National Taiwan University **Finite Element Method**

Department of Civil Engineering Instructors: C.-S. CHEN

Lab Assignment 10, 06/06/2019, 1800 -- 2000

Due 2000

Lab Grading Policy: Attendance 20%, Score 80%, Bonus 40%

In case you have difficulty in finishing the exercises on time, you should upload them before **2100 on Saturday** and a penalty of 20% discount will be applied on your score. No late submission after 2100 on Saturday is permitted. We will in general post the reference solutions **by Sunday**.

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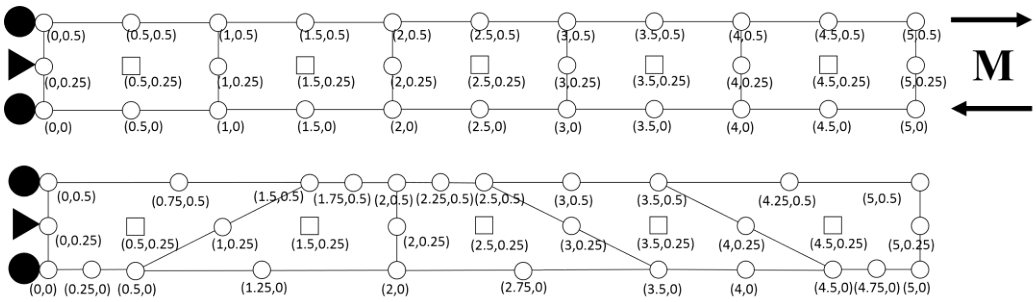
1. (40%) Reconsider the example in Lab 9. The finite element mesh and element and nodal numbering are shown in the following figure. Use the properties given in Lab 9.



Extend your Q8 2x2 reduced integration implementation with a `stressRecovery` function for stress recovery. You should compute stresses at four Gauss points and extrapolate them to compute the nodal stresses. Finally, averaging these element nodal stresses in all elements at a common node to represent the stress at the node. Plot the element-wise discontinuous stress contour and smooth stress contour from average nodal stresses.

Below are sample outputs and contour plots:

(please plot two figures, discontinuous contour and smooth stress contour)

R-Q8R

Stresses at Gauss points

Element	gp	Sxx	Syy	Sxy
1	1	-8.3138e+03	7.5861e-10	-4.0338e-09
1	2	-8.3138e+03	-6.9035e-11	-4.1085e-09
1	3	8.3138e+03	1.6380e-10	-4.0268e-09
1	4	8.3138e+03	-8.0215e-10	-4.0968e-09
2	1	-8.3138e+03	-4.9921e-10	-3.9656e-09
2	2	-8.3138e+03	8.1619e-10	-4.3059e-09
2	3	8.3138e+03	7.5112e-11	-3.8401e-09
2	4	8.3138e+03	-3.9794e-11	-4.1330e-09
3	1	-8.3138e+03	7.6747e-10	-2.5363e-09
3	2	-8.3138e+03	-7.7751e-10	-2.7473e-09
3	3	8.3138e+03	-3.8548e-10	-2.6647e-09
3	4	8.3138e+03	1.0099e-10	-2.9723e-09
4	1	-8.3138e+03	-4.0264e-10	-3.5009e-09
4	2	-8.3138e+03	-8.9570e-10	-3.4718e-09
4	3	8.3138e+03	-6.1770e-10	-3.6441e-09
4	4	8.3138e+03	8.6830e-10	-3.3667e-09
5	1	-8.3138e+03	-9.6305e-10	-9.9276e-10
5	2	-8.3138e+03	5.7527e-10	-1.0242e-09
5	3	8.3138e+03	1.5582e-10	-1.0347e-09
5	4	8.3138e+03	-2.1010e-09	-9.5551e-10

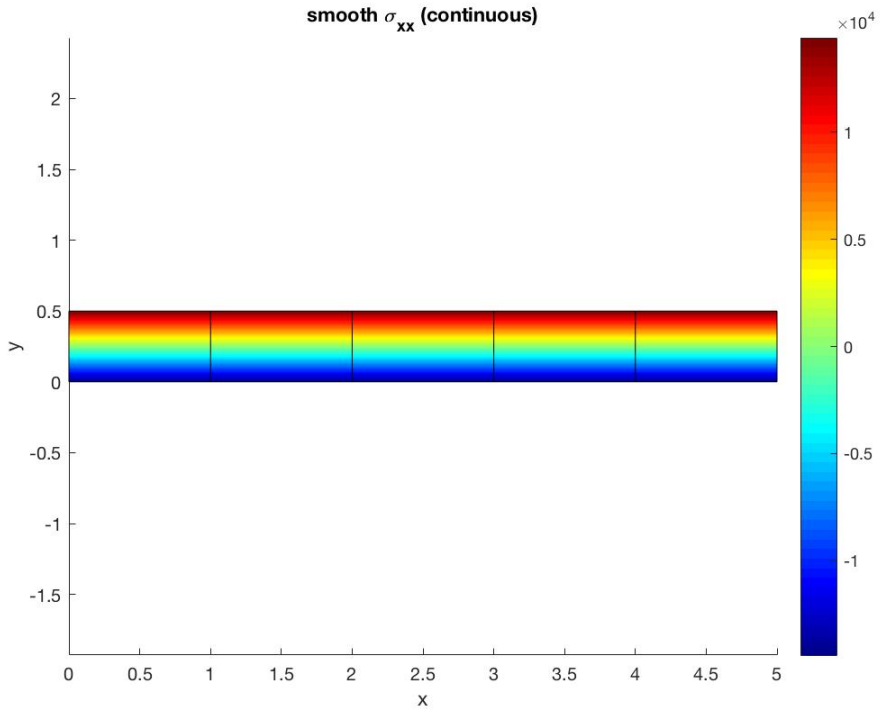
Element nodal Stresses

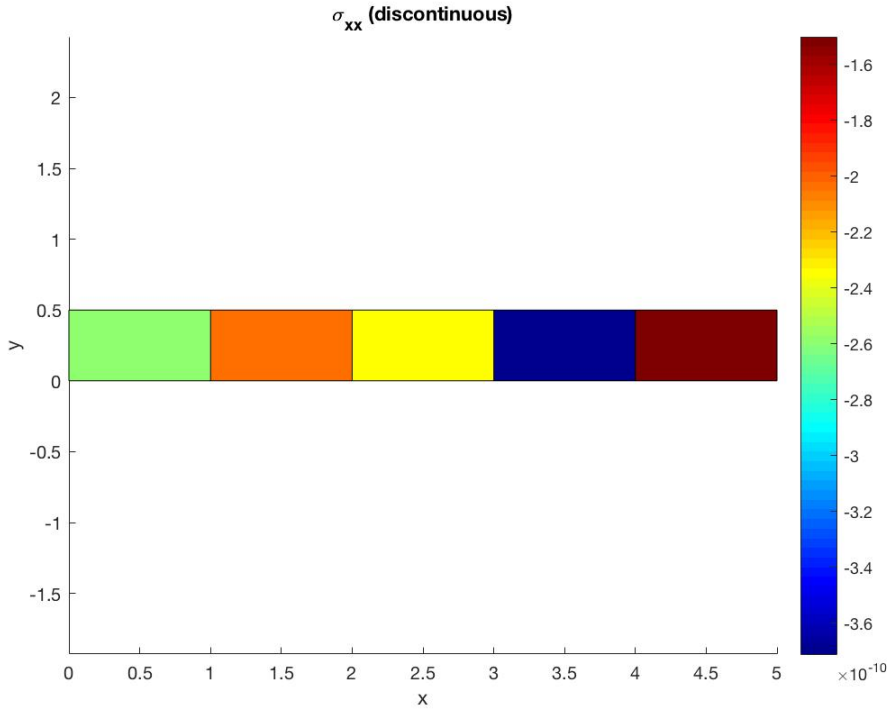
Element	node	Sxx	Syy	Sxy
1	1	-1.4400e+04	1.8731e-09	-3.9640e-09
1	2	-1.4400e+04	-6.9749e-10	-4.1851e-09
1	3	1.4400e+04	8.4287e-10	-3.9519e-09
1	4	1.4400e+04	-1.9673e-09	-4.1649e-09
1	5	-1.4400e+04	5.8781e-10	-4.0745e-09
1	6	-2.5648e-10	7.2691e-11	-4.0685e-09
1	7	1.4400e+04	-5.6220e-10	-4.0584e-09
1	8	-2.5557e-10	-4.7081e-11	-4.0644e-09
2	1	-1.4400e+04	-1.3097e-09	-3.6950e-09
2	2	-1.4400e+04	1.7298e-09	-4.6858e-09
2	3	1.4400e+04	-3.1492e-10	-3.4776e-09
2	4	1.4400e+04	2.4714e-10	-4.3862e-09
2	5	-1.4400e+04	2.1004e-10	-4.1904e-09
2	6	-1.8554e-10	7.0742e-10	-4.0817e-09
2	7	1.4400e+04	-3.3890e-11	-3.9319e-09
2	8	-2.2555e-10	-5.3127e-10	-4.0406e-09
3	1	-1.4400e+04	1.7187e-09	-2.2300e-09
3	2	-1.4400e+04	-1.6283e-09	-2.9243e-09
3	3	1.4400e+04	-2.7823e-10	-2.4523e-09
3	4	1.4400e+04	-1.0671e-10	-3.3140e-09
3	5	-1.4400e+04	4.5201e-11	-2.5772e-09
3	6	-2.7467e-10	-9.5328e-10	-2.6883e-09
3	7	1.4400e+04	-1.9247e-10	-2.8831e-09
3	8	-1.9554e-10	8.0601e-10	-2.7720e-09
4	1	-1.4400e+04	-8.2039e-10	-3.6017e-09
4	2	-1.4400e+04	-1.0449e-09	-3.3570e-09
4	3	1.4400e+04	-1.1929e-09	-3.8497e-09
4	4	1.4400e+04	2.0104e-09	-3.1750e-09
4	5	-1.4400e+04	-9.3265e-10	-3.4794e-09
4	6	-5.1023e-10	-1.1189e-09	-3.6034e-09
4	7	1.4400e+04	4.0878e-10	-3.5124e-09
4	8	-2.3192e-10	5.9502e-10	-3.3884e-09

5	1	-1.4400e+04	-1.0133e-09	-1.0013e-09
5	2	-1.4400e+04	1.1956e-09	-1.0255e-09
5	3	1.4400e+04	9.2461e-10	-1.0739e-09
5	4	1.4400e+04	-3.4398e-09	-9.0650e-10
5	5	-1.4400e+04	9.1135e-11	-1.0134e-09
5	6	-1.9827e-10	1.0601e-09	-1.0497e-09
5	7	1.4400e+04	-1.2576e-09	-9.9019e-10
5	8	-1.0186e-10	-2.2266e-09	-9.5390e-10

Average nodal Stresses

node	Sxx	Syy	Sxy
1	-1.4400e+04	1.8731e-09	-3.9640e-09
2	-1.4400e+04	5.8781e-10	-4.0745e-09
3	-1.4400e+04	-1.0036e-09	-3.9400e-09
4	-1.4400e+04	2.1004e-10	-4.1904e-09
5	-1.4400e+04	1.7242e-09	-3.4579e-09
6	-1.4400e+04	4.5201e-11	-2.5772e-09
7	-1.4400e+04	-1.2244e-09	-3.2630e-09
8	-1.4400e+04	-9.3265e-10	-3.4794e-09
9	-1.4400e+04	-1.0291e-09	-2.1791e-09
10	-1.4400e+04	9.1135e-11	-1.0134e-09
11	-1.4400e+04	1.1956e-09	-1.0255e-09
12	-2.5557e-10	-4.7081e-11	-4.0644e-09
13	-2.4102e-10	-2.2929e-10	-4.0546e-09
14	-1.9054e-10	7.5671e-10	-3.4268e-09
15	-2.5329e-10	-1.7913e-10	-3.0383e-09
16	-3.0604e-10	-1.6727e-09	-2.2786e-09
17	-1.9827e-10	1.0601e-09	-1.0497e-09
18	1.4400e+04	-1.9673e-09	-4.1649e-09
19	1.4400e+04	-5.6220e-10	-4.0584e-09
20	1.4400e+04	5.4501e-10	-4.1691e-09
21	1.4400e+04	-3.3890e-11	-3.9319e-09
22	1.4400e+04	-2.1081e-10	-3.3958e-09
23	1.4400e+04	-1.9247e-10	-2.8831e-09
24	1.4400e+04	8.6611e-10	-2.8137e-09
25	1.4400e+04	4.0878e-10	-3.5124e-09
26	1.4400e+04	-2.3164e-09	-2.3781e-09
27	1.4400e+04	-1.2576e-09	-9.9019e-10
28	1.4400e+04	9.2461e-10	-1.0739e-09


smooth σ_{xx} (continuous)
$\times 10^4$
1
0.5
0
-0.5
-1
2
1.5
1
0.5
0
-0.5
-1
-1.5
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x


σ_{xx} (discontinuous)
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
-3
-3.2
-3.4
-3.6
$\times 10^{-10}$
2
1.5
1
0.5
0
-0.5
-1
-1.5
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x

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Stresses at Gauss points

Element	gp	Sxx	Syy	Sxy
1	1	-8.3559e+03	7.5885e+00	-3.0251e+01
1	2	-8.3301e+03	5.5038e+01	3.3459e+01
1	3	8.3601e+03	-4.2973e+01	-1.4428e+01
1	4	8.3615e+03	2.3843e+01	4.3536e+01
2	1	-8.3557e+03	-4.0524e+01	-2.9785e+01
2	2	-8.2772e+03	-2.5747e+01	-1.9725e+01
2	3	8.3075e+03	1.9541e+02	9.1431e+01
2	4	8.2177e+03	-1.4925e+01	-5.7677e+01
3	1	-8.6145e+03	-9.9354e+01	1.9402e+02
3	2	-7.6650e+03	3.6905e+02	-4.4781e+02
3	3	7.8860e+03	-2.8019e+02	3.0764e+02
3	4	8.5640e+03	-1.8349e+02	-7.4478e+01
4	1	-8.0776e+03	-2.1776e+02	1.5870e+02
4	2	-9.0724e+03	-3.4406e+02	5.3369e+02
4	3	8.0995e+03	1.0083e+02	-4.7236e+01
4	4	8.5609e+03	3.3859e+02	-4.0036e+02
5	1	-7.5717e+03	4.2658e+02	-5.6181e+02
5	2	-8.4156e+03	-6.7916e+01	9.7943e+01
5	3	8.4206e+03	9.5563e+01	-1.0963e+02
5	4	7.9834e+03	-2.6109e+02	3.0079e+02

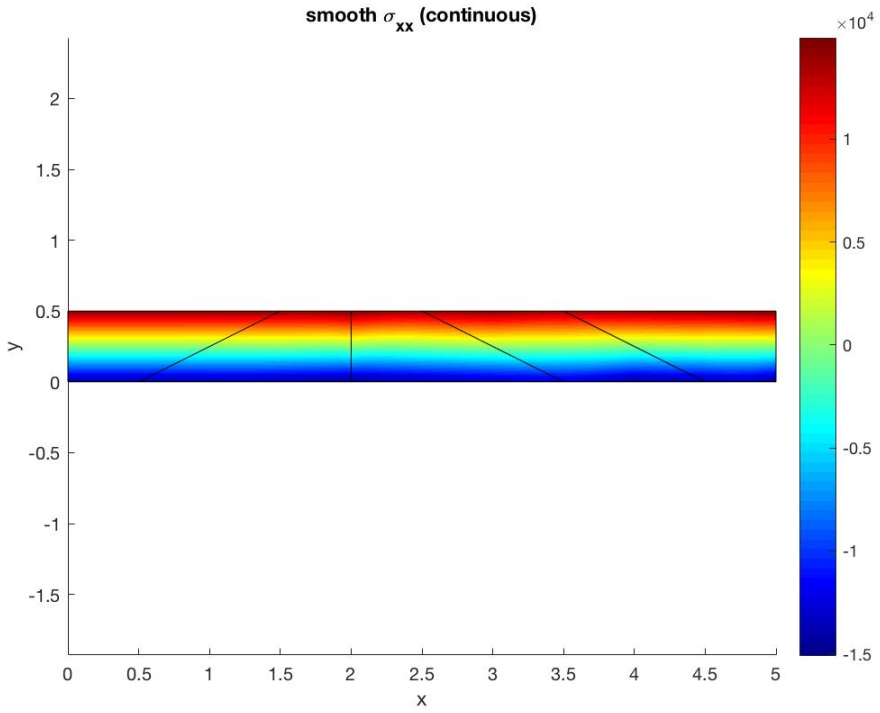
Element nodal Stresses

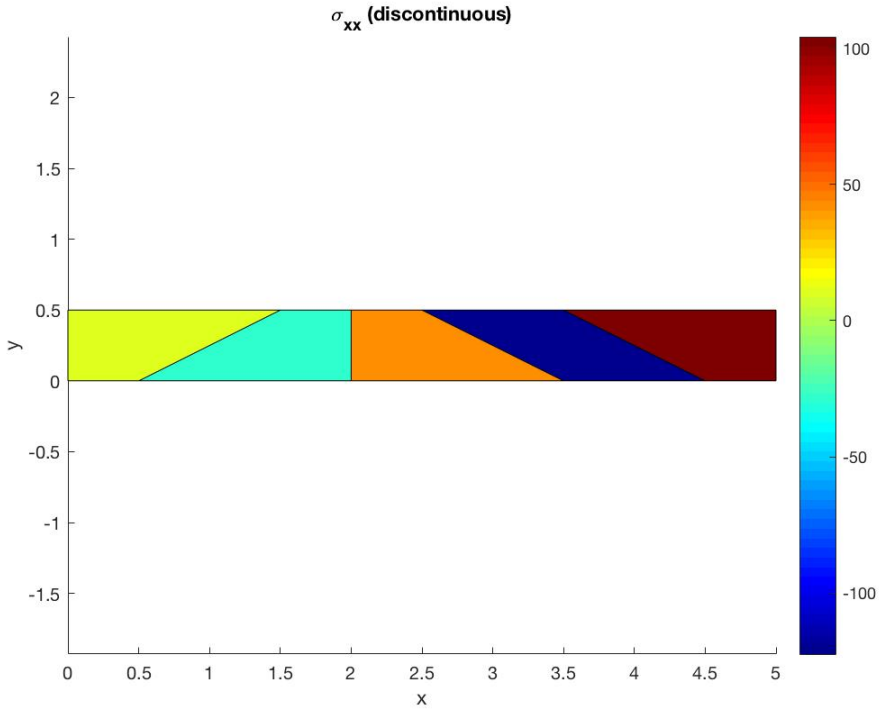
Element	node	Sxx	Syy	Sxy
1	1	-1.4488e+04	-3.1038e+01	-9.6880e+01
1	2	-1.4426e+04	1.2359e+02	9.0608e+01
1	3	1.4465e+04	-1.1861e+02	-6.9473e+01
1	4	1.4485e+04	6.9558e+01	1.0806e+02
1	5	-1.4457e+04	4.6275e+01	-3.1364e+00
1	6	1.9476e+01	2.4883e+00	1.0567e+01
1	7	1.4475e+04	-2.4527e+01	1.9294e+01
1	8	-1.6589e+00	1.9260e+01	5.5906e+00
2	1	-1.4449e+04	-2.9104e+01	-4.6301e+00
2	2	-1.4320e+04	-1.2748e+02	-7.5357e+01
2	3	1.4412e+04	3.7954e+02	2.0532e+02
2	4	1.4250e+04	-1.0874e+02	-1.4109e+02
2	5	-1.4385e+04	-7.8294e+01	-3.9993e+01
2	6	4.5948e+01	1.2603e+02	6.4983e+01
2	7	1.4331e+04	1.3540e+02	3.2115e+01
2	8	-9.9775e+01	-6.8922e+01	-7.2861e+01
3	1	-1.5468e+04	-3.1571e+02	6.6441e+02
3	2	-1.2791e+04	8.5384e+02	-1.0964e+03
3	3	1.3112e+04	-6.2893e+02	8.6120e+02
3	4	1.5318e+04	-1.0319e+02	-4.4980e+02
3	5	-1.4130e+04	2.6906e+02	-2.1601e+02
3	6	1.6025e+02	1.1246e+02	-1.1761e+02
3	7	1.4215e+04	-3.6606e+02	2.0570e+02
3	8	-7.4972e+01	-2.0945e+02	1.0730e+02
4	1	-1.3732e+04	-3.9011e+02	2.2315e+02
4	2	-1.5793e+04	-5.3819e+02	8.8652e+02
4	3	1.4287e+04	1.6170e+02	-1.3355e+02
4	4	1.4748e+04	6.4419e+02	-7.3131e+02
4	5	-1.4763e+04	-4.6415e+02	5.5483e+02
4	6	-7.5297e+02	-1.8824e+02	3.7648e+02
4	7	1.4518e+04	4.0295e+02	-4.3243e+02
4	8	5.0816e+02	1.2704e+02	-2.5408e+02

5	1	-1.2785e+04	9.7331e+02	-1.2624e+03
5	2	-1.5059e+04	-4.2278e+02	5.5878e+02
5	3	1.4915e+04	3.9998e+02	-4.7920e+02
5	4	1.3345e+04	-7.5738e+02	9.1012e+02
5	5	-1.3922e+04	2.7526e+02	-3.5181e+02
5	6	-7.1978e+01	-1.1402e+01	3.9790e+01
5	7	1.4130e+04	-1.7870e+02	2.1546e+02
5	8	2.8031e+02	1.0797e+02	-1.7614e+02

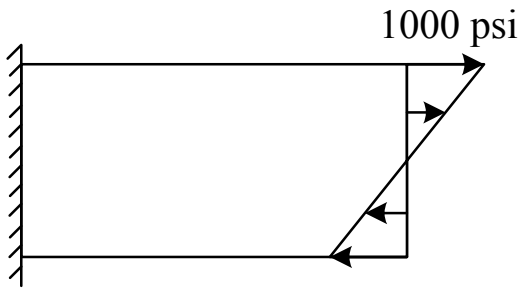
Average nodal Stresses

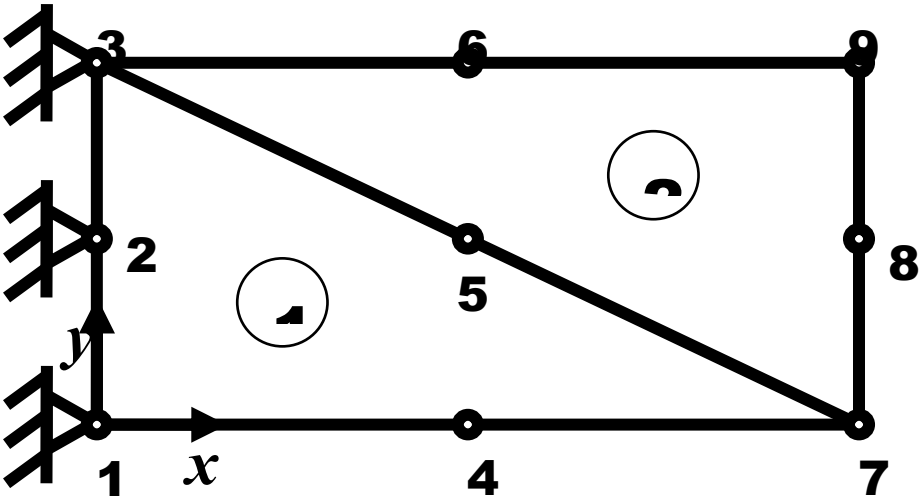
node	Sxx	Syy	Sxy
1	-1.4488e+04	-3.1038e+01	-9.6880e+01
2	-1.4457e+04	4.6275e+01	-3.1364e+00
3	-1.4438e+04	4.7242e+01	4.2989e+01
4	-1.4385e+04	-7.8294e+01	-3.9993e+01
5	-1.4894e+04	-2.2160e+02	2.9453e+02
6	-1.4130e+04	2.6906e+02	-2.1601e+02
7	-1.3262e+04	2.3187e+02	-4.3664e+02
8	-1.4763e+04	-4.6415e+02	5.5483e+02
9	-1.4289e+04	2.1756e+02	-1.8795e+02
10	-1.3922e+04	2.7526e+02	-3.5181e+02
11	-1.5059e+04	-4.2278e+02	5.5878e+02
12	-1.6589e+00	1.9260e+01	5.5906e+00
13	-4.0149e+01	-3.3217e+01	-3.1147e+01
14	-1.4512e+01	-4.1713e+01	8.6143e+01
15	3.3421e+02	1.1975e+02	-1.8585e+02
16	-2.3633e+02	-4.0138e+01	1.0017e+02
17	-7.1978e+01	-1.1402e+01	3.9790e+01
18	1.4485e+04	6.9558e+01	1.0806e+02
19	1.4475e+04	-2.4527e+01	1.9294e+01
20	1.4357e+04	-1.1368e+02	-1.0528e+02
21	1.4331e+04	1.3540e+02	3.2115e+01
22	1.4865e+04	1.3817e+02	-1.2224e+02
23	1.4215e+04	-3.6606e+02	2.0570e+02
24	1.3930e+04	7.6306e+00	6.4948e+01
25	1.4518e+04	4.0295e+02	-4.3243e+02
26	1.3816e+04	-2.9784e+02	3.8829e+02
27	1.4130e+04	-1.7870e+02	2.1546e+02
28	1.4915e+04	3.9998e+02	-4.7920e+02


smooth σ_{xx} (continuous)
$\times 10^4$
1
0.5
0
-0.5
-1
-1.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x


σ_{xx} (discontinuous)
100
50
0
-50
-100
2
1.5
1
0.5
0
-0.5
-1
-1.5
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x

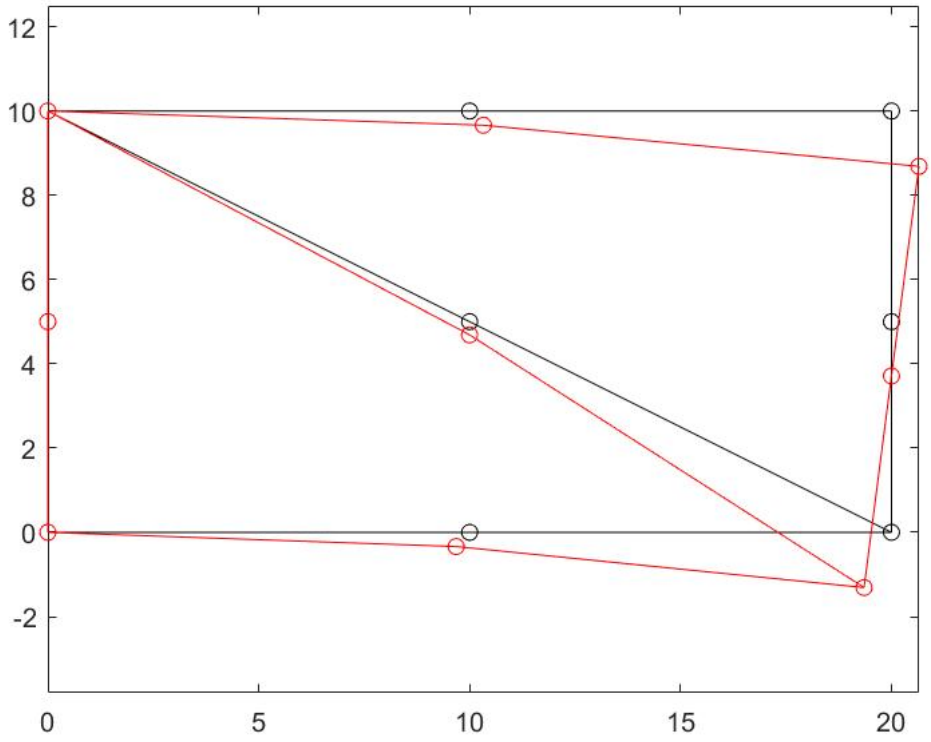
2. (40%) Consider a thin plate subjected to a bending moment. Discretize the model with 2 T6 elements shown below. Use the isoparametric formulation and three-point integration. Calculate the external force matrix by hands and specify their values at the corresponding nodes directly. Report the displacements and plot the undeformed and deformed configurations.



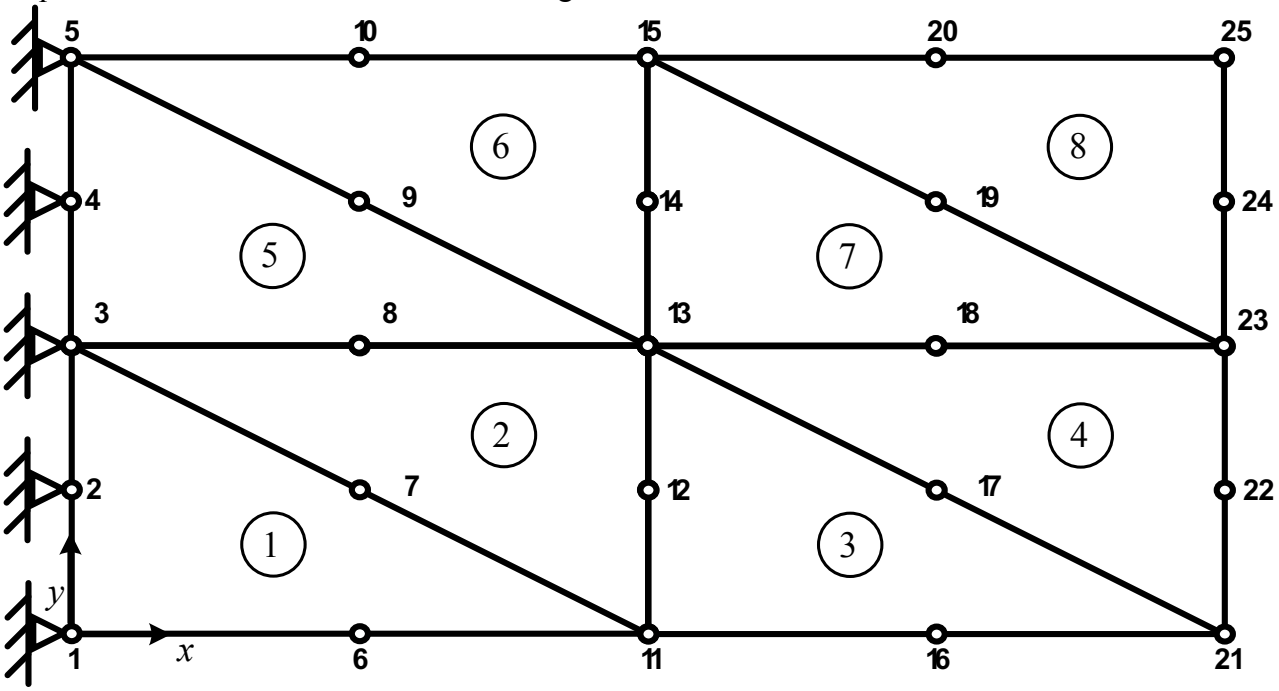


Below are the sample output and the sample plot:

```
Displacements
Node          UX          UY
   1   0.0000e+00    0.0000e+00
   2   0.0000e+00    0.0000e+00
   3   0.0000e+00    0.0000e+00
   4   -3.2109e-04    -3.3718e-04
   5   -2.3449e-06    -3.1243e-04
   6   3.2553e-04    -3.3665e-04
   7   -6.4951e-04    -1.3105e-03
   8   -3.9366e-07    -1.2878e-03
   9   6.5157e-04    -1.3135e-03
```



3. **Bonus (20%):** Consider the same thin plate. Discretize the model with 8 elements shown below. Use the isoparametric formulation and three-point integration. Calculate the external force matrix by hands and specify their values at the corresponding nodes directly. Report the displacements and plot the undeformed and deformed configurations.



Below are the sample output and the sample plot:

Displacements

Node	UX	UY
1	0.0000e+00	0.0000e+00
2	0.0000e+00	0.0000e+00
3	0.0000e+00	0.0000e+00
4	0.0000e+00	0.0000e+00
5	0.0000e+00	0.0000e+00
6	-1.5986e-04	-9.6298e-05
7	-8.0065e-05	-7.8171e-05
8	-9.0874e-07	-7.2163e-05
9	7.8939e-05	-7.7937e-05
10	1.6386e-04	-9.6280e-05
11	-3.2805e-04	-3.4093e-04
12	-1.6345e-04	-3.2233e-04
13	1.0228e-06	-3.1641e-04
14	1.6350e-04	-3.2341e-04
15	3.2665e-04	-3.4241e-04
16	-4.9396e-04	-7.5224e-04
17	-2.4672e-04	-7.3348e-04
18	4.2573e-07	-7.2713e-04
19	2.4707e-04	-7.3338e-04
20	4.9375e-04	-7.5216e-04
21	-6.6040e-04	-1.3294e-03
22	-3.3013e-04	-1.3107e-03
23	8.1776e-08	-1.3044e-03
24	3.3039e-04	-1.3105e-03
25	6.6067e-04	-1.3293e-03