

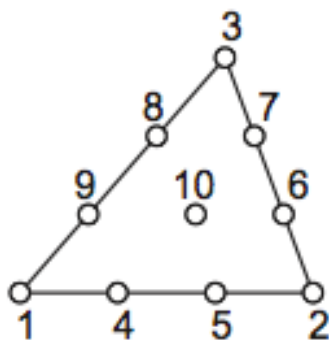
Homework 11, 06/05/2019 Due: 06/12/2019

**A4 professional format and you should document your codes, collecting at the BEGINNING of class (09:09 am)**

**(late submission within 24 hours: score\*0.9; late submission before post of solution: score\*0.8 (the solution will be posted usually within a week))**

**Total 80% + 40% (Bonus)**

1. (40%) Consider a cubic triangular element shown below. Find the shape functions in the triangular coordinates for the nodes 3, 8, 9, 10 and plot these shape functions in MATLAB.



➤ Sol :

Shape functions (20%)

$$N_3 = \frac{9}{2} \xi_3 \left( \xi_3 - \frac{2}{3} \right) \left( \xi_3 - \frac{1}{3} \right)$$

$$N_8 = \frac{27}{2} \xi_1 \xi_3 \left( \xi_3 - \frac{1}{3} \right)$$

$$N_9 = \frac{27}{2} \xi_1 \xi_3 \left( \xi_1 - \frac{1}{3} \right)$$

$$N_{10} = 27 \xi_1 \xi_2 \xi_3$$

MATLAB Code: (4%)

```
%HW11 problem 1 (T10)
```

```
clc; clear;
```

```
close all;
```

```
nodeCoordinate = [0 0;3 0;1.8 3;1 0;2 0;2.6 1;2.2 2;1.2 2;0.6 1;1.6 1];
```

```
node = {'1','2','3','4','5','6','7','8','9','10'};
```

```

xi1 = zeros(101);
xi2 = zeros(101);
j=1;
for i=0:0.01:1
    xi1(j,:) = linspace(0,1-i,101);
    xi2(j,:) = i.*ones(1,101);
    j=j+1;
end
xi3=1-xi2-xi1;
%shpae Function
N1=(9/2).*xi1.*(xi1-2/3).*(xi1-1/3);
N2=(9/2).*xi2.*(xi2-2/3).*(xi2-1/3);
N3=(9/2).*xi3.*(xi3-2/3).*(xi3-1/3);
N4=(27/2).*xi1.*xi2.*(xi1-1/3);
N5=(27/2).*xi1.*xi2.*(xi2-1/3);
N6=(27/2).*xi2.*xi3.*(xi2-1/3);
N7=(27/2).*xi2.*xi3.*(xi3-1/3);
N8=(27/2).*xi1.*xi3.*(xi3-1/3);
N9=(27/2).*xi1.*xi3.*(xi1-1/3);
N10=27.*xi1.*xi2.*xi3;
x =
N1.*nodeCoordinate(1,1)+N2.*nodeCoordinate(2,1)+N3.*nodeCoordinate(3,1)+N4.*nodeC
oordinate(4,1)+N5.*nodeCoordinate(5,1)+N6.*nodeCoordinate(6,1)+N7.*nodeCoordinate(
7,1)+N8.*nodeCoordinate(8,1)+N9.*nodeCoordinate(9,1)+N10.*nodeCoordinate(10,1);
y =
N1.*nodeCoordinate(1,2)+N2.*nodeCoordinate(2,2)+N3.*nodeCoordinate(3,2)+N4.*nodeC
oordinate(4,2)+N5.*nodeCoordinate(5,2)+N6.*nodeCoordinate(6,2)+N7.*nodeCoordinate(
7,2)+N8.*nodeCoordinate(8,2)+N9.*nodeCoordinate(9,2)+N10.*nodeCoordinate(10,2);

%% 3D PLOT
% N3
figure()
plot3([0 3 1.8 0],[0 0 3 0],[0 0 0 0],'r-');
axis([0 4 0 4 -0.5 1]);

```

```

hold on; grid on;
title('N3'); xlabel('x'); ylabel('y')
mesh(x,y,N3);
for k=1:10
    text(nodeCoordinate(k,1),nodeCoordinate(k,2)+0.3,-0.2,node{k},'color','r');
end

```

```
% N8
```

```

figure()
plot3([0 3 1.8 0],[0 0 3 0],[0 0 0 0],'r-');
axis([0 4 0 4 -0.5 1]);
hold on; grid on;
title('N8'); xlabel('x'); ylabel('y')
mesh(x,y,N8);
for k=1:10
    text(nodeCoordinate(k,1),nodeCoordinate(k,2)+0.3,-0.2,node{k},'color','r');
end

```

```
% N9
```

```

figure()
plot3([0 3 1.8 0],[0 0 3 0],[0 0 0 0],'r-');
axis([0 4 0 4 -0.5 1]);
hold on; grid on;
title('N9'); xlabel('x'); ylabel('y')
mesh(x,y,N9);
for k=1:10
    text(nodeCoordinate(k,1),nodeCoordinate(k,2)+0.3,-0.2,node{k},'color','r');
end

```

```
% N10
```

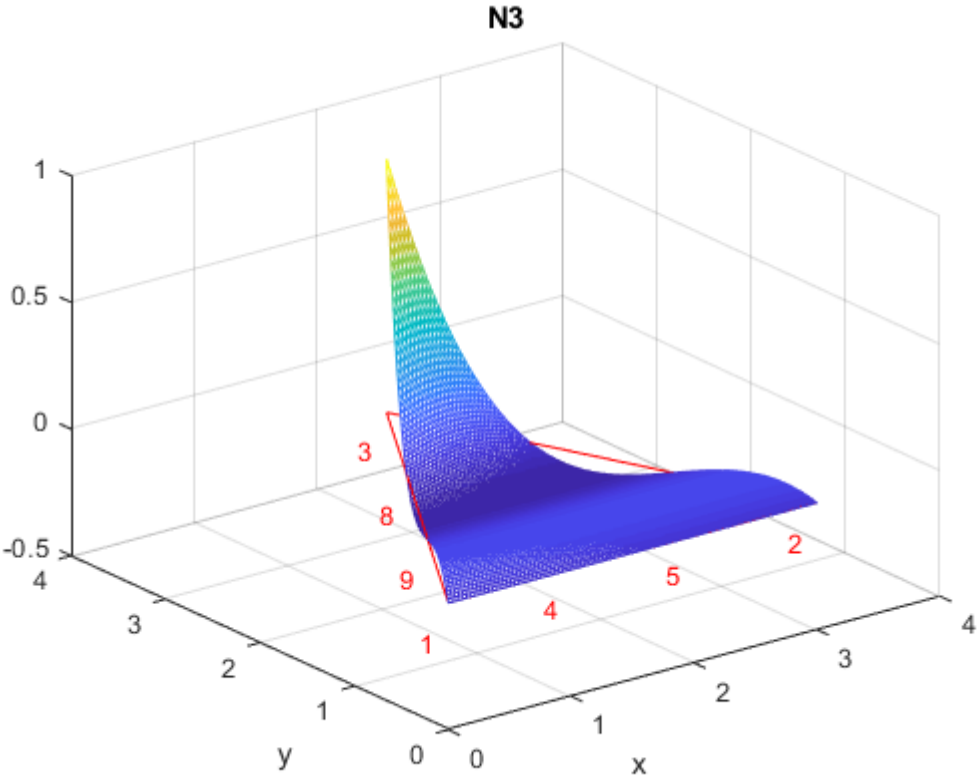
```

figure()
plot3([0 3 1.8 0],[0 0 3 0],[0 0 0 0],'r-');
axis([0 4 0 4 -0.5 1]);
hold on; grid on;

```

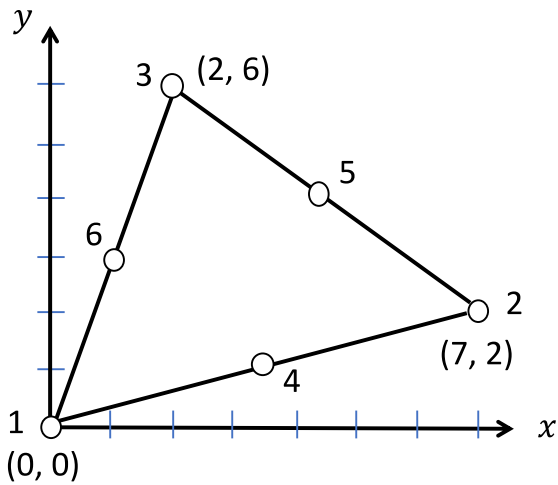
```
title('N10'); xlabel('x'); ylabel('y')
mesh(x,y,N10);
for k=1:10
    text(nodeCoordinate(k,1),nodeCoordinate(k,2)+0.3,-0.2,node{k},'color','r');
end
```

Figures (4%/each figure)





2. (40%) Consider the quadratic triangle element shown below. Calculate the Jacobian matrix using T3 shape functions (subparametric formulation) and calculate  $\frac{\partial N_4}{\partial x}$  and  $\frac{\partial N_4}{\partial y}$  at the point (2, 4).



➤ Sol :

T3 shape functions are as below:

(20%)

$$N_1 = \frac{1}{19}x^2 + \frac{2}{19}y^2 + \frac{2}{19}xy$$

$$N_2 = \frac{1}{19}x^2 + \frac{2}{19}y^2 + \frac{2}{19}xy$$

$$N_3 = \frac{1}{19}x^2 + \frac{2}{19}y^2 + \frac{2}{19}xy$$

$$x = \sum_{i=1}^3 N_i x_i = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix} = 7x_2 + 2x_3$$

$$y = \sum_{i=1}^3 N_i y_i = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 6 \end{bmatrix} = 2y_2 + 6y_3$$

$$\Rightarrow \begin{cases} N_2(x,y) = \frac{3x-y}{19} \\ N_3(x,y) = \frac{-2x+7y}{38} \end{cases}$$

$$J^e = \begin{bmatrix} \frac{\partial x}{\partial x_2} & \frac{\partial y}{\partial x_2} \\ \frac{\partial x}{\partial x_3} & \frac{\partial y}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 2 & 6 \end{bmatrix}$$

For T6:

(20%)

$$N_4 = 4 \left( 1 - \frac{x}{7} - \frac{y}{2} \right)^2$$

$$\begin{bmatrix} \frac{\partial N_4}{\partial x} \\ \frac{\partial N_4}{\partial y} \end{bmatrix} = (J^e)^{-1} \begin{bmatrix} \frac{\partial N_4}{\partial x_2} \\ \frac{\partial N_4}{\partial x_3} \end{bmatrix} = \frac{1}{38} \begin{bmatrix} 6 & -2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 4 - 8 \frac{x}{7} - 4 \frac{y}{2} \\ 3 \end{bmatrix} = \begin{bmatrix} 44 \\ 361 \\ -40 \\ 361 \end{bmatrix}$$

$$\left( \because \frac{\partial N_4}{\partial x_2}(2,4) = \frac{2}{19}, \quad \frac{\partial N_4}{\partial x_3}(2,4) = \frac{12}{19} \right)$$

3. **(Bonus 40%)** Consider the quadratic triangle element shown in Problem 2. Calculate the Jacobian matrix using a T6 (isoparametric formulation) and calculate  $\frac{\partial N_1}{\partial x}$ ,  $\frac{\partial N_1}{\partial y}$ ,  $\frac{\partial N_4}{\partial x}$  and  $\frac{\partial N_4}{\partial y}$  at the point (2, 4).

➤ Sol :

T6 shape functions