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" F "

National Taiwan University	Finite Element Method
F " "E "G """"""""""""""""""""""""	"E0 0E G
." "35."423	
" "D " " ."Time: 90 minutes	
" "G " " 試卷隨答案卷繳回	

Total 100%

1. 57 "C " " " " " " " " " " " "
"

$$\frac{d^2u}{dx^2} + u = 4x, 0 < x < 1$$

" " " $u(0) = 0, u(1) = 10$

(a) 37 "F " " " " " " " " 0
(b) 42 " " " " " " " " " " " " " "
" " " "I " 0

$$\frac{d^2u}{dx^2} + u - 4x = 0$$

$$\int w\frac{d^2u}{dx^2}dx + \int wudx - 4\int wxdx = 0$$

$$\int w\frac{d}{dx}(\frac{du}{dx})dx + \int wudx - 4\int wxdx = 0$$

$$w\frac{du}{dx}\bigg|_{x=0}^{x=1} - \int_0^1 \frac{dw}{dx}(\frac{du}{dx})dx + \int_0^1 wudx - 4\int_0^1 wxdx = 0$$ 9

Find $u(x)$ among the smooth functions that satisfy $u(0)=2,\ u(1)=1$ such that

$$\int_0^1 \frac{dw}{dx}(\frac{du}{dx})dx = \int_0^1 wudx - 4\int_0^1 wxdx = 0 \qquad \forall w \quad w(0)=0\ , w(1)=0$$

8 4

7 7

let $u=\alpha_2 x^2+\alpha_1 x+\alpha_0$ $\quad w=\beta_2 x^2+\beta_1 x+\beta_0$

$u(0)=0 \Rightarrow \alpha_0=0$ $\quad w(0)=0 \Rightarrow \beta_0=0$

$u(1)=1 \Rightarrow \alpha_1=1-\alpha_2$ $\quad w(1)=0 \Rightarrow \beta_2=-\beta_1$

$$\int_0^1 (2\beta_2\,x+\beta_1)(2\alpha_2\,x+\alpha_1)dx=\int_0^1(\alpha_2x^2+\alpha_1x+\alpha_0)(\beta_2x^2+\beta_1x+\beta_0)dx-4\int_0^1(\beta_2x^2+\beta_1x+\beta_0)xdx=0$$

$$\int_0^1 (2\beta_2\,x-\beta_2)(2\alpha_2\,x+1-\alpha_2)dx=\int_0^1[\alpha_2x^2+(1-\alpha_2)x][\beta_2x^2-\beta_2x]dx-4\int_0^1(\beta_2x^2-\beta_2x)xdx=0 \quad 7$$

$$\frac{\alpha_2}{3}-\frac{2\alpha_2+15}{60}=0 \qquad \Rightarrow \alpha_2=\frac{5}{6},\alpha_1=\frac{1}{6} \qquad \Rightarrow u(x)=\frac{5}{6}x^2+\frac{1}{6}x \quad 7$$

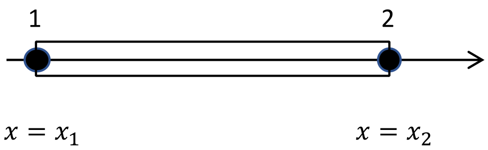
2. 57 "C " " " " " " " " " " " "
"

$$x\frac{d^2u}{dx^2}+\frac{du}{dx}-4x=0, 1<x<2$$

" " "$u(1)=u(2)=0$

(a) 37 "F " " " " " " " " 0

(b) 42 "H " " " " " " " " " " " " " "
" " " H "3 " " "



H "3 'C" " " "

$$x\frac{d^2u}{dx^2}+\frac{du}{dx}-4x=0 \qquad 1<x<2$$

$$\frac{d}{dx}(x\frac{du}{dx})-4x=0$$

$$\int^2 w\frac{d}{dx}(x\frac{du}{dx})-4\int_1^2 wxdx=0$$

$dx=0$ $\quad w(x\frac{du}{dx})\Big|_{x=1}^{x=2}-\int_1^2\frac{dw}{dx}(x\frac{du}{dx})-4\int_1^2 wx$ 9
"

Find $u(x)$ among the smooth functions that satisfy $u(1)=u(2)=0$ such that

$$\int_1^2\frac{dw}{dx}(x\frac{du}{dx})=-4\int_1^2 wxdx \qquad \forall w \quad w(1)=0\ ,w(2)=0$$ "

8 4

$$\frac{d}{dx}(EA\frac{du}{dx})+\mathrm{b(x)}=0 \qquad where \quad b(\mathrm{x})=-4\mathrm{x} \qquad EA=\mathrm{x}$$

shape → $$u^e(\mathrm{x})=\frac{1}{x_2-x_1}\begin{bmatrix}x_2-x & x-x_1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}$$ 4

→ $$B=\frac{1}{x_2-x_1}\begin{bmatrix}-1 & 1\end{bmatrix}$$ 4

→ $$\mathbf{K}=\int_{x_1}^{x_2}\mathbf{B}^{\mathrm{T}}\mathbf{A}E\mathbf{B}dx \qquad \mathbf{f}_{\mathrm{b}}^{\mathrm{e}}=\int_{x_1}^{x_2}\mathbf{N}^{\mathrm{T}}b(\mathrm{x})dx$$

$$\begin{bmatrix}x\\x_1\end{bmatrix}(-4x)dx \quad :: \quad \frac{1}{(x_2-x_1)^2}\int_{x_1}^{x_2}\begin{bmatrix}-1\\1\end{bmatrix}x\begin{bmatrix}-1 & 1\end{bmatrix}dx \quad = \int_{x_1}^{x_2}\frac{1}{x_2-x_1}\begin{bmatrix}x_2-\\x-\end{bmatrix}$$

$$\begin{bmatrix}+x_2\\2x_2\end{bmatrix} \quad = \frac{x_2+x_1}{2(x_2-x_1)}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} \quad = \frac{2(\mathrm{x}_1-\mathrm{x}_2)}{3}\begin{bmatrix}2x_1\\x_1+\end{bmatrix}$$

3. 52 " " " " " " " " C CD" A'

" 7

```
disp(size(rand(5)))
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> disp(size(rand(5)))
     5     5

fx >>
```

" 32

```
A1 = [9.9, 9900];
A2 = [8.8,  7.7 ; ...
      8800, 7700];
formatSpec = 'X is %4.2f meters or %8.3f mm\n';
fprintf(formatSpec,A1,A2)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
X is 9.90 meters or 9900.000 mm
X is 8.80 meters or 8800.000 mm
X is 7.70 meters or 7700.000 mm
fx >>
```

7 7

" 37

```
A = [1 2 3 4; 5 6 7 8; 9 10 11 12; 13 14 15 16];
disp(A(:,2:3))
disp(A(:,:))
disp(A(:))
```