National Taiwan University **Finite Element Method**
Department of Civil Engineering Instructors: C.-S. CHEN

Lab Assignment 9, 05/30/2019, 1800 -- 2000

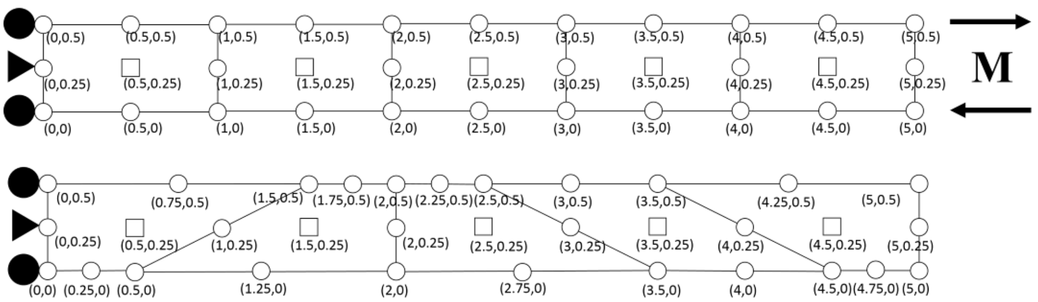
Due 2000

Lab Grading Policy: Attendance 20%, Score 80%, Bonus 40%

In case you have difficulty in finishing the exercises on time, you should upload them before **2100 on Saturday** and a penalty of 20% discount will be applied on your score. No late submission after 2100 on Saturday is permitted. We will in general post the reference solutions **by Sunday**.

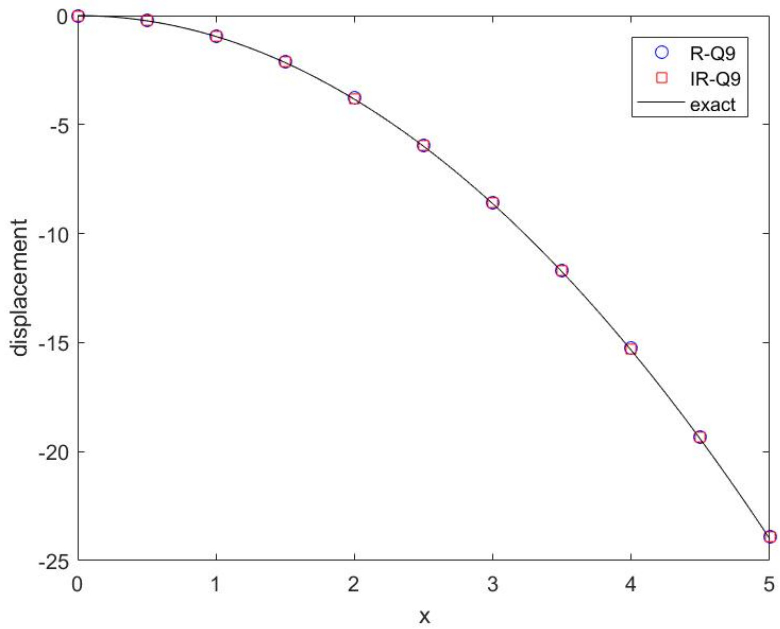
Download `Q9Simple.zip` from the course website and unzip it. You will find a folder containing `Q9Simple.m` script file with six functions, `formStiffnessRec.m` `guass2d.m` `outputDisplacements.m` `shapeFunctionQ9.m` `solution.m`.

1. **(40%)** Consider a pure bending problem with two meshes shown below and model the problem with Q9 elements using 3x3 Gauss integration.

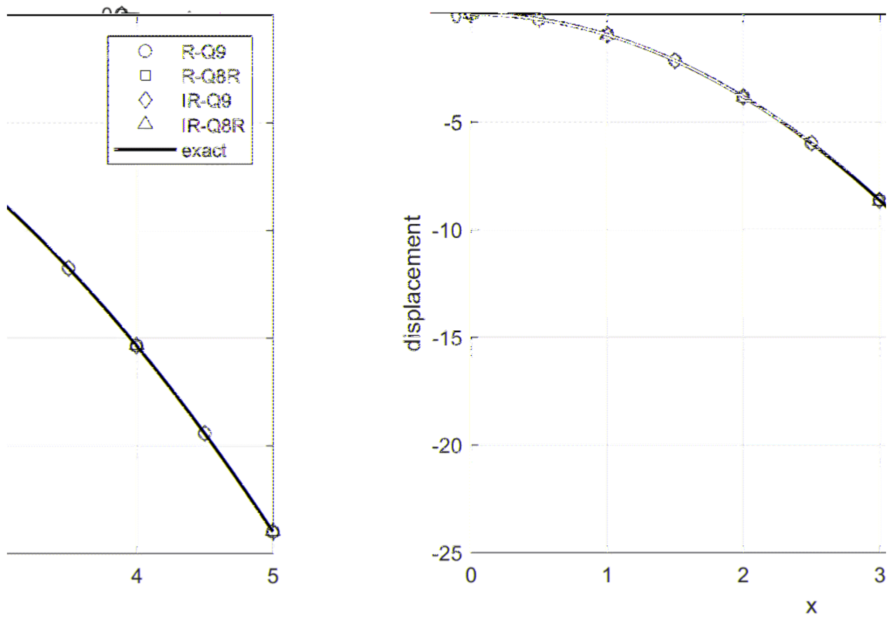


thickness = 1, and M = 600 which gives a vertical

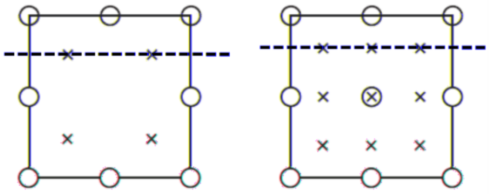
24 units. Report the displacement along the centerline and compare your Q9 results with the exact solution from the beam theory $\frac{Mx^2}{2EI}$. Below is a sample plot:



2. **(40%)** Extend your implementation in Problem 1 to include the eight-node quadrilateral element with 2x2 reduced integration and consider the same pure bending problem again. Report the displacement along the centerline and compare your Q8 and Q9 (from Problem 1) results with the exact solution from the beam theory $\frac{Mx^2}{2EI}$. Below is a sample plot:



3. **(Bonus 40%)** Extend your implementation to compute the stresses at Gauss points and report the axial stress σ_{xx} along the upper line of quadrature points (靠近上邊界的高斯點) for the same pure bending problem.



Compare your Q9 full integration and Q8 reduced integration results with the exact solutions from the elementary beam theory. Notice that the exact solutions differ as you change the Gauss point locations (that is, you will have different exact solutions for 3x3 and 2x2 upper line of quadrature points). Below is the sample plot: