National Taiwan University
Department of Civil Engineering

Finite Element Method
Instructor: C.-S. CHEN

Homework 8, 05/09/2019   Due: 05/15/2019

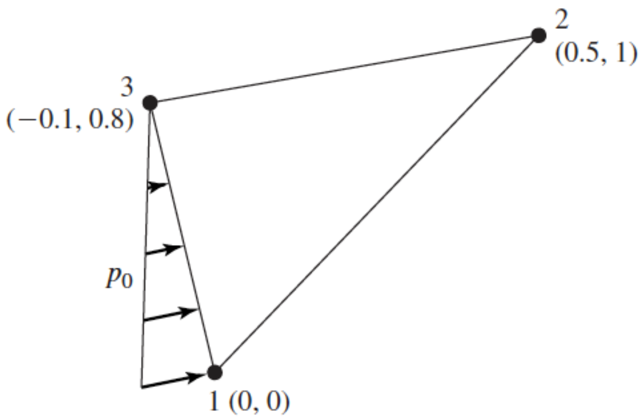
A4 professional format, collecting at the BEGINNING of class (09:09 am)

(late submission within 24 hours: score*0.9; late submission before post of solution: score*0.8

(the solution will be posted usually within a week))

Total 80%

1. **(30%)** The T3 element shown below is subjected to the linearly varying pressure as shown below. Determine the equivalent nodal forces.



(Sol):
1D integral

$$l=\sqrt{0.8^2+0.1^2}$$

$$t_{x1}=\frac{0.8}{l}p_0 \qquad t_{y1}=\frac{0.1}{l}p_0$$

$$t_{x3}=0 \qquad t_{y3}=0$$

$$N_1=1-\xi \quad N_3=\xi$$

$$b(x)=\mathbf{N}^e\mathbf{b} \qquad \mathbf{b}=\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\mathbf{f}_{\Omega}^{e} = t\int_0^1 \mathbf{N}^{eT}\mathbf{N}^{e} bJd\xi = t\int_0^1 \begin{bmatrix} N_1 & 0 \\ 0 & N_1 \\ 0 & 0 \\ 0 & 0 \\ N_3 & 0 \\ 0 & N_3 \end{bmatrix} \begin{bmatrix} N_1 t_{x1} + N_3 t_{x3} \\ N_1 t_{y1} + N_3 t_{y3} \end{bmatrix} l d\xi$$

$$= t\int_0^1 \begin{bmatrix} (1-\xi) & 0 \\ 0 & (1-\xi) \\ 0 & 0 \\ 0 & 0 \\ \xi & 0 \\ 0 & \xi \end{bmatrix} \begin{bmatrix} (1-\xi)0.8p_0 \\ (1-\xi)0.1p_0 \end{bmatrix} d\xi$$

$$= \begin{bmatrix} \frac{1}{3}0.8 \\ \frac{1}{3}0.1 \\ 0 \\ 0 \\ \frac{1}{6}0.8 \\ \frac{1}{6}0.1 \end{bmatrix} p_0 t \quad = \frac{p_0 t}{60} \begin{bmatrix} 16 \\ 2 \\ 0 \\ 0 \\ 8 \\ 1 \end{bmatrix}$$

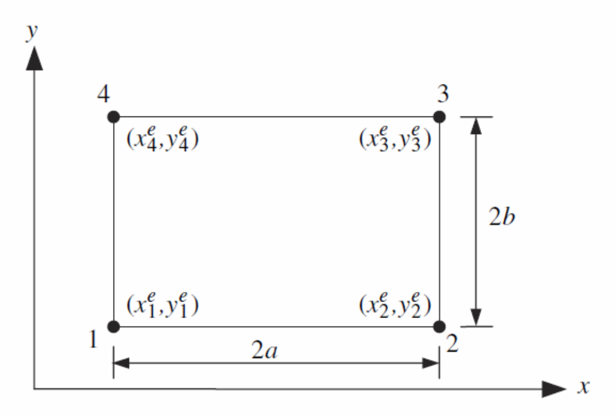
2. (30%) Consider a constant distributed body force, $\bar{\mathbf{b}} = \begin{bmatrix} \bar{b}_x \\ \bar{b}_y \end{bmatrix}$

(Sol):

$$f_{\Omega}^{e} = t\int_{\Omega^e} \mathbf{N}^{eT} \cdot b^e d\Omega = tab\int_{-1}^{1}\int_{-1}^{1}(\mathbf{N}^{Q4})d\xi d\eta = tab\begin{bmatrix} \overline{b_x} \\ \overline{b_y} \\ \overline{b_x} \\ \overline{b_y} \\ \overline{b_x} \\ \overline{b_y} \\ \overline{b_x} \\ \overline{b_y} \end{bmatrix}$$

3. (20%) For a plane-stress rectangular element shown below, the nodal displacements are given by

$$\begin{bmatrix} u_{x1}^e \\ u_{y1}^e \\ u_{x2}^e \\ u_{y2}^e \\ u_{x3}^e \\ u_{y3}^e \\ u_{x4}^e \\ u_{y4}^e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.005 \\ 0.0025 \\ 0.0025 \\ -0.0025 \\ 0 \\ 0 \end{bmatrix} \text{in.}$$



Let $a = 2$ in., $b = 1$ in., $E = 30\times 10^6$ psi, and $\nu = 0.3$. Determine the element strains and stresses at the centroid of the element and at the four corner nodes.

(Sol):

$$\mathbf{B}^e = \frac{1}{4}\begin{bmatrix} -\frac{1-\eta}{a} & 0 & \frac{1-\eta}{a} & 0 & \frac{1+\eta}{a} & 0 & -\frac{1+\eta}{a} & 0 \\ 0 & -\frac{1-\xi}{b} & 0 & -\frac{1+\xi}{b} & 0 & \frac{1+\xi}{b} & 0 & \frac{1-\xi}{b} \\ -\frac{1-\xi}{b} & -\frac{1-\eta}{a} & -\frac{1+\xi}{b} & \frac{1-\eta}{a} & \frac{1+\xi}{b} & \frac{1+\eta}{a} & \frac{1-\xi}{b} & -\frac{1+\eta}{a} \end{bmatrix} \quad (4\%)$$

$$\boldsymbol{\varepsilon}^e = \mathbf{B}^e\mathbf{d}^e$$
$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$$

(3% each)

Centroid of the element： (2%)

$$(\xi,\eta)=(0,0)$$

$$\boldsymbol{\varepsilon}=\begin{bmatrix}0.0009375\\-0.0012500\\-0.0006250\end{bmatrix}\qquad \boldsymbol{\sigma}=\begin{bmatrix}18544\\-31937\\-7212\end{bmatrix}(psi)$$

Node 1： (2%)

$$(\xi,\eta)=(-1,-1)$$

$$\boldsymbol{\varepsilon}=\begin{bmatrix}0.0012500\\0.0000000\\0.0006250\end{bmatrix}\qquad \boldsymbol{\sigma}=\begin{bmatrix}41209\\12363\\7212\end{bmatrix}(psi)$$

Node 2： (2%)

$$(\xi,\eta)=(1,-1)$$

$$\boldsymbol{\varepsilon}=\begin{bmatrix}0.001250\\-0.002500\\-0.000625\end{bmatrix}\qquad \boldsymbol{\sigma}=\begin{bmatrix}16484\\-70055\\-7212\end{bmatrix}(psi)$$

Node 3： (2%)

$$(\xi,\eta)=(1,1)$$

$$\boldsymbol{\varepsilon}=\begin{bmatrix}0.000625\\-0.002500\\-0.001875\end{bmatrix}\qquad \boldsymbol{\sigma}=\begin{bmatrix}-4121\\-76236\\-21635\end{bmatrix}(psi)$$

Node 4： (2%)

$$(\xi,\eta)=(-1,1)$$

$$\boldsymbol{\varepsilon}=\begin{bmatrix}0.000625\\0.000000\\-0.000625\end{bmatrix}\qquad \boldsymbol{\sigma}=\begin{bmatrix}20604\\6181\\-7212\end{bmatrix}(psi)$$