National Taiwan University | **Finite Element Method**
Department of Civil Engineering | Instructors: C.-S. CHEN

Lab Assignment 4, 04/11/2019, 1800 -- 2000

Due 2000

Lab Grading Policy: Attendance 20%, Score 80%, Bonus 20%

You are expected to complete the basic part during the Lab. In case you have difficulty in finishing the basic part on time, you should upload them before 2100 on Saturday and a penalty of 20% discount will be applied on your score. You are encouraged to complete the bonus part (no penalty applied). Basic and/or bonus parts should be submitted by **2100 on Saturday and no late submission is permitted**. We will in general post the reference solutions by **Sunday**.

Download `BarIsoParametric.zip` from the course website and unzip it. You will find a folder containing `problem1dIsoEx1.m` `problem1dIsoEx2.m` `problem1dIsoEx3.m` files with four functions, `solution.m` `outputDisplacementsReactionsPretty.m` `guass1d.m` `shapeFucntionL2.m`.

Some background knowledge on stress calculations in finite elements

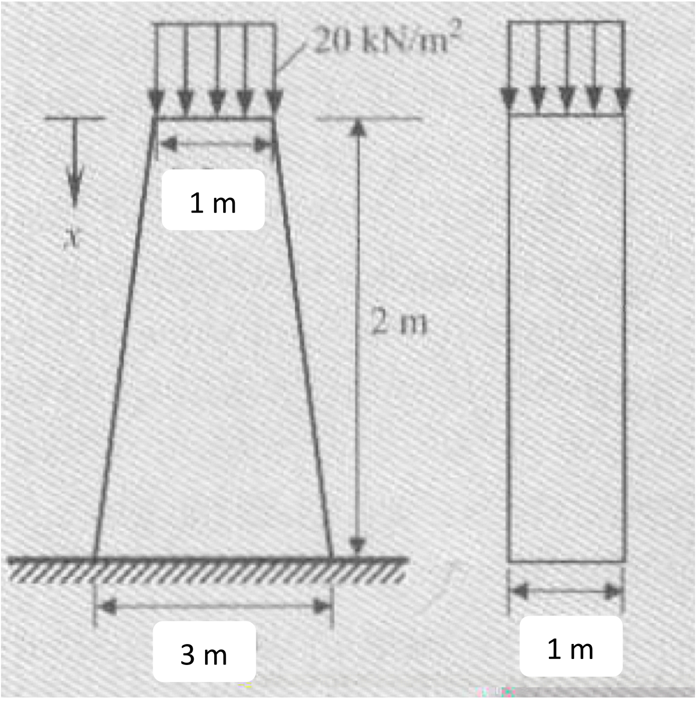
Stress calculations (or so called stress recovery) are an important topic in finite element analysis. We will talk about various ways to "recover" the stresses when we deal with 2D problems. For now, let us compute stress simply through nodal displacements:

$$\sigma^e(x) = E^e(x)\mathbf{B}^e\mathbf{d}^e$$

For the isoparametric formulation, you will need to compute $\mathbf{B}^e$ by substituting the locations of the nodes in natural coordinates and multiplying it with the inverse of Jacobian:

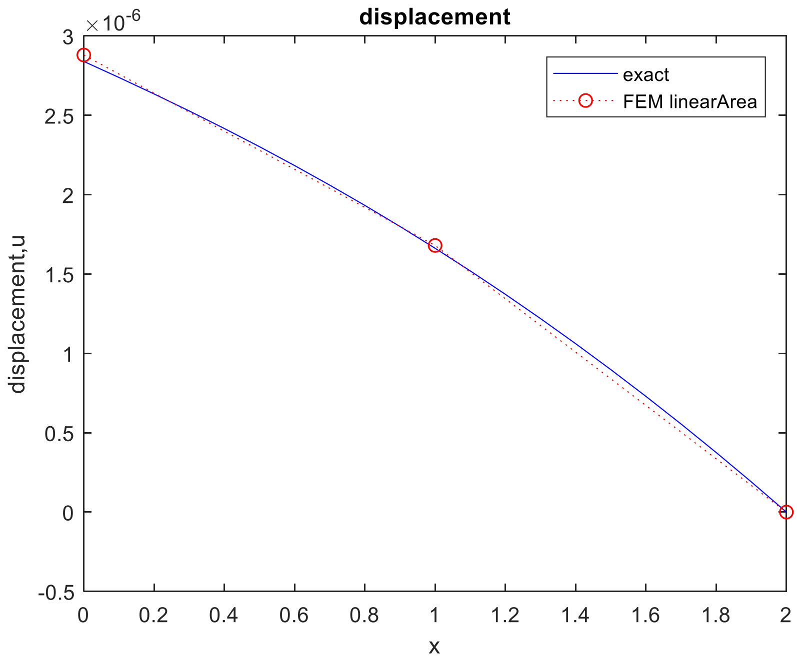
$$\mathbf{B}^e = \frac{d\mathbf{N}^e}{dx} = \frac{d\mathbf{N}^L}{d\xi}\frac{d\xi}{dx} = \frac{d\mathbf{N}^L}{d\xi}\frac{1}{J}$$

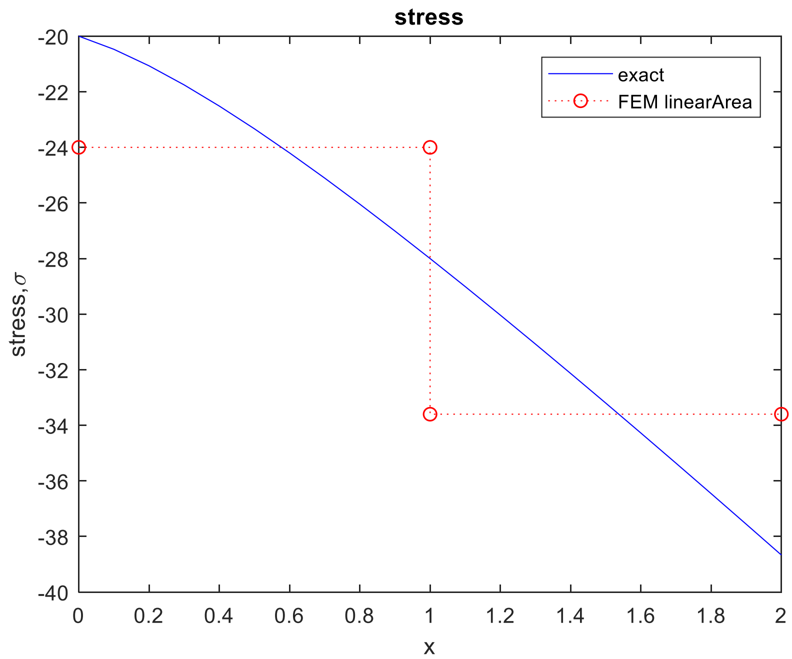
1. **(40%)** Consider the concrete pier problem in HW1 (shown below). The load $20\ \text{kN/m}^2$ represent the weight of bridge and a distribution of the traffic on the bridge. The concrete weighs $24\ \text{kN/m}^3$ and its modulus is $E = 2\times10^7$ N/ ^{2}. Modify the 1D isoparametric MATLAB codes and solve for the axial displacement and stress using 2 uniformly spacing linear elements with the area is a function of x, i.e. $A(x) = A_0(1+x)$.



Compare your finite element results (displacements and stresses) with the exact solution. Notice the discontinuity nature of stress between elements. Sample MATLAB plots look like:

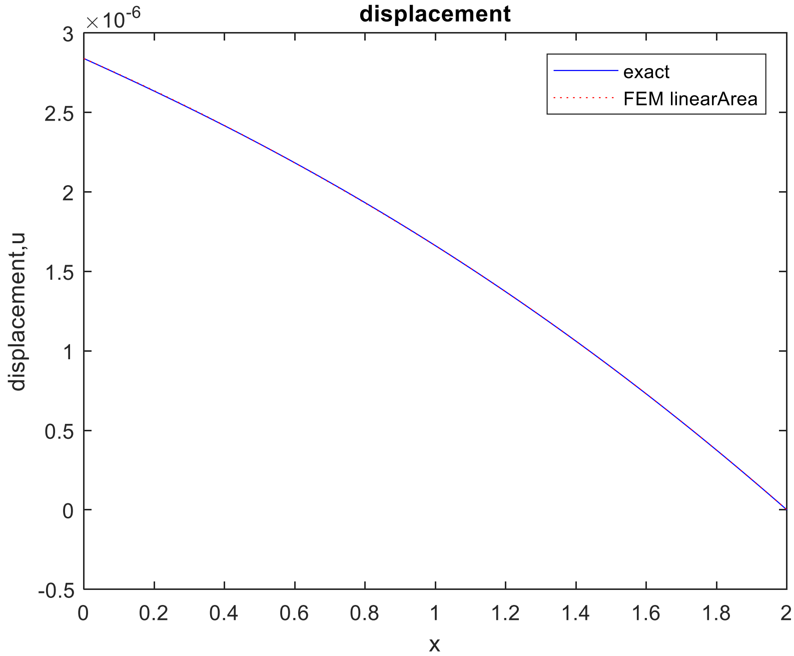
```
Displacements
node     displacements
 1:        2.8800e-06
 2:        1.6800e-06
 3:        0.0000e+00
Reactions
node     reactions
 3:        -1.1600e+02
```

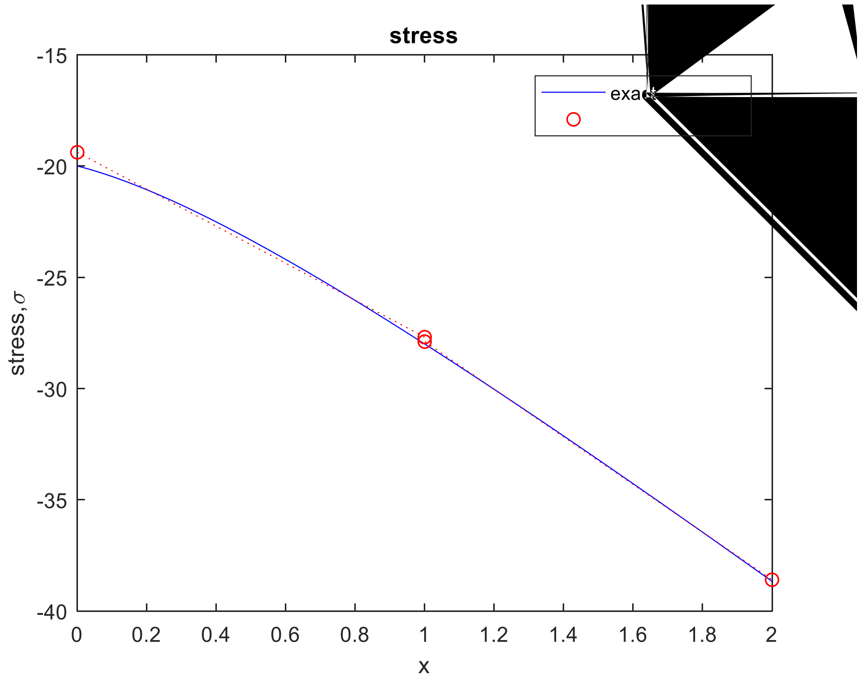




2. **(40%)** Redo Problem 1 using 2 uniformly spacing quadratic elements with mid-node in the center and the area is a function of x, i.e. $A(x) = A_0(1+x)$. Sample MATLAB plots look like:

```
Displacements
node     displacements
 1:         2.8391e-06
 2:         2.3025e-06
 3:         1.6622e-06
 4:         8.9797e-07
 5:         0.0000e+00
Reactions
node     reactions
 5:         -1.1600e+02
```

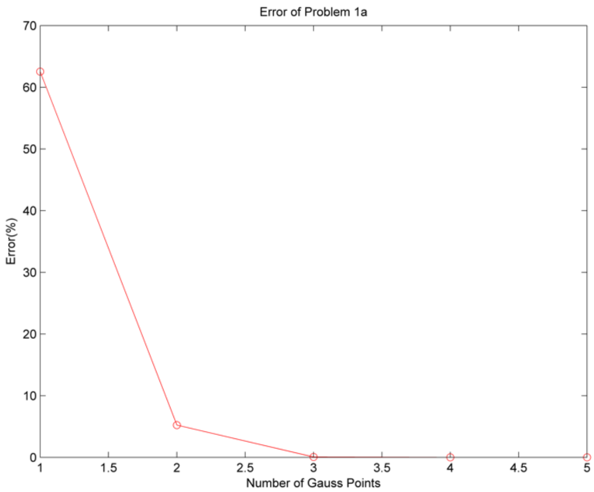




3. **(Bonus 20%)** Use `gauss1d` to integrate the following polynomial:

$$f(x) = \int_{1}^{2\sqrt[3]{}} (2 + 7x + 6x^2 + 5x^3 + 4x^4 + 3x^5 + 2x^6 + x^7)dx$$

and plot the error vs. the number of Gauss points. Observe that you can do a very decent approximation using three Gauss points and using four Gauss points to integrate the polynomial exactly. A sample MATLAB plot looks like:


Error of Problem 1a
Error(%)
Number of Gauss Points
70
60
50
40
30
20
10
0
1
1.5
2
2.5
3
3.5
4
4.5
5