National Taiwan University **Finite Element Method**

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Homework 9, 05/16/2019 Due: 05/22/2019

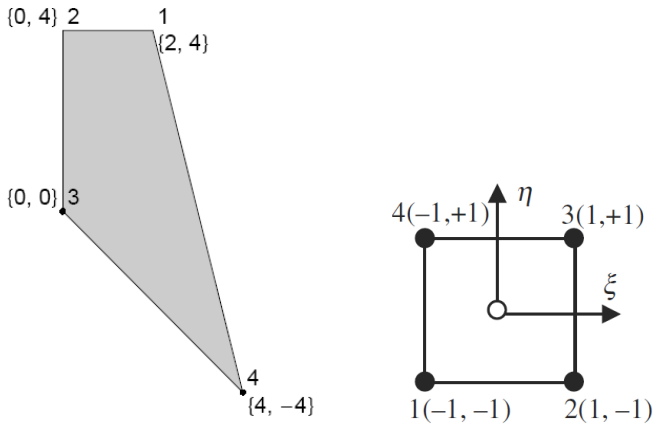
A4 professional format, collecting at the BEGINNING of class (09:10 am)

(late submission within 24 hours: score*0.9; late submission before post of solution: score*0.8

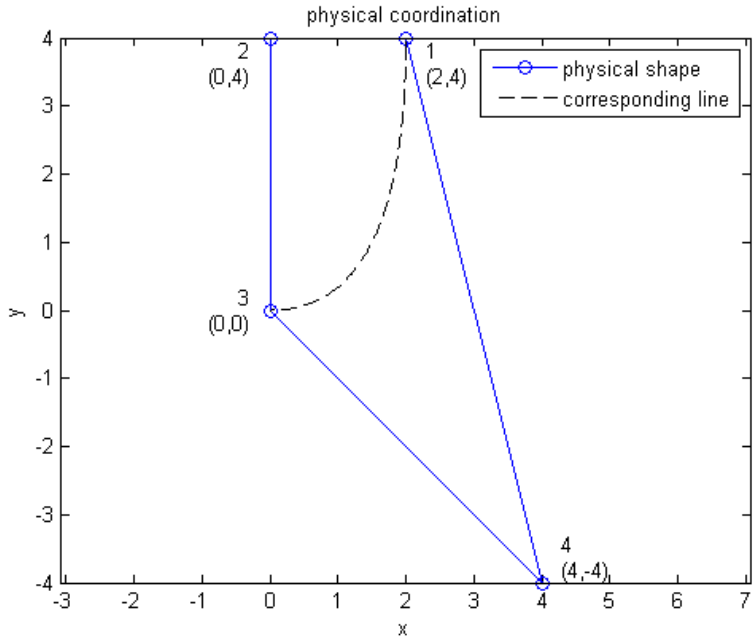
(the solution will be posted usually within a week))

Total 70%

1. **(40%)** A four node quadrilateral element is shown in the figure below (you should maintain the nodal ordering):



Take the diagonal of the element in the parent coordinate where $\xi = \eta$, and find and draw the corresponding curved line in the physical coordinate. Below is a sample plot:



(Sol):

$$N^{Q4}(\xi,\eta)=\begin{bmatrix} N_1^{Q4} & N_2^{Q4} & N_3^{Q4} & N_4^{Q4}\end{bmatrix}=\frac{1}{4}\begin{bmatrix}(1-\xi)(1-\eta) & (1+\xi)(1-\eta) & (1+\xi)(1+\eta) & (1-\xi)(1+\eta)\end{bmatrix}$$

$$x(\xi,\eta)=\begin{bmatrix} N_1^{Q4} & N_2^{Q4} & N_3^{Q4} & N_4^{Q4}\end{bmatrix}\begin{bmatrix} x_1^e \\ x_2^e \\ x_3^e \\ x_4^e\end{bmatrix}=\frac{1}{2}(1-\xi)(3+\eta)$$

$$\mathrm{y}(\xi,\eta)=\begin{bmatrix} N_1^{Q4} & N_2^{Q4} & N_3^{Q4} & N_4^{Q4}\end{bmatrix}\begin{bmatrix} y_1^e \\ y_2^e \\ y_3^e \\ y_4^e\end{bmatrix}=\xi\eta+\xi-3\eta+1$$

For $\xi=\eta$

$$x=\frac{1}{2}(1-\xi)(3+\xi)$$

$$\mathrm{y}=(1-\xi)^2$$

(15%)

(1) Draw $(x,y)=(\frac{1}{2}(1-\xi)(3+\xi),(1-\xi)^2)\quad -1\le\xi\le 1$

(2) $\xi=1-\sqrt{y},x=\frac{1}{2}\sqrt{y}(4-\sqrt{y})$

$4x^2+4xy+y^2-16y\quad 0,\quad 0\le y\le 4,0\le x\le 2$

Choose one (10%)

Figure (10%)
Code (5%)

2. (30%) Consult the lecture note (pp. 16, 05/17/2019) and compute (1, 1) and (1, 2) components of $\mathbf{K}^e(\xi_1,\eta_1)$.

(Sol):

$$K^e(\xi_1,\eta_1) = w_1 w_2 B^{e^T}(\xi_1,\eta_1) D^e B^e(\xi_1,\eta_1) \left| J^e(\xi_1,\eta_1) \right|$$

$$(\xi_1,\eta_1) = (-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$$

$$w_1 = 1$$

$$D^e = \frac{3\times10^7}{0.91}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$$

$$J^e = \begin{bmatrix} 0 & -0.4472 \\ 1 & 0.0528 \end{bmatrix}$$

$$B^e = \begin{bmatrix} -0.4409 & 0 & -0.0591 & 0 & 0.1181 & 0 & 0.3819 & 0 \\ 0 & 0.8819 & 0 & -0.8819 & 0 & -0.2363 & 0 & 0.2363 \\ 0.8819 & -0.4409 & -0.8819 & -0.0591 & -0.2363 & 0.1181 & 0.2363 & 0.3819 \end{bmatrix}$$

$$K^e(1,1) = 0.6879\times10^7(\frac{N}{m})$$ (15%)

$$K^e(1,2) = -0.3726\times10^7(\frac{N}{m})$$ (15%)