National Taiwan University **Finite Element Method**

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Homework 2, 03/06/2019 Due: 03/20/2019

A4 professional format, collecting at the BEGINNING of class (09:09 am)

(late submission within 24 hours: score*0.9; late submission before post of solution: score*0.8

(the solution will be posted usually within a week))

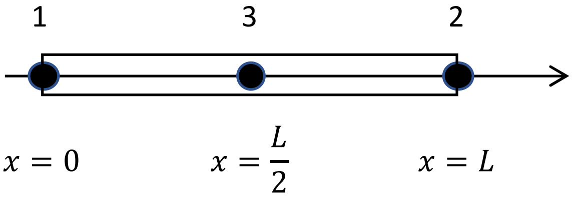
Total 60%

1. **(60%)** (a) (10%) Use the relation $\mathbf{N}^e(x) = \mathbf{p}(x)\left(\mathbf{M}^e\right)^{-1}$ as we have done for a two-node linear element to derive the shape functions for a three-node quadratic element illustrated below where node 3 is in the middle point of the element.

   (b) (10%) Plot the values of shape functions vs. x using MATLAB.

   (c) (20%) Derive the element stiffness matrix $\mathbf{K}^e$ for the element with a constant cross-sectional area A^e E^e.

   (d) (20%) Derive the external body force matrix $\mathbf{f}_\Omega^e$ subjected to **linear distribution of body forces** for the element with the value of b_1 at node 1 and b_2 at node 2.



(Sol):

(a)

$$u^e(x) = \alpha_0^e + \alpha_1^e x + \alpha_2^e x^2 = \begin{bmatrix}1 & x & x^2\end{bmatrix}\begin{bmatrix}\alpha_0^e \\ \alpha_1^e \\ \alpha_2^e\end{bmatrix} \Rightarrow \boldsymbol{p}(x) = \begin{bmatrix}1 & x & x^2\end{bmatrix}$$

$$\begin{bmatrix}u_1^e \\ u_2^e \\ u_3^e\end{bmatrix} = \begin{bmatrix}1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2\end{bmatrix}\begin{bmatrix}\alpha_0^e \\ \alpha_1^e \\ \alpha_2^e\end{bmatrix} \Rightarrow \boldsymbol{M}^e = \begin{bmatrix}1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 \\ 1 & L & L^2 \\ 1 & L/2 & L^2/4\end{bmatrix}$$

$$\boldsymbol{N}^e = \boldsymbol{p}(x)(\boldsymbol{M}^e)^{-1} = \begin{bmatrix}1 & x & x^2\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 0 \\ 1 & L & L^2 \\ 1 & L/2 & L^2/4\end{bmatrix}\right)^{-1}$$

$$= \begin{bmatrix}\dfrac{2x^2 - 3Lx + L^2}{L^2} & \dfrac{2x^2 - Lx}{L^2} & \dfrac{-4x^2 + 4Lx}{L^2}\end{bmatrix}$$

(b)

(c)

$$\boldsymbol{B}^e = \frac{d\boldsymbol{N}}{dx} = \frac{1}{L^2}\begin{bmatrix} 4x-3L & 4x-L & -8x+4L \end{bmatrix}$$

$$\boldsymbol{K}^e = \int_0^L \boldsymbol{B}^{eT} A^e E^e \boldsymbol{B}^e dx$$

$$= \frac{A^e E^e}{3L}\begin{bmatrix} 7 & 1 & -8 \\ 1 & 7 & -8 \\ -8 & -8 & 16 \end{bmatrix}$$

(d)

$$b(x) = b_1 + \frac{b_2 - b_1}{L} x$$

$$\mathrm{f}_\Omega^e = \int_0^L \boldsymbol{N}^{eT} b(x) dx = \frac{L}{6}\begin{bmatrix} b_1 \\ b_2 \\ 2(b_1+b_2) \end{bmatrix}$$

Note:

1. If your answer matrixes are not following the node order, you should point out to show me in your answer. (-3% each subproblem)
2. If your node orders are not identical, but the following procedures are correct. (-15%)

Matlab Code Check