National Taiwan University **Finite Element Method**

Department of Civil Engineering Instructor: C.-S. CHEN

Homework 1, 02/27/2019 Due: 03/06/2019

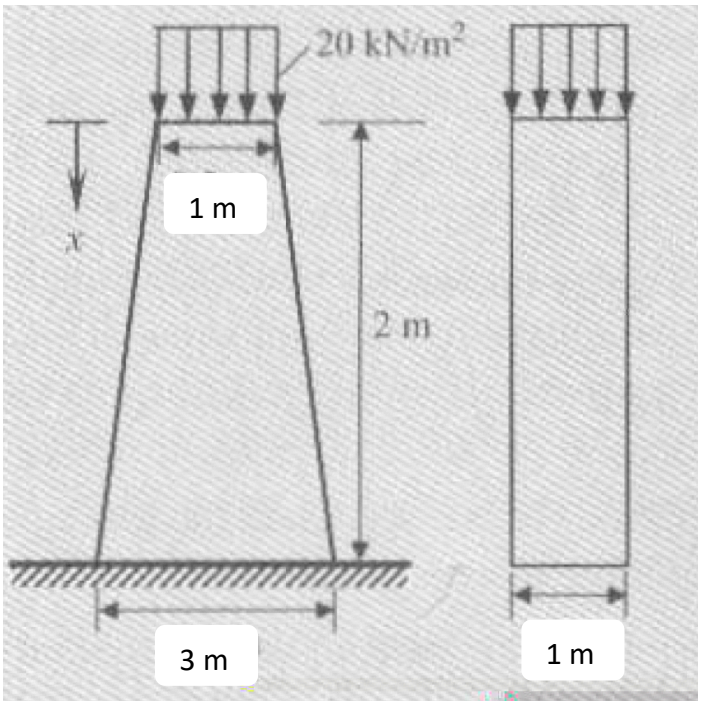
A4 professional format, collecting at the BEGINNING of class (09:09 am)

(late submission within 24 hours: score*0.9; late submission before post of solution: score*0.8

(the solution will be posted usually within a week))

Total 80%

1. **(50%)** A bridge is supported by several concrete piers, and the geometry and loads of a typical pier are shown in the figure below. The load $20\ \text{kN/m}^2$ represent the weight of bridge and a distribution of the traffic on the bridge. The concrete weighs $24\ \text{kN/m}^3$ and its modulus is $E = 2\ \ 10^7\ \text{kN/m}^2$. We wish to analyze the pier for displacements and stresses.
   (a) (8%) Derive the strong form (governing equation and boundary conditions).
   (b) (8%) Use what you have learned from Engineering Mathematics and derive the exact solutions of displacements and stresses for the problem.
   (c) (8%) Derive the weak form with admissible trial solutions and weight functions.
   (d) (8%) Obtain the displacements and stresses to the weak form by using trial solution and weight function of a linear polynomial.
   (e) (8%) Obtain the displacements and stresses to the weak form by using trial solution and weight function of a quadratic polynomial.
   (f) (10%) Plot the comparisons of exact solutions from (b) with approximate displacements and stresses from (c) and (d).



2. **(30%)** An engineering analysis problem is formulated in terms of the following ordinary differential equation:

$$\frac{d\ u}{dx} = x \qquad < x <$$

$$u \quad = u \quad =$$

(a) (15%) Develop the corresponding weak form from the strong form.

(b) (15%) Obtain the solution to the weak form by using trial solution and weight function of a cubic form:

$$u(x) = x(1 \quad x)(\ _0 + \ _1 x)$$

$$w(x) = x(1-x)(\beta_0 + \beta_1 x)$$