(x, y) = \frac{1}{2A^e} (a_i + b_i x + c_i y)$$

$$\begin{cases} a_i = x_j^e y_k^e - x_k^e y_j^e \\ b_i = y_j^e - y_k^e \\ c_i = x_k^e - x_j^e \end{cases}$$



3.7.2 E M : 3

We ow follow t ee-ste o e u es es i e eviously to evelo ele e t t i es fo T ele e t.



1. o st u tio of s e fu tio s t i $\mathbf{N}^e$

$$(\text{ . 9}) \quad \mathbf{u}^e(x, y) = \mathbf{N}^e(x, y) \mathbf{d}^e$$

2. Fo ul tio of t e st i - is l e e t t i $\mathbf{B}^e$

$$(\text{ . 4}) \quad \boldsymbol{\varepsilon}^e = \nabla_S \mathbf{u}^e = \nabla_S \mathbf{N}^e \mathbf{d}^e = \mathbf{B}^e \mathbf{d}^e$$

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The element shape functions

If the shape functions vary linearly with the element, we use the shape function to interpolate the field:

$$(\text{.64}) \quad \mathbf{b}(\xi, \eta) = \mathbf{N}^e(\xi, \eta) \mathbf{b}^e$$

where $\mathbf{b}^e$ is the element nodal values:

$$\mathbf{b}^e = \begin{bmatrix} b_{x1}^e \\ b_{y1}^e \\ b_{x2}^e \\ b_{y2}^e \\ b_{x3}^e \\ b_{y3}^e \end{bmatrix}$$

$$(\text{.6}) \quad \mathbf{f}_{\Omega}^e = t \left(\int_{\Omega^e} \mathbf{N}^{eT} \mathbf{N}^e d\Omega \right) \mathbf{b}^e$$

Knowing the element values we follow directly the relation for T :

$$(\text{.66}) \quad \int_{\Omega^e} (N_1^e)^a (N_2^e)^b (N_3^e)^c d\Omega = \frac{a! b! c!}{(a+b+c+2)!} 2A^e$$

The element nodal values thus yield:

$$(\text{.67}) \quad \mathbf{f}_{\Omega}^e = \frac{tA^e}{12} \begin{bmatrix} 2b_{x1} + b_{x2} + b_{x3} \\ 2b_{y1} + b_{y2} + b_{y3} \\ b_{x1} + 2b_{x2} + b_{x3} \\ b_{y1} + 2b_{y2} + b_{y3} \\ b_{x1} + b_{x2} + 2b_{x3} \\ b_{y1} + b_{y2} + 2b_{y3} \end{bmatrix}$$

R: for these elements $b_{x1} = b_{x2} = b_{x3} = \bar{b}_x$ and $b_{y1} = b_{y2} = b_{y3} = \bar{b}_y$, (.67) reduces to

(.6).