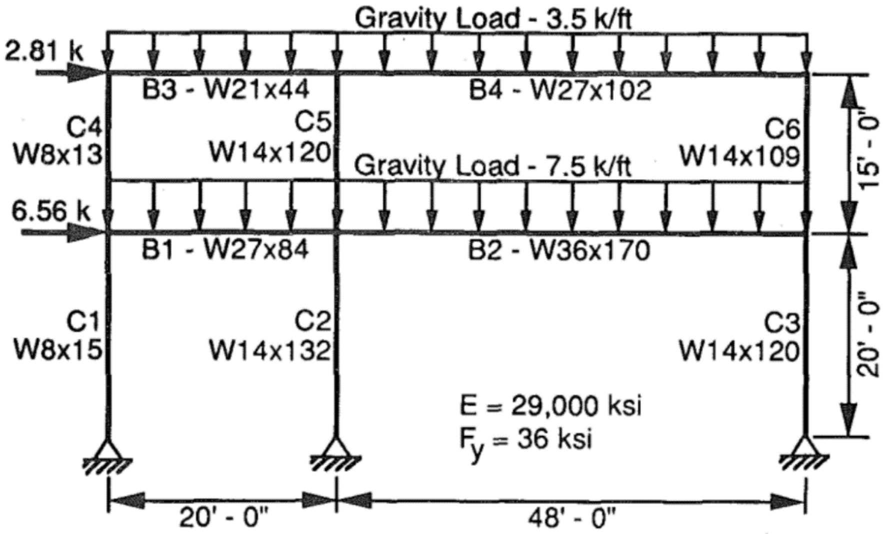
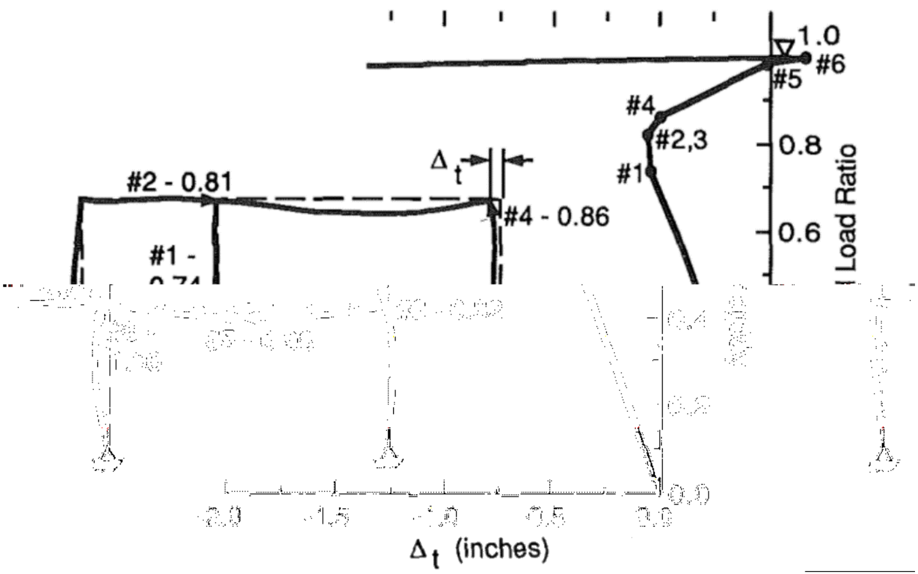


FIG. 5. Loads and Dimensions for Frame U-P36H



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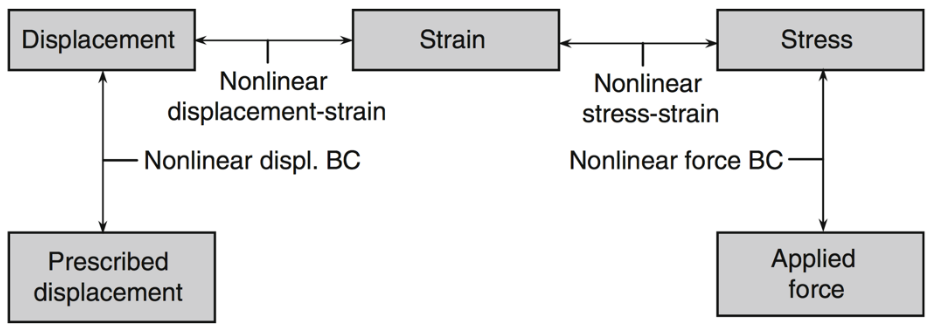
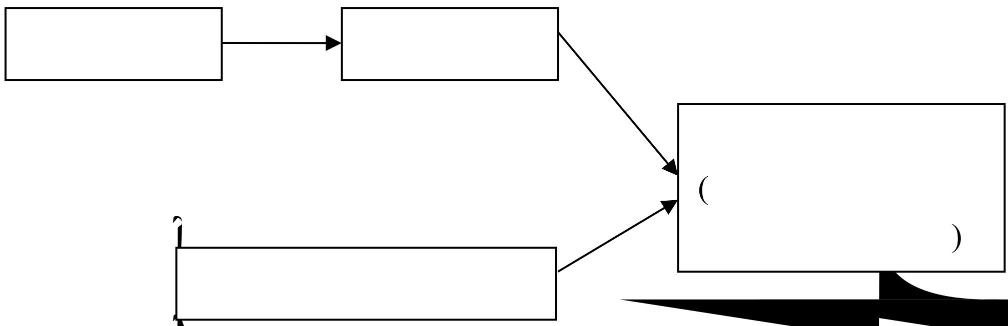




, (.(.) (.))

(.) $\mathbf{w}^T(\mathbf{K}\mathbf{d} -$) = 0

K_F is $f(d_F)$

(.) $\mathbf{K}_F\mathbf{d}_F = \mathbf{f}_F - \mathbf{K}_{EF}^T\bar{\mathbf{d}}_E \quad \Rightarrow \quad \mathbf{d}_F = (\mathbf{K}_F)^{-1}(\mathbf{f}_F - \mathbf{K}_{EF}^T\bar{\mathbf{d}}_E)$

) **K** **f**

d. **cannot** **d** **K**

f . **d** **K**

f $\mathbf{K}_F\mathbf{d}_F$ $\mathbf{f}_F - \mathbf{K}_{EF}\bar{\mathbf{d}}_E$.

nonlinear algebraic equations.

,

. .

, - .

, - .

5.3 Solution Methods for Nonlinear Algebraic Equations

- - (-

) $\mathbf{K}_F \mathbf{d}_F = \mathbf{f}_F$ ($\bar{\mathbf{d}}_E = \mathbf{0}$) $\mathbf{d}_F$

$\mathbf{K}_F$ $\mathbf{d}_F$.

.

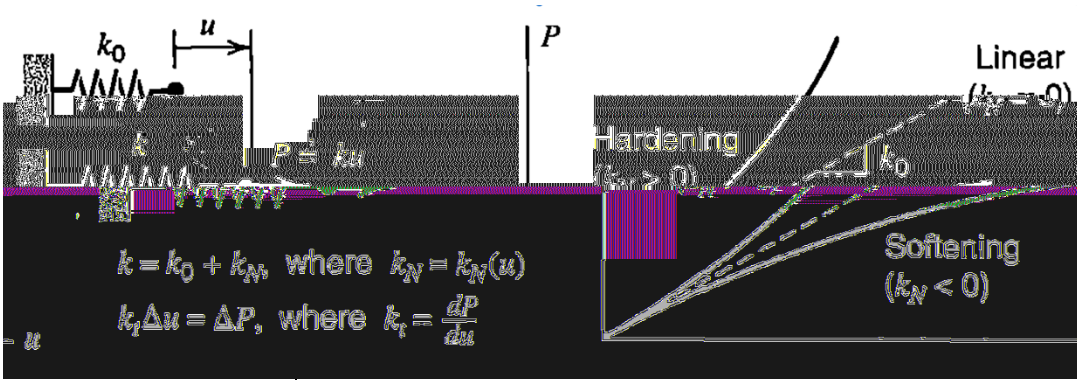
- ,

,

(.1) $g(u,x)=0$ $u=u(x)$

u .

.



P u

(.) $ku = P$ ($+k_N)u = P$ $k_N = k_N(u)$

- . . . - . . .

$\mathbf{K}_F \mathbf{d}_F = \mathbf{f}_F$, k, ku,

u, u P . u

, .

tangent stiffness,

(.) $k_t = \frac{dP}{du} = k + \frac{k_N}{u} u$

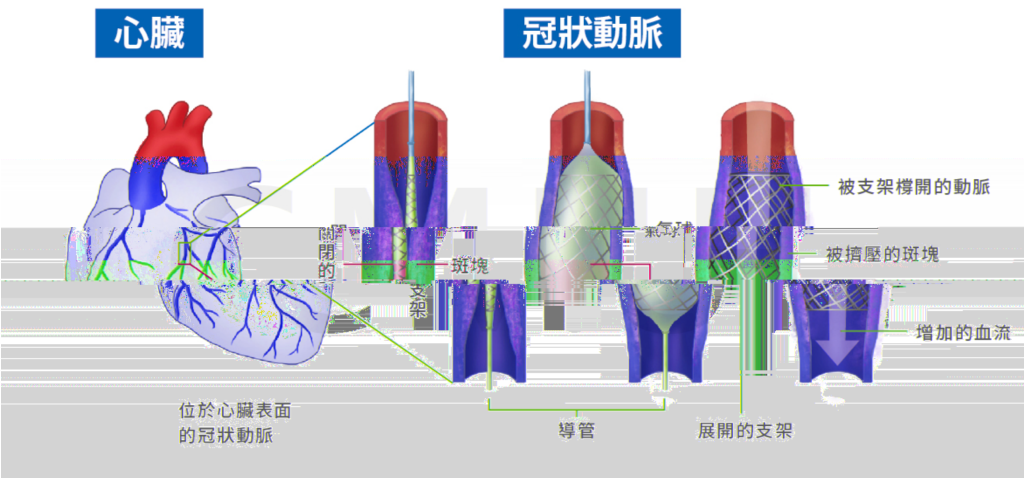
____ P u .

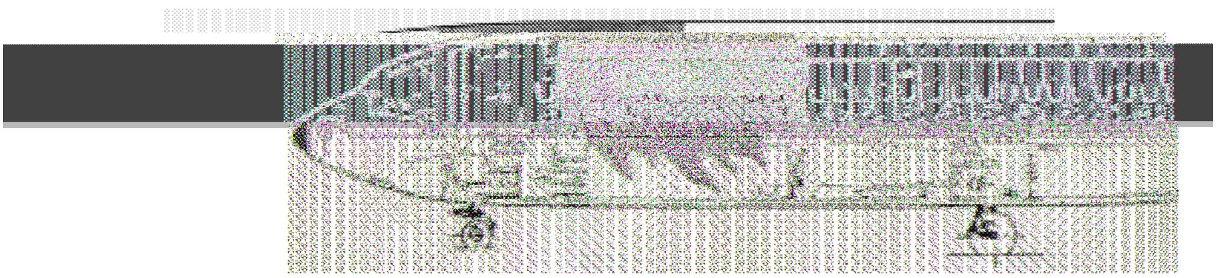
Remarks

1. **tangent stiffness** k_t **NOT** k (, **tangent stiffness matrix** $\mathbf{K}_T$ $\mathbf{K}_F$)

. - , u, P, u. , $\mathbf{K}_F$ $\mathbf{f}_F$ $\mathbf{d}_F$.

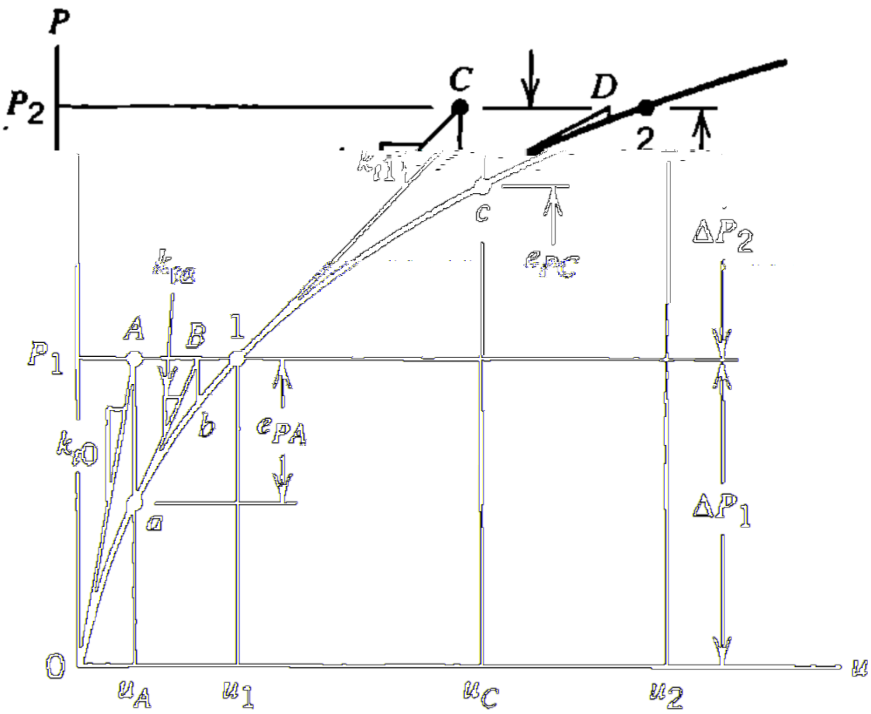
(. ., ________________ , ,).





pa gure 1.2: 1988 Aloha Airlines accident–the shaded area illustrates the fuselage lost at cruising altitude.

5.3.1 Newton-Raphson Method for Single DOF



, , P

u , .

. (.), $u = 0$. P_1

$_1$.

k_{t0} ,

$\Delta P = P_1$.

(.) $k_t \, \Delta u = \Delta P$ $\Delta u = \left(k_{t0}\right)^{-1} \Delta P_1$ $u_A = 0 + \Delta u$

u_A u_1 .

P_1 .

Q: ( ) e_{PA}

A:

(.)

Finite Element Method

0 /0 / 01

Remark: ku_A

______________________, .

equilibrium iterations

( ). P_1

, ***a*** **tangent**

a. , u_B

$$(\ .\ )\quad k_{ta}\Delta u = e_{PA} \qquad \Delta u = \left(k_{ta}\right)^{-1} e_{PA} \qquad u_B = u_A + \Delta u$$

P_1.

Q: ( ) e_{PB}

A:

(.)

b

Δu $u_B + \Delta u$.

,

, Δu ,

u_1.

ΔP_2 ,

u_2 P_2.

. (.) (.)

$$(\ .\ )\quad k_{t1}\Delta u = \Delta P_2 \qquad \Delta u = \left(k_{t1}\right)^{-1} \Delta P_2 \qquad u_C = u_1 + \Delta u$$

$$(\ .\ )\quad e_{PC} = P_2 - ku_C \qquad k = k(u) \qquad u_C.$$

, , ,

1, , , ...

P u . ____

________________________________.

Finite Element Method

0 /0 / 01

Remark: -

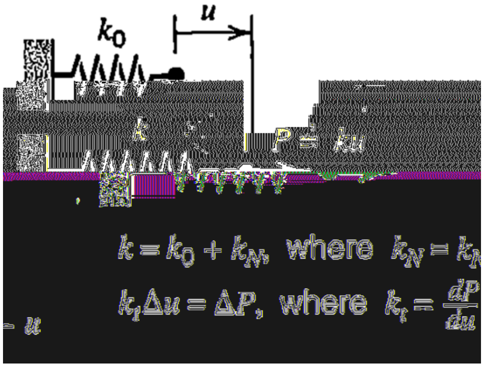
prediction (. ., . (.)) *the tangent stiffness*

. **correction** (. ., . (.))

.

-

$$k_0 = 50, \quad k_N = \ \ 5u, \qquad P = \ \ .$$



(Solution)

$$k = k_0 + k_N = 50 - 5u$$

$$P = ku = (50 - 5u)u$$

Q:

A:

$$P_1 = P = 100$$

1

Prediction (. (.))

$$k_{t0} = 50$$

$$\Delta \ = \qquad \Delta u = \frac{1}{50}100 \qquad u_A = 0 + \Delta u = 2.0$$

Correction (. (.))

$$k = 50 - 5u_A = 40$$

$$e_{PA} = P_1 - ku_A = 20$$

Prediction (. (.))

$$k_{ta} = 50 - 10u_A = 30$$

$$k_{ta}\Delta u = e_{PA} \qquad \Delta u = \frac{1}{30}20 = 0.6667 \qquad u_B = u_A + \Delta u = 2.6667$$

Correction (. (.))

$$k = 50 \quad 5u_B = 36.6667$$

$$e_{PB} = P_1 - ku_B = 2.2222$$

Q

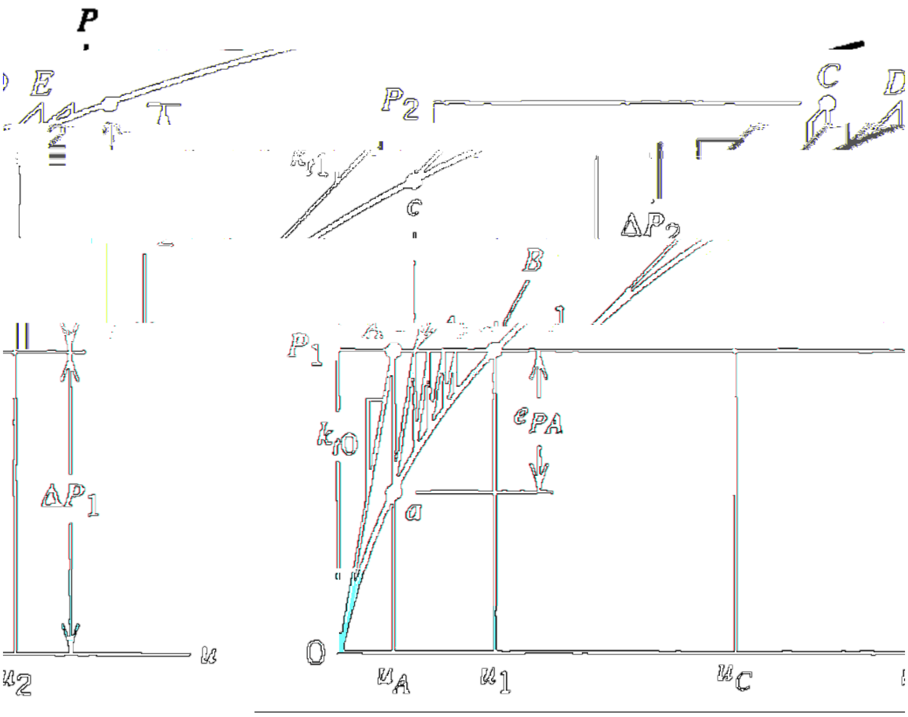
A

```
%
% a single nonlinear spring (Newton-Raphson method)
%
u = 0;
P = 100;
e = P;
conv = e ^ 2 / P ^ 2;
tol = 1.0e-12;
iter = 0;
while conv > tol && iter < 20
    iter = iter + 1;
    uold = u;
    kt = 50 - 10*uold;
    delta_u = kt\e;
    u = uold + delta_u;
    k = 50 - 5*u;
    e = P - k*u;
    conv = e ^ 2 / P ^ 2;
    fprintf('%3d %10.5f %7.5e \n',iter, e, conv);
end
```

```
  1   20.00000 4.00000e-02
  2    2.22222 4.93827e-04
  3    0.04535 2.05676e-07
  4    0.00002 4.21494e-14
```

$$\frac{\|e_P\|}{\|P\|} \qquad 1 \times 10^{-12}.$$

6.3.2 Modified Newton-Raphson Method for Single DOF



- -

k_t

Δu , .

, k_{t0}

P_1 k_{t1} k_{t1} P_2 .

```
%
% a single nonlinear spring (modified Newton-Raphson method)
%
u = 0;
P = 100;
e = P;
conv = e ^ 2 / P ^ 2;
tol = 1.0e-12;
iter = 0;
while conv > tol && iter < 30
    iter = iter + 1;
    uold = u;
    kt = 50;
    delta_u = kt\e;
    u = uold + delta_u;
    k = 50 - 5*u;
    e = P - k*u;
```

```
    conv = e ^ 2 / P ^ 2;
    fprintf('%3d %10.5f %7.5e \n',iter, e, conv);
end
```

```
  1   20.00000 4.00000e-02
  2    8.80000 7.74400e-03
  3    4.37888 1.91746e-03
  4    2.29435 5.26403e-04
  5    1.23276 1.51970e-04
  6    0.67106 4.50328e-05
  7    0.36786 1.35317e-05
  8    0.20241 4.09698e-06
  9    0.11161 1.24558e-06
 10    0.06161 3.79552e-07
 11    0.03403 1.15801e-07
 12    0.01880 3.53555e-08
 13    0.01039 1.07986e-08
 14    0.00574 3.29889e-09
 15    0.00317 1.00791e-09
 16    0.00175 3.07964e-10
 17    0.00097 9.41013e-11
 18    0.00054 2.87541e-11
 19    0.00030 8.78635e-12
 20    0.00016 2.68485[illegible]-12
 21    0.0000[illegible]e-13
```

1 $\dfrac{\|e_P\|}{\|P\|}$ 1×10^{-12}.

Remark - . - - , - , , .

5.3.3 Newton-Raphson Method for Multiple DOFs

. , **u**

$$(.10) \quad \mathbf{K}(\mathbf{u})\mathbf{u} = \mathbf{f} \quad \Rightarrow \quad \Psi(\mathbf{u}) = \mathbf{f}$$

. (.), $\mathbf{K}(\mathbf{u}) = \mathbf{K}_F,\ \mathbf{u} = \mathbf{d}_F,\ \mathbf{f} = \mathbf{f}_F,\ \bar{\mathbf{d}}_E = \mathbf{0}$.

i $\mathbf{u}^i$.

-

(.10) $\Psi(\mathbf{u}^{i+1}) \approx \Psi(\mathbf{u}^i) + \mathbf{K}_T^i(\mathbf{u}^i)\cdot\Delta\mathbf{u}^i$

i

(.11) $\mathbf{K}_T^i(\mathbf{u}^i) \equiv \left(\dfrac{\partial\Psi}{\partial\mathbf{u}}\right)^i$

$\Delta\mathbf{u}^i$ i 1. . (.10)

(.10),

(.1) $\mathbf{K}_T^i\Delta\mathbf{u}^i = \mathbf{f} - \Psi(\mathbf{u}^i)$

Remark ( ) ,

(1) , $\mathbf{K}_T^i$(^{i}), , $\mathbf{u}^i$

( ) , $\Delta\mathbf{u}^i$, , $\mathbf{u}$

( ) - ,

. .

, $\Delta\mathbf{u}^i$,

(.1) $\mathbf{u}^{i+1} = \mathbf{u}^i + \Delta\mathbf{u}^i$

,

(.1) $\mathbf{R}^{i+1} = \mathbf{f} - \Psi(\mathbf{u}^{i+1})$

, , $\mathbf{u}^{i+1}$,

, . ,

.

$\mathbf{K}(\mathbf{u})\mathbf{u} = \mathbf{f}$

$$\mathbf{K}_T = \left(\frac{\partial\Psi}{\partial\mathbf{u}}\right) = \left(\frac{\partial(\mathbf{K}\mathbf{u})}{\partial\mathbf{u}}\right) = \mathbf{K} + \frac{\partial\mathbf{K}}{\partial\mathbf{u}}\mathbf{u}$$

Finite Element Method

0 /0 / 01

K **u**

. ,

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\Psi_1(u_1, u_2) = K_{11}u_1 + K_{12}u_2$$
$$\Psi_2(u_1, u_2) = K_{21}u_1 + K_{22}u_2$$

Q . (**hint** $\mathbf{K}_T = \left(\frac{\partial \Psi}{\partial \mathbf{u}} \right) = \left(\frac{\partial (\mathbf{Ku})}{\partial \mathbf{u}} \right) = \mathbf{K} + \frac{\partial \mathbf{K}}{\partial \mathbf{u}} \mathbf{u}$)

A

Finite Element Method

0 /0 / 01

n ,

$$\mathbf{K}_T = \mathbf{K} + \frac{\partial \mathbf{K}}{\partial \mathbf{u}}\mathbf{u} \quad \Rightarrow \left(K_T\right)_{ij} = K_{ij} + \sum_{m=1}^{n} \frac{\partial K_{im}}{\partial u_j} u_m$$

Remarks:

1. ( )

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