National Taiwan University **Finite Element Method**

Department of Civil Engineering Instructor: C.-S. CHEN

Homework 5, 04/11/2019 Due: 04/17/2019

A4 professional format, collecting at the BEGINNING of class (09:09 am)

(late submission within 24 hours: score*0.9; late submission before post of solution: score*0.8

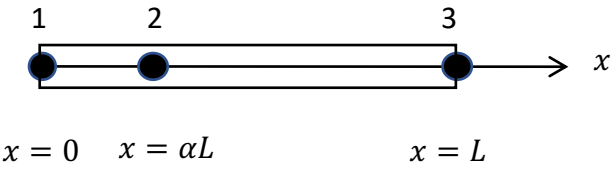
(the solution will be posted usually within a week))

Total 100%

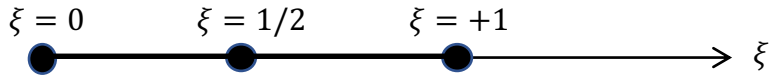
1. **(10%)** Evaluate the integral $\int_3^7 \frac{1}{1.1+x}dx$ by hand using three Gauss points.

2. **(60%)** Theory of linear elastic fracture mechanics tells us that stresses near a crack tip are inversely proportional to $\sqrt{x}$, where is the distance from a crack tip. It happens that we can accomplish this by simply placing the mid-side nodes of a quadratic element to their quarter points. A 1D element shown in following figure is the best to illustrate the singular behavior.
   (a) Derive the mapping relationship between the physical coordinate (Cartesian space) x and the parametric coordinate ξ. That is, when $x_1^e = 0,\ x_2^e = \alpha L,\ x_3^e = L$.
   (b) Let us first consider a standard 1D quadratic element with a mid-side node ($\alpha = \frac{1}{2}$). Show the expected linear expression for the strain $\frac{du}{dx}$ using the isoparametric formulation.
   (c) Now if we move the mid-side node at the Cartesian space only to the quarter point ($\alpha = \frac{1}{4}$). Show that we now have a desirable singular expression for the strain $\frac{du}{dx}$ using the isoparametric formulation.

Physical element

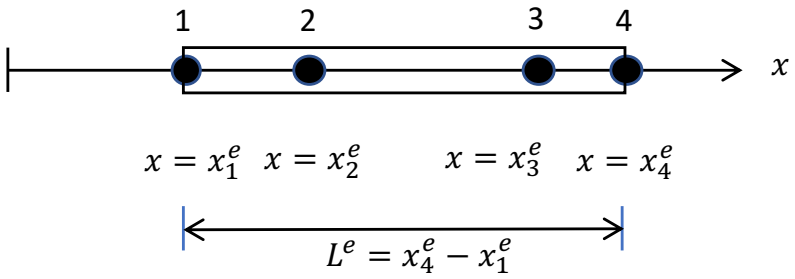


Mapped element



3. (30%) Consider a cubic element below. (a) Derive the Jacobian and (b) show that when the nodes in the physical space are equally distributed, the Jacobian reduces to $\frac{L^e}{2}$.

Ph ical elemen



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