National Taiwan University **Finite Element Method**

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Lab Assignment 8, 05/23/2019, 1800 -- 2000

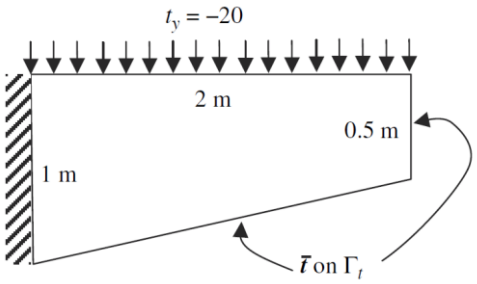
Due 2000

Lab Grading Policy: Attendance 20%, Score 80%, Bonus 20%

In case you have difficulty in finishing the exercises on time, you should upload them before **2100 on Saturday** and a penalty of 20% discount will be applied on your score. No late submission after 2100 on Saturday is permitted. We will in general post the reference solutions **by Sunday**.

Download `Q4Taper.zip` from the course website and unzip it. You will find a folder containing `problem2dTaperE1.m` and `problem2dTaperE4.m` files with seven functions, `drawingMesh.m` `formStiffness2D.m` `guass2d.m` `Jacobian.m` `outputDisplacements.m` `shapeFunctionQ4.m` `solution.m`.

1. **(80%)** Consider the same trapezoidal panel given in the Lecture. The vertical left edge is fixed. The bottom and the right vertical edges are traction free. Traction $t_y = -20$ Nm^{-1} is applied on the top horizontal edge. Material properties are Young's modulus $E = 3\times10^7$ Pa, Poisson's ratio $\nu = 0.3$ and thickness = 1 m. Plane stress conditions are considered.



Write an interactive preprocessor to discretize the domain using `NxM` bilinear Q4 quadrilateral elements and output the displacements. Below is a sample run:

```
Enter the discretization N and M: 8 6
```

```
Displacements
Node          UX          UY
   1   0.0000e+00   0.0000e+00
   2   -1.5059e-06   -1.5078e-06
   3   -2.7139e-06   -3.5074e-06
   4   -3.5266e-06   -6.3540e-06
   5   -3.8586e-06   -9.8898e-06
   6   -3.7282e-06   -1.3937e-05
   7   -3.1931e-06   -1.8308e-05
   8   -2.3350e-06   -2.2810e-05
```

9	-1.2876e-06	-2.7261e-05
10	0.0000e+00	0.0000e+00
11	-1.0115e-06	-1.2789e-06
12	-1.7865e-06	-3.3574e-06
13	-2.1936e-06	-6.2442e-06
14	-2.2521e-06	-9.8186e-06
15	-1.9892e-06	-1.3896e-05
16	-1.4504e-06	-1.8291e-05
17	-7.0034e-07	-2.2808e-05
18	1.7661e-07	-2.7263e-05
19	0.0000e+00	0.0000e+00
20	-5.3390e-07	-1.1651e-06
21	-8.4436e-07	-3.2900e-06
22	-8.8144e-07	-6.2222e-06
23	-6.8454e-07	-9.8154e-06
24	-2.9706e-07	-1.3904e-05
25	2.4671e-07	-1.8305e-05
26	9.0417e-07	-2.2825e-05
27	1.6263e-06	-2.7276e-05
28	0.0000e+00	0.0000e+00
29	-4.0174e-08	-1.1386e-06
30	1.2035e-07	-3.3139e-06
31	4.5058e-07	-6.2753e-06
32	8.8374e-07	-9.8700e-06
33	1.3831e-06	-1.3952e-05
34	1.9281e-06	-1.8346e-05
35	2.4963e-06	-2.2858e-05
36	3.0696e-06	-2.7303e-05
37	0.0000e+00	0.0000e+00
38	4.9698e-07	-1.2001e-06
39	1.1564e-06	-3.4352e-06
40	1.8461e-06	-6.3971e-06
41	2.4914e-06	-9.9731e-06
42	3.0859e-06	-1.4036e-05
43	3.6205e-06	-1.8411e-05
44	4.0929e-06	-2.2905e-05
45	4.5156e-06	-2.7344e-05
46	0.0000e+00	0.0000e+00
47	1.1375e-06	-1.3870e-06
48	2.3453e-06	-3.6598e-06
49	3.3440e-06	-6.5774e-06
50	4.1786e-06	-1.0119e-05
51	4.8462e-06	-1.4148e-05
52	5.3499e-06	-1.8495e-05
53	5.7087e-06	-2.2965e-05
54	5.9709e-06	-2.7396e-05
55	0.0000e+00	0.0000e+00
56	2.1521e-06	-1.8027e-06
57	3.7152e-06	-3.9475e-06
58	4.9889e-06	-6.8171e-06
59	5.9858e-06	-1.0303e-05
60	6.7007e-06	-1.4284e-05
61	7.1437e-06	-1.8591e-05
62	7.3556e-06	-2.3033e-05
63	7.4366e-06	-2.7452e-05

16x10
Displacements

Node	UX	UY
1	0.0000e+00	0.0000e+00
2	-8.1102e-07	-7.7366e-07

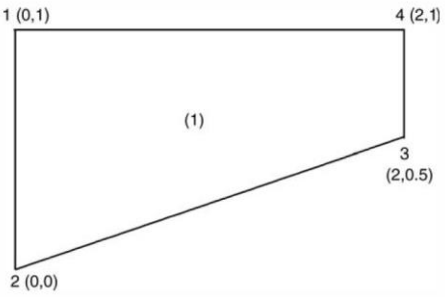
3	-1.5368e-06	-1.5167e-06
4	-2.2082e-06	-2.4536e-06
5	-2.7922e-06	-3.6056e-06
6	-3.2662e-06	-4.9724e-06
7	-3.6191e-06	-6.5368e-06
8	-3.8475e-06	-8.2776e-06
9	-3.9533e-06	-1.0172e-05
10	-3.9409e-06	-1.2198e-05
11	-3.8167e-06	-1.4332e-05
12	-3.5881e-06	-1.6549e-05
13	-3.2640e-06	-1.8826e-05
14	-2.8556e-06	-2.1135e-05
15	-2.3762e-06	-2.3452e-05
16	-1.8436e-06	-2.5751e-05
17	-1.2856e-06	-2.8013e-05
18	0.0000e+00	0.0000e+00
19	-6.3849e-07	-6.2488e-07
20	-1.2523e-06	-1.3874e-06
21	-1.7801e-06	-2.3335e-06
22	-2.2170e-06	-3.4997e-06
23	-2.5551e-06	-4.8804e-06
24	-2.7906e-06	-6.4584e-06
25	-2.9234e-06	-8.2121e-06
26	-2.9558e-06	-1.0119e-05
27	-2.8919e-06	-1.2156e-05
28	-2.7370e-06	-1.4300e-05
29	-2.4978e-06	-1.6526e-05
30	-2.1821e-06	-1.8811e-05
31	-1.7996e-06	-2.1127e-05
32	-1.3621e-06	-2.3449e-05
33	-8.8511e-07	-2.5751e-05
34	-3.8623e-07	-2.8013e-05
35	0.0000e+00	0.0000e+00
36	-4.9485e-07	-5.3588e-07
37	-9.5379e-07	-1.2851e-06
38	-1.3398e-06	-2.2471e-06
39	-1.6387e-06	-3.4292e-06
40	-1.8497e-06	-4.8239e-06
41	-1.9745e-06	-6.4136e-06
42	-2.0160e-06	-8.1771e-06
43	-1.9779e-06	-1.0092e-05
44	-1.8643e-06	-1.2137e-05
45	-1.6797e-06	-1.4287e-05
46	-1.4300e-06	-1.6519e-05
47	-1.1217e-06	-1.8808e-05
48	-7.6316e-07	-2.1128e-05
49	-3.6439e-07	-2.3453e-05
50	6.3383e-08	-2.5756e-05
51	5.0775e-07	-2.8016e-05
52	0.0000e+00	0.0000e+00
53	-3.4865e-07	-4.8042e-07
54	-6.5470e-07	-1.2186e-06
55	-8.9392e-07	-2.1932e-06
56	-1.0567e-06	-3.3919e-06
57	-1.1446e-06	-4.8000e-06
58	-1.1630e-06	-6.3992e-06
59	-1.1171e-06	-8.1694e-06
60	-1.0114e-06	-1.0090e-05
61	-8.5002e-07	-1.2138e-05
62	-6.3707e-07	-1.4292e-05
63	-3.7723e-07	-1.6527e-05

64	-7.6063e-08	-1.8818e-05
65	2.5979e-07	-2.1138e-05
66	6.2259e-07	-2.3463e-05
67	1.0044e-06	-2.5765e-05
68	1.3968e-06	-2.8023e-05
69	0.0000e+00	0.0000e+00
70	-1.9940e-07	-4.4904e-07
71	-3.5132e-07	-1.1848e-06
72	-4.4144e-07	-2.1729e-06
73	-4.6684e-07	-3.3873e-06
74	-4.3308e-07	-4.8069e-06
75	-3.4793e-07	-6.4129e-06
76	-2.1789e-07	-8.1867e-06
77	-4.7668e-08	-1.0109e-05
78	1.5900e-07	-1.2158e-05
79	3.9863e-07	-1.4312e-05
80	6.6749e-07	-1.6547e-05
81	9.6127e-07	-1.8837e-05
82	1.2750e-06	-2.1156e-05
83	1.6033e-06	-2.3479e-05
84	1.9407e-06	-2.5780e-05
85	2.2828e-06	-2.8035e-05
86	0.0000e+00	0.0000e+00
87	-4.5252e-08	-4.3767e-07
88	-3.8900e-08	-1.1823e-06
89	2.4339e-08	-2.1872e-06
90	1.3866e-07	-3.4159e-06
91	2.9347e-07	-4.8438e-06
92	4.7946e-07	-6.4531e-06
93	6.9029e-07	-8.2271e-06
94	9.2183e-07	-1.0148e-05
95	1.1710e-06	-1.2195e-05
96	1.4350e-06	-1.4346e-05
97	1.7112e-06	-1.6578e-05
98	1.9966e-06	-1.8866e-05
99	2.2878e-06	-2.1182e-05
100	2.5816e-06	-2.3501e-05
101	2.8752e-06	-2.5799e-05
102	3.1678e-06	-2.8053e-05
103	0.0000e+00	0.0000e+00
104	1.1714e-07	-4.4597e-07
105	2.9044e-07	-1.2131e-06
106	5.1397e-07	-2.2385e-06
107	7.7060e-07	-3.4785e-06
108	1.0451e-06	-4.9100e-06
109	1.3286e-06	-6.5181e-06
110	1.6165e-06	-8.2887e-06
111	1.9058e-06	-1.0205e-05
112	2.1942e-06	-1.2247e-05
113	2.4798e-06	-1.4393e-05
114	2.7609e-06	-1.6620e-05
115	3.0359e-06	-1.8903e-05
116	3.3031e-06	-2.1214e-05
117	3.5616e-06	-2.3529e-05
118	3.8108e-06	-2.5823e-05
119	4.0538e-06	-2.8077e-05
120	0.0000e+00	0.0000e+00
121	2.9390e-07	-4.7804e-07
122	6.4970e-07	-1.2838e-06
123	1.0426e-06	-2.3308e-06
124	1.4424e-06	-3.5759e-06

125	1.8328e-06	-5.0045e-06
126	2.2094e-06	-6.6068e-06
127	2.5701e-06	-8.3701e-06
128	2.9130e-06	-1.0278e-05
129	3.2370e-06	-1.2313e-05
130	3.5408e-06	-1.4452e-05
131	3.8236e-06	-1.6672e-05
132	4.0851e-06	-1.8947e-05
133	4.3257e-06	-2.1253e-05
134	4.5465e-06	-2.3561e-05
135	4.7500e-06	-2.5851e-05
136	4.9426e-06	-2.8105e-05
137	0.0000e+00	0.0000e+00
138	4.9804e-07	-5.4562e-07
139	1.0612e-06	-1.4068e-06
140	1.6320e-06	-2.4677e-06
141	2.1677e-06	-3.7071e-06
142	2.6675e-06	-5.1263e-06
143	3.1316e-06	-6.7178e-06
144	3.5602e-06	-8.4700e-06
145	3.9525e-06	-1.0367e-05
146	4.3079e-06	-1.2391e-05
147	4.6258e-06	-1.4520e-05
148	4.9063e-06	-1.6731e-05
149	5.1504e-06	-1.8999e-05
150	5.3602e-06	-2.1296e-05
151	5.5398e-06	-2.3598e-05
152	5.6948e-06	-2.5882e-05
153	5.8352e-06	-2.8137e-05
154	0.0000e+00	0.0000e+00
155	7.5753e-07	-6.8082e-07
156	1.5758e-06	-1.5985e-06
157	2.3010e-06	-2.6451e-06
158	2.9593e-06	-3.8702e-06
159	3.5594e-06	-5.2737e-06
160	4.1049e-06	-6.8501e-06
161	4.5963e-06	-8.5877e-06
162	5.0334e-06	-1.0471e-05
163	5.4156e-06	-1.2482e-05
164	5.7430e-06	-1.4598e-05
165	6.0163e-06	-1.6798e-05
166	6.2378e-06	-1.9055e-05
167	6.4115e-06	-2.1344e-05
168	6.5443e-06	-2.3638e-05
169	6.6462e-06	-2.5917e-05
170	6.7316e-06	-2.8170e-05
171	0.0000e+00	0.0000e+00
172	1.2688e-06	-9.7040e-07
173	2.2203e-06	-1.8303e-06
174	3.0617e-06	-2.8594e-06
175	3.8258e-06	-4.0617e-06
176	4.5176e-06	-5.4455e-06
177	5.1385e-06	-7.0029e-06
178	5.6879e-06	-8.7225e-06
179	6.1651e-06	-1.0589e-05
180	6.5693e-06	-1.2583e-05
181	6.9009e-06	-1.4685e-05
182	7.1613e-06	-1.6870e-05
183	7.3540e-06	-1.9115e-05
184	7.4847e-06	-2.1393e-05
185	7.5630e-06	-2.3680e-05

```
  186   7.6041e-06    -2.5952e-05
  187   7.6304e-06    -2.8204e-05
```

2. **(Bonus, 20%)** You should finish Problem 1 first. Conduct a convergence study on Problem 1 by setting N = M and change N from 1, 2, 4, 16, 32, 64, 128. Monitor the deflection along the y direction on node 4. Use the answer from N = 128 as your reference solution and plot the difference in % in the y-axis and N in the x-axis.



Below is a sample plot: