

Tunneling spin and heat transport in ultracold atomic systems

Yuta Sekino, Yuya Ominato, Hiroyuki Tajima, Shun Uchino, & Mamoru Matsuo, arXiv:2312.04280

Yuta Sekino (RIKEN iTHEMS/ RIKEN CPR)



In collaboration with

Yuya Ominato (Waseda), Hiroyuki Tajima (Univ. Tokyo)

Shun Uchino (Waseda), Mamoru Matsuo (UCAS, Beijing)

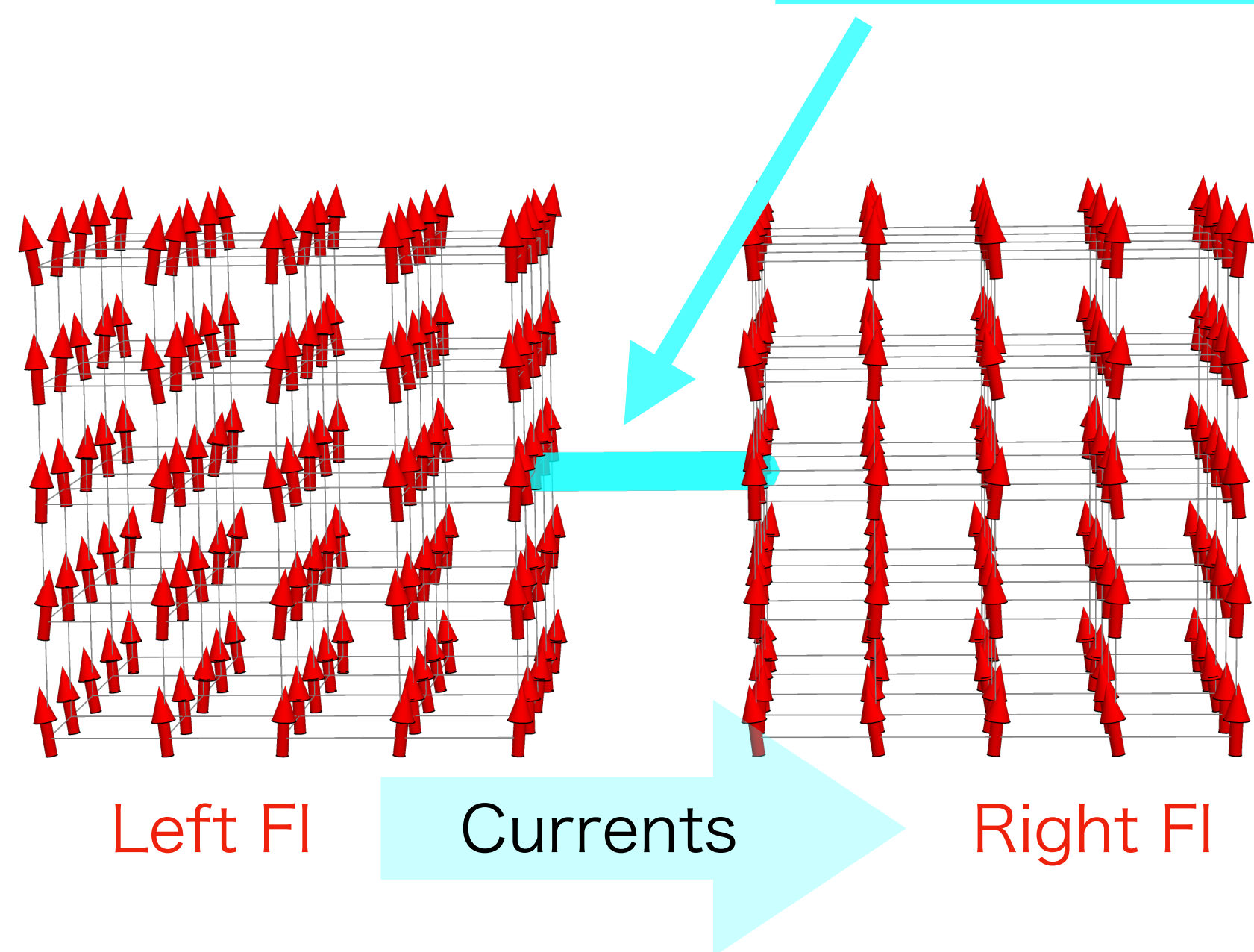
Takehome message: Tunneling transport by criticality

Anomalous tunneling spin and heat transport near magnonic critical points of ferromagnets

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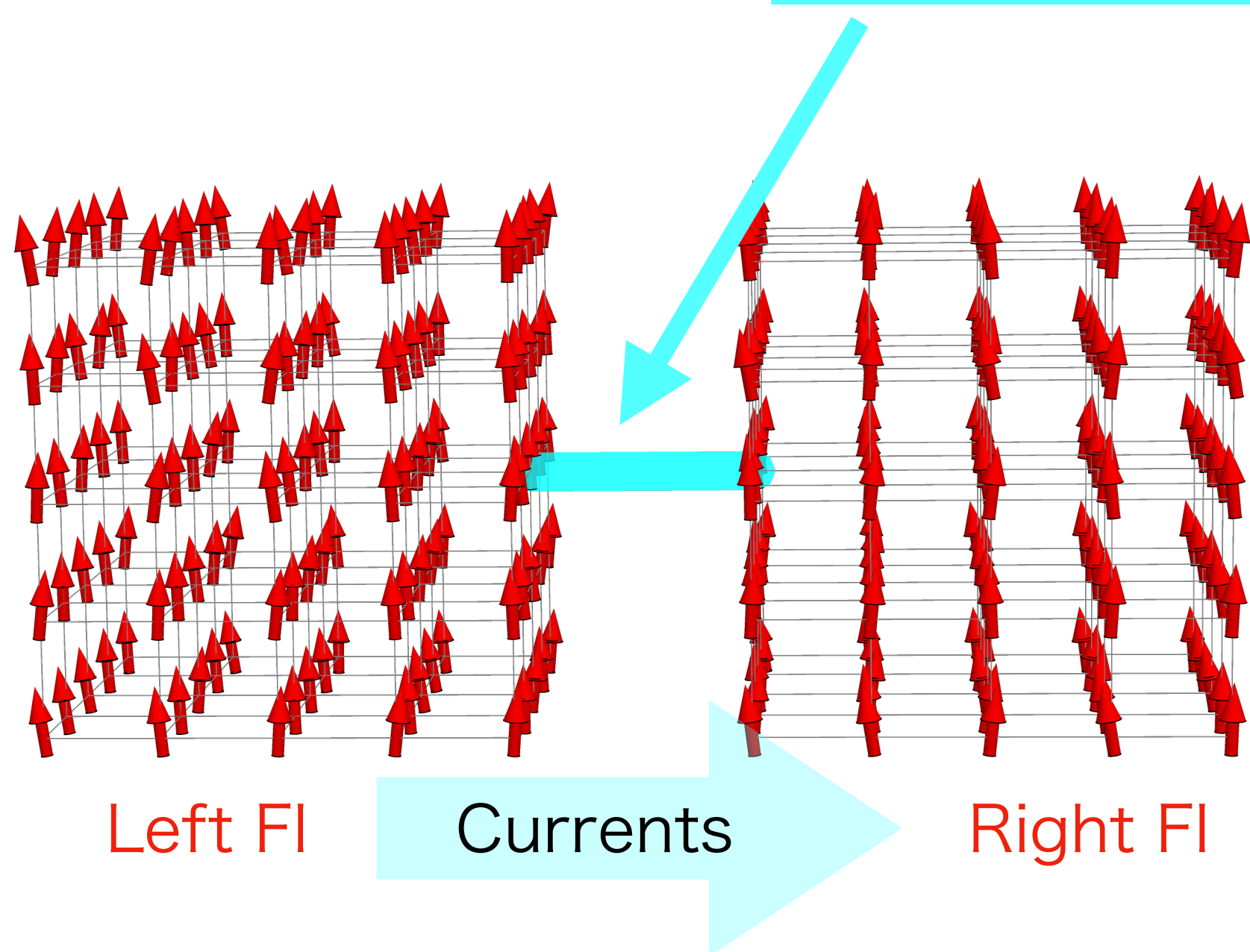
Two ferromagnetic insulators (FIs) realized with **cold atoms** connected via a quantum point contact



Takehome message: Tunneling transport by criticality

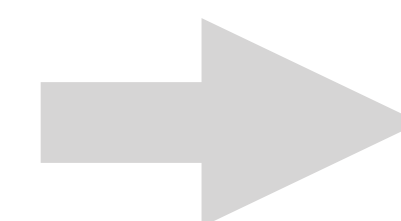
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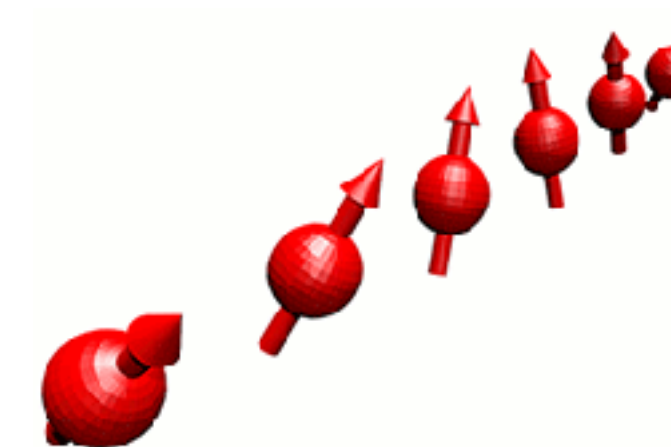
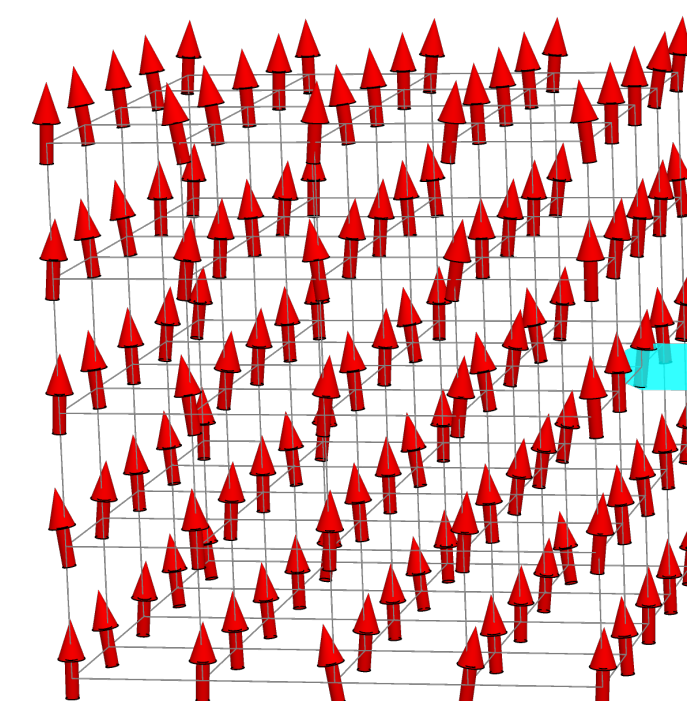


Gapless point of magnons in ferromagnets

Spontaneous breaking
of $O(3)$ symmetry in FIs



Magnon as **gapless Nambu-Goldstone mode**

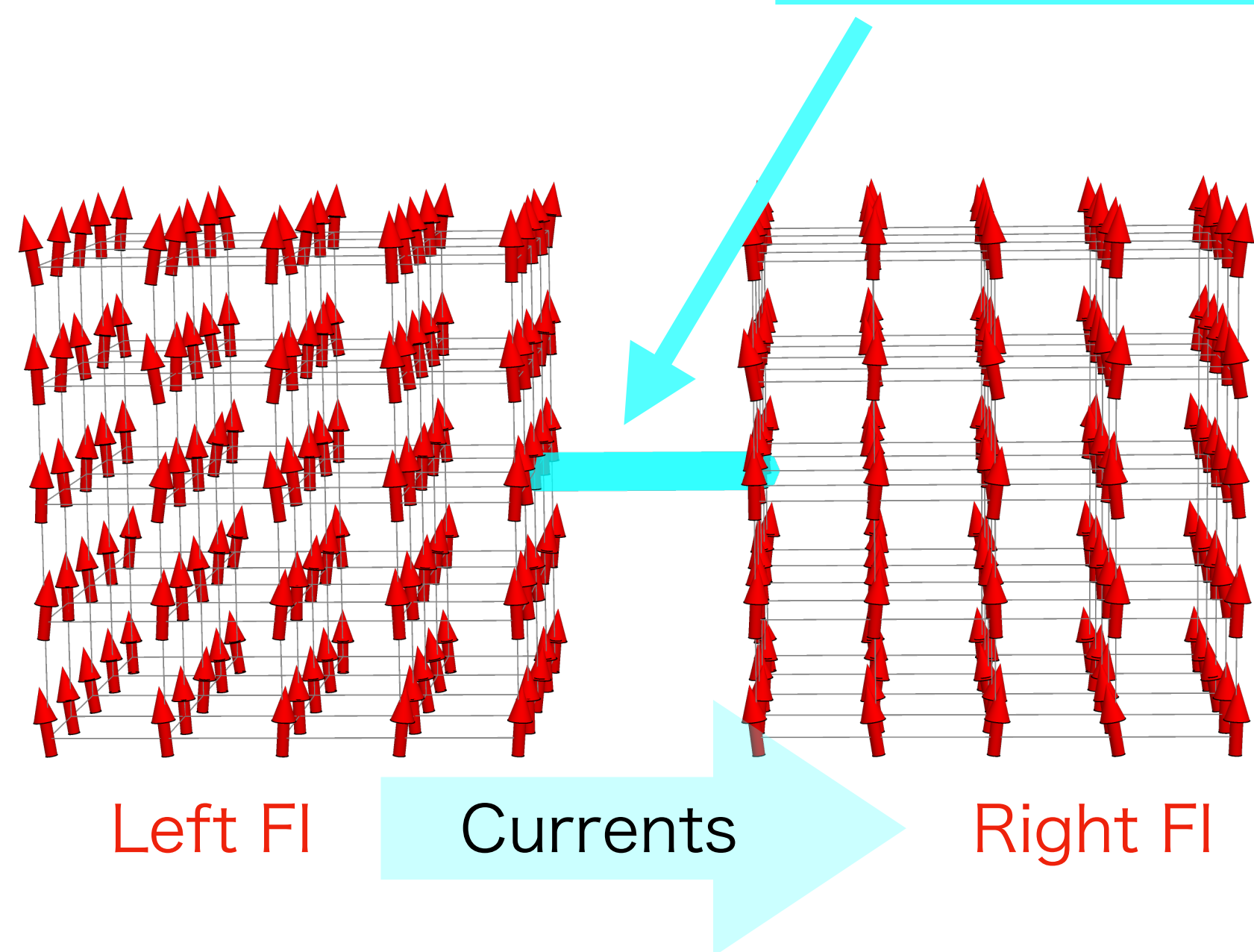


Animation by R. Pradip, KIT.
https://www.ips.kit.edu/2786_Giant-Spin-Phonon-Interaction.php

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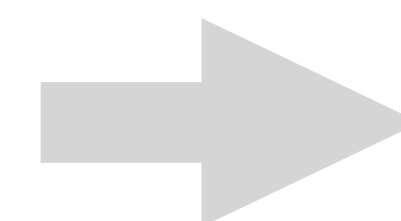
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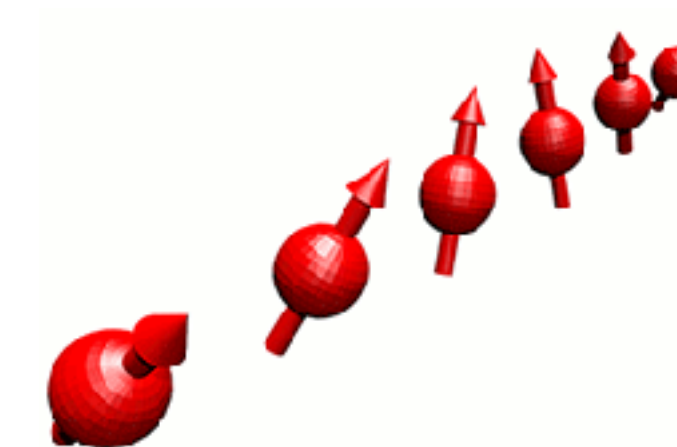
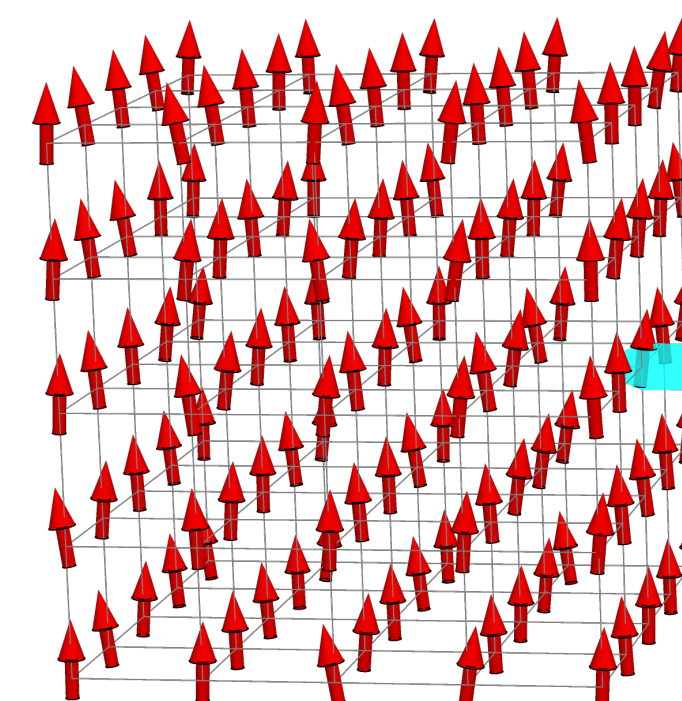


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Anomalous enhancement of spin & heat conductances resulting from **the magnonic criticality**

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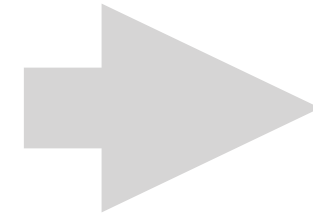
1. Introduction of cold atoms and tunneling transport
2. Magnon and its criticality in ferromagnetic insulators
3. Anomalous tunneling transport of magnons

Introduction of cold atoms and tunneling transport

Introduction: Quantum transport with cold atoms

Cold atoms: Highly controllable many-body systems of atoms

Parameters tunable by lasers & magnetic fields



Various many-body systems

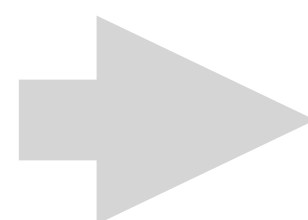
- Interactions b/w atoms
- Spatial dimension
- Lattice structure

- **Ferromagnetic Heisenberg spins**
- Antiferromagnetic Heisenberg spins
- Superfluids of Fermi gases

Introduction: Quantum transport with cold atoms

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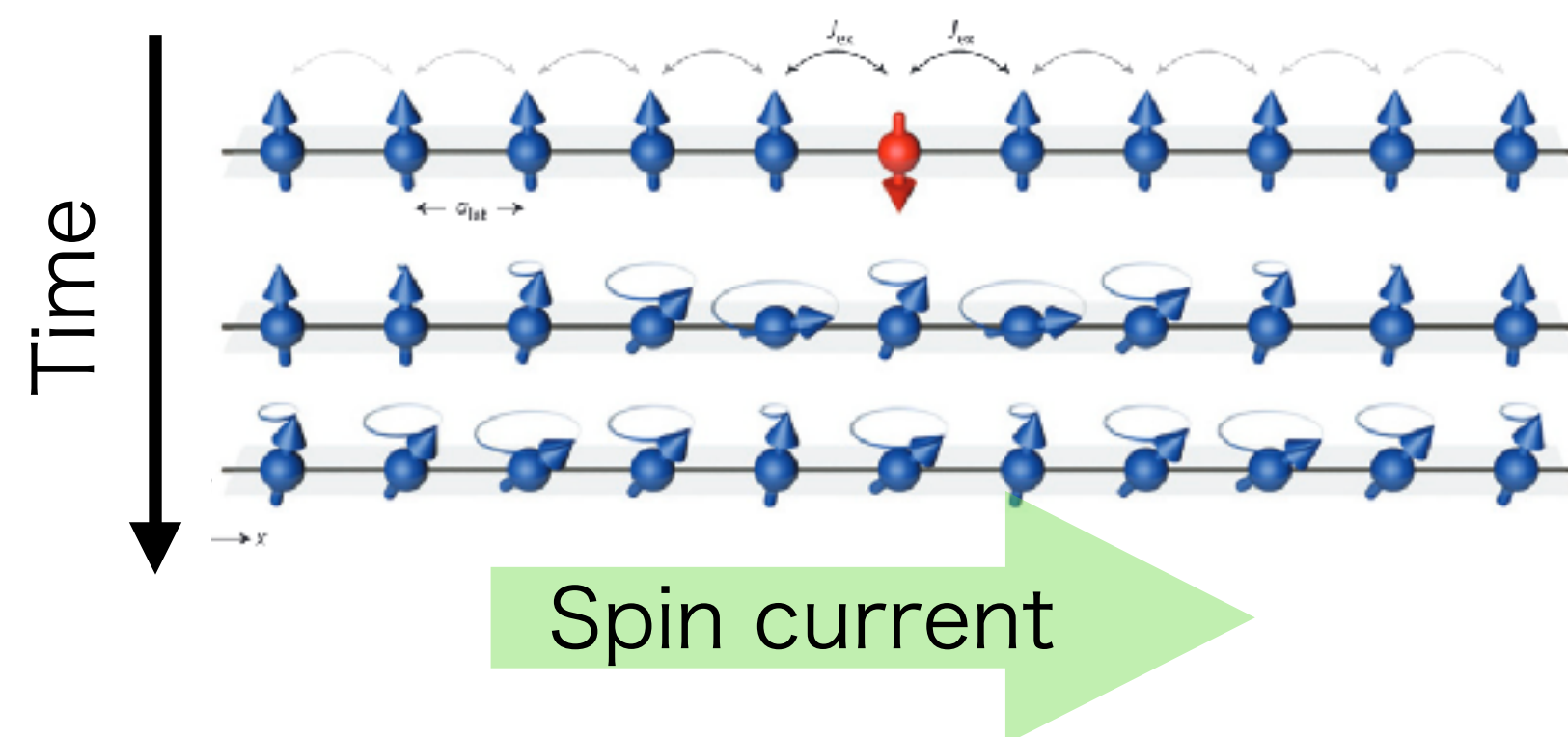


Various many-body systems

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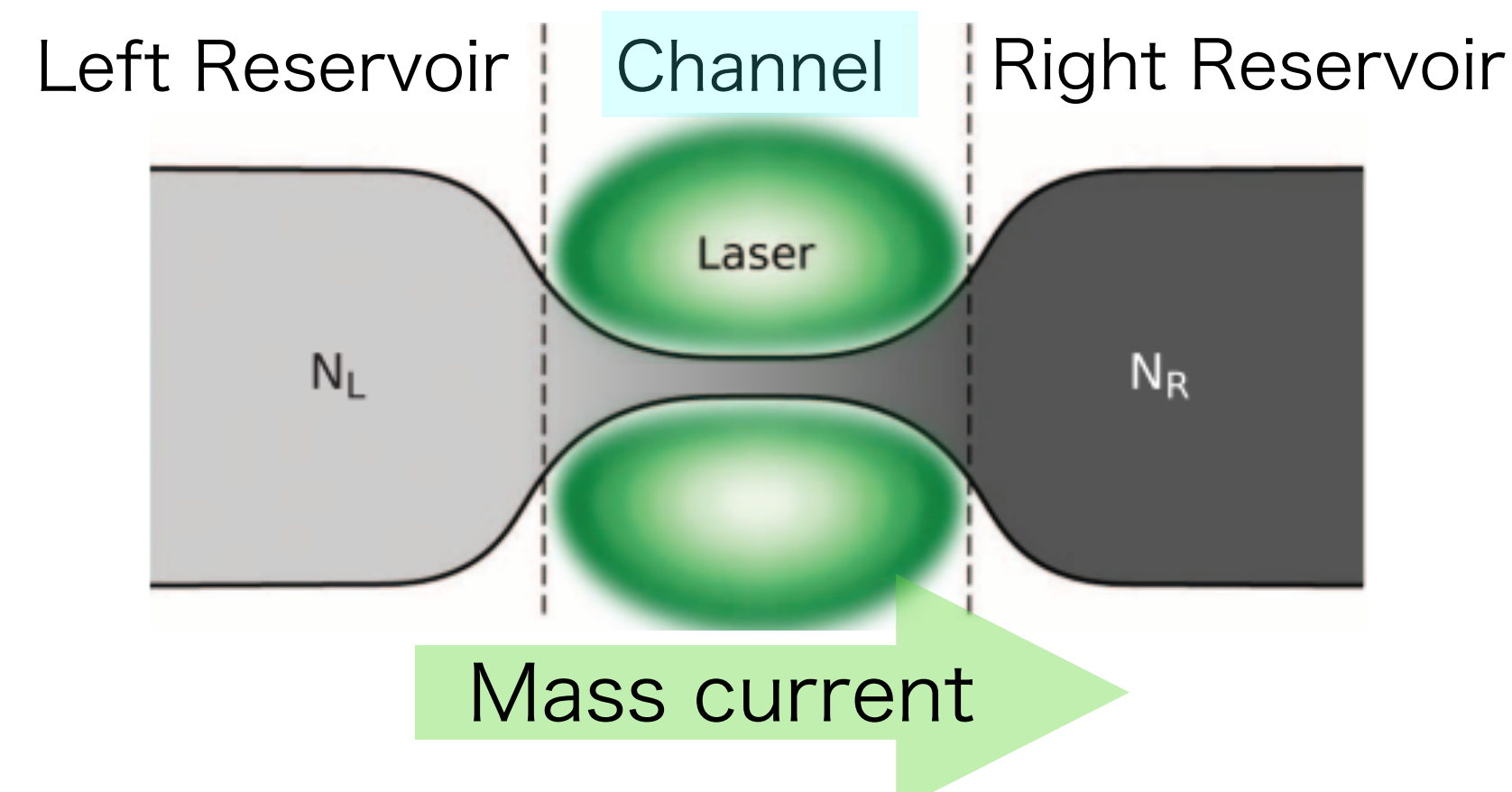
Cold atoms as platforms for **quantum transport**

Bulk spin transport in ferromagnetic Heisenberg spins



MPI: Fukuhara et al., Nat. Phys (2013); Nature (2013);
 Hild et al., PRL (2014); Wei et al., Science (2022)
 MIT: Jepsen et al., Nature (2020); PRX (2021); Nat. Phys. (2022).

Tunneling mass transport b/w Fermi gases



ETH: Brantut et al., Science **337**, 1069-1071 (2012); ...
 LENS: Valtolina et al., Science **350**, 1505-1508 (2015); ...
 Kyoto (w/ synthetic dim): Ono et al., Nat Commun **12**, 6724 (2021)

Extending tunneling transport to spin systems

We propose tunneling transport b/w **quantum magnets** with ultracold atoms

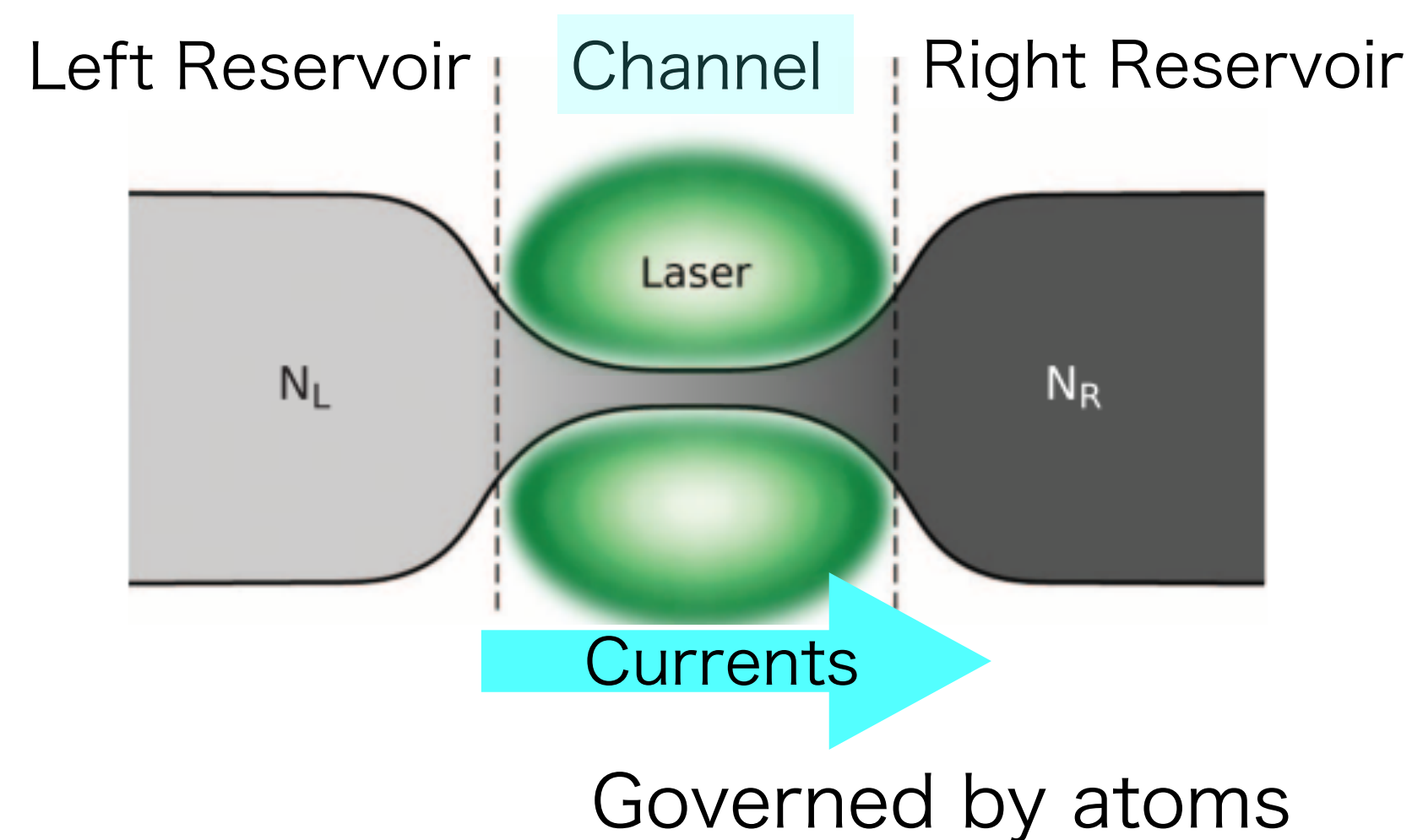
YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

Between Fermi atomic gases

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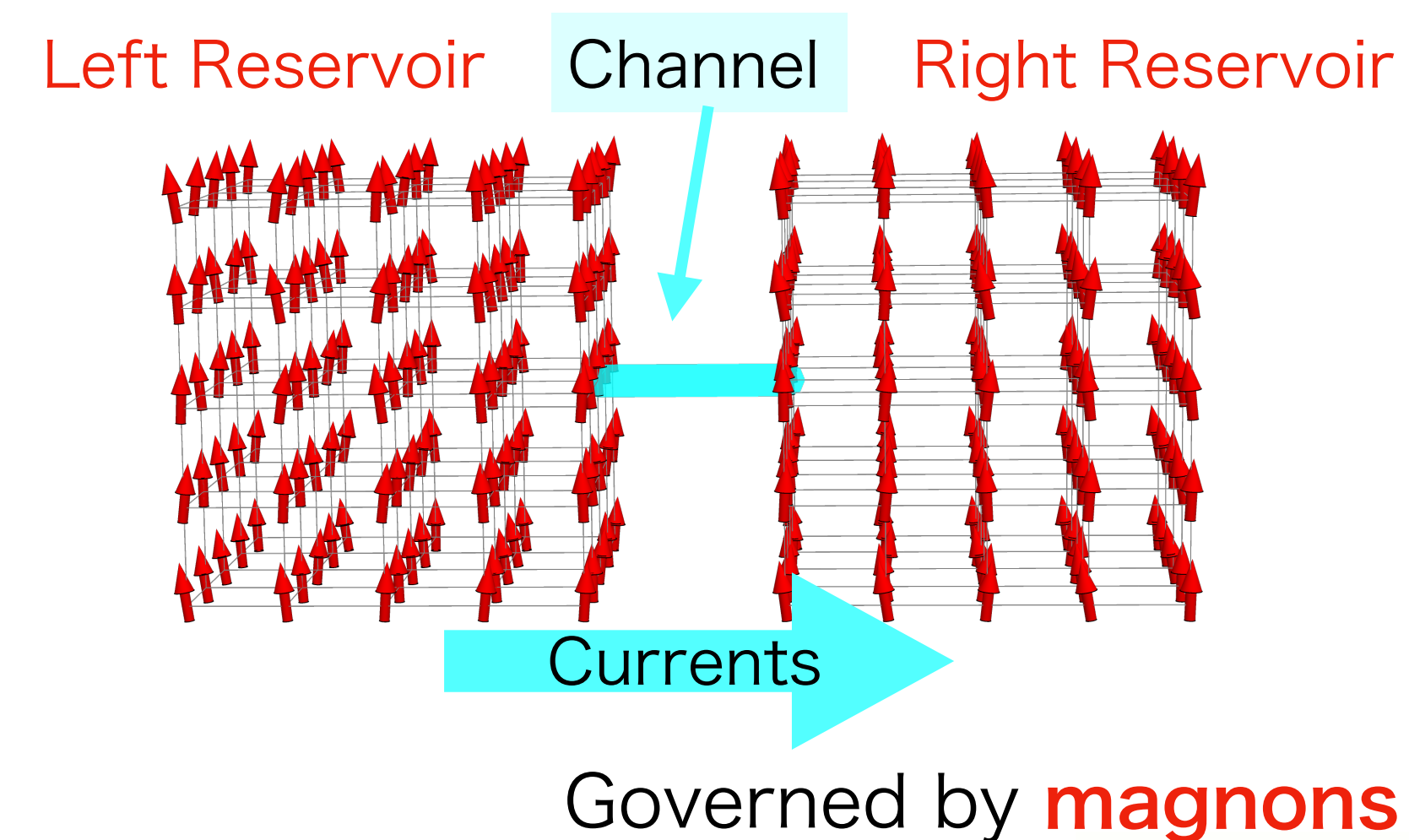
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Between **ferromagnetic Heisenberg spins**

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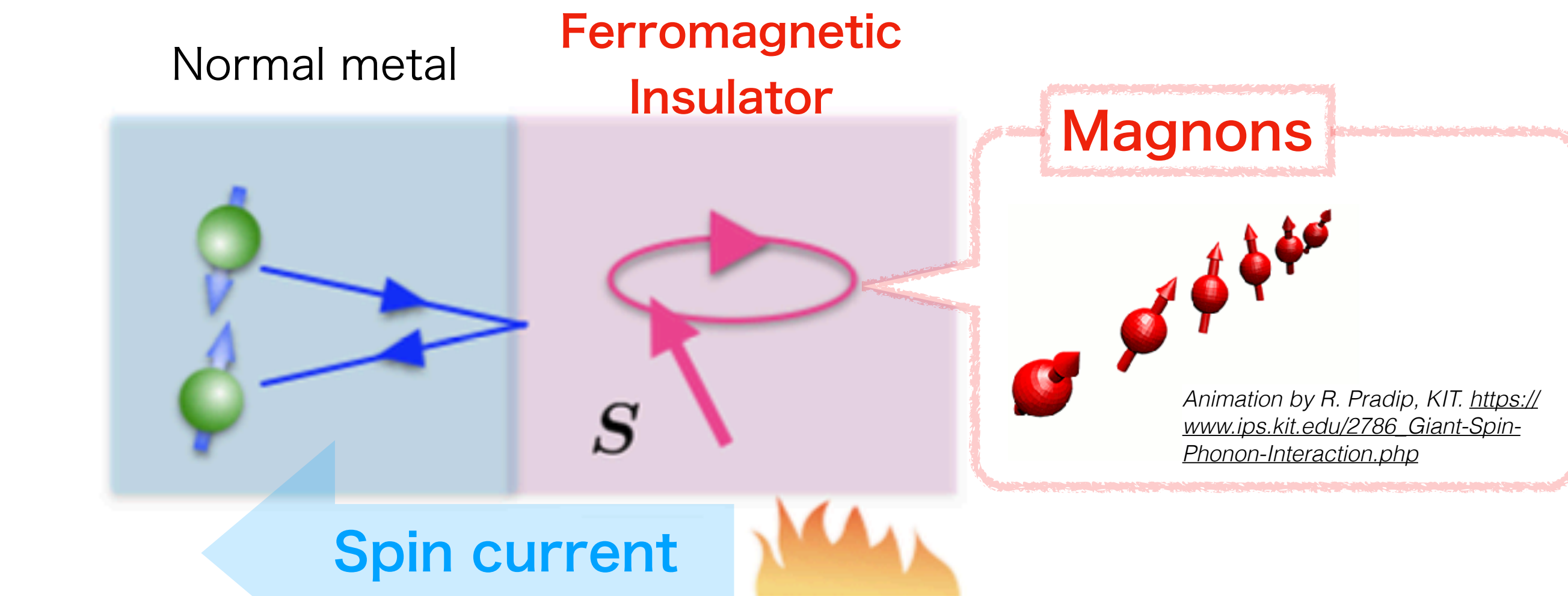
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Motivation from solid-state physics

Spin and **heat** tunneling transport with **magnons** is one of the hot topics in spintronics focusing on efficient **spin-heat** conversion for devices applications

► **Spin Seebeck effect**: **spin-current** generation by **temperature bias ΔT**

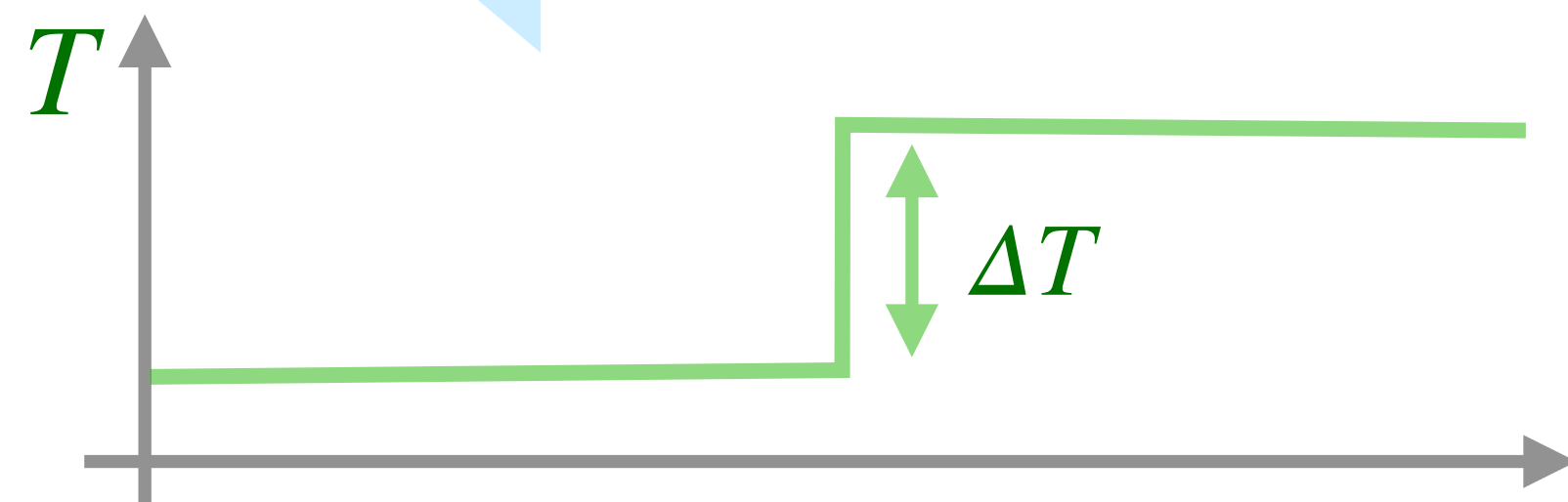


Observation:

Uchida et al., Nature **455**, 778–781 (2008)

Review of spin caloritronics:

Bauer et al., Nature Materials **11**, 391–399 (2012)

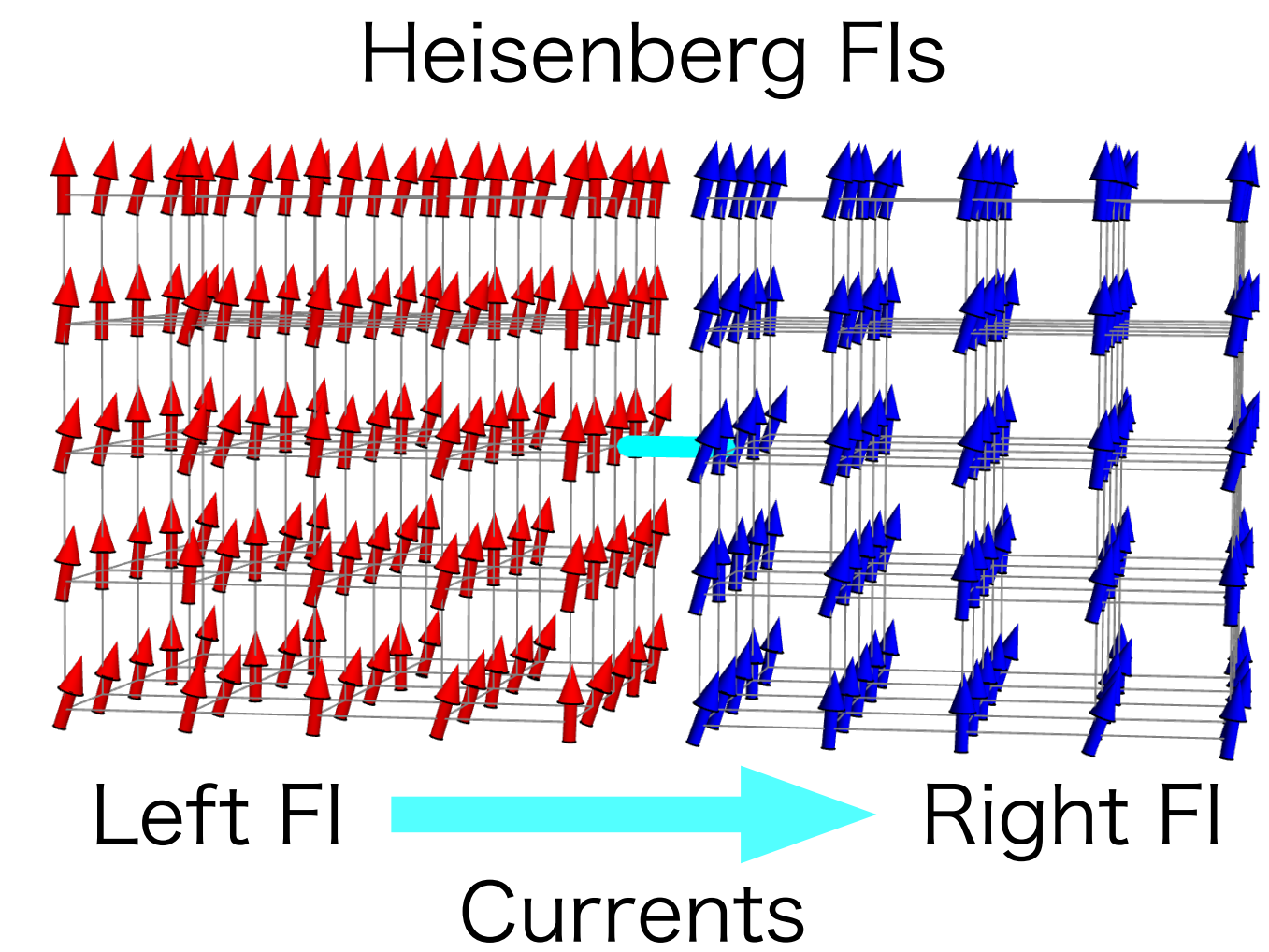


Spin & heat transport is basic, important issues in **both cold-atomic and solid-state physics**

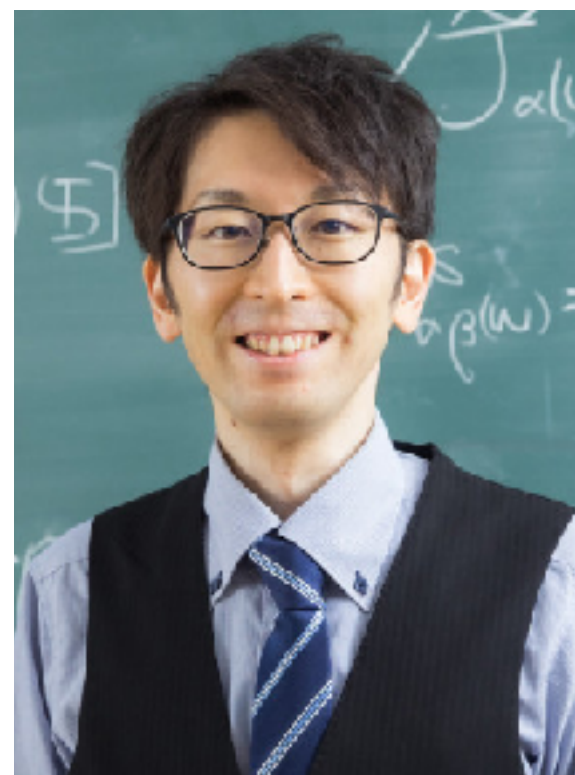
Quantum simulation of magnonic transport

To bridge **cold atoms and spintronics**, we propose tunneling spin & heat transport of magnons by utilizing high controllability of cold atoms

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280



Cold-atomic physics



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RIKEN



H. Tajima
U. Tokyo



S. Uchino
Waseda Univ.



Solid-state physics



Y. Ominato
Waseda Univ.



M. Matsuo
UCAS, China

Why tunneling spin & heat transport w/ cold atoms?

1. Ultraclean systems

- No impurity

- No roughness & lattice mismatch

Why tunneling spin & heat transport w/ cold atoms?

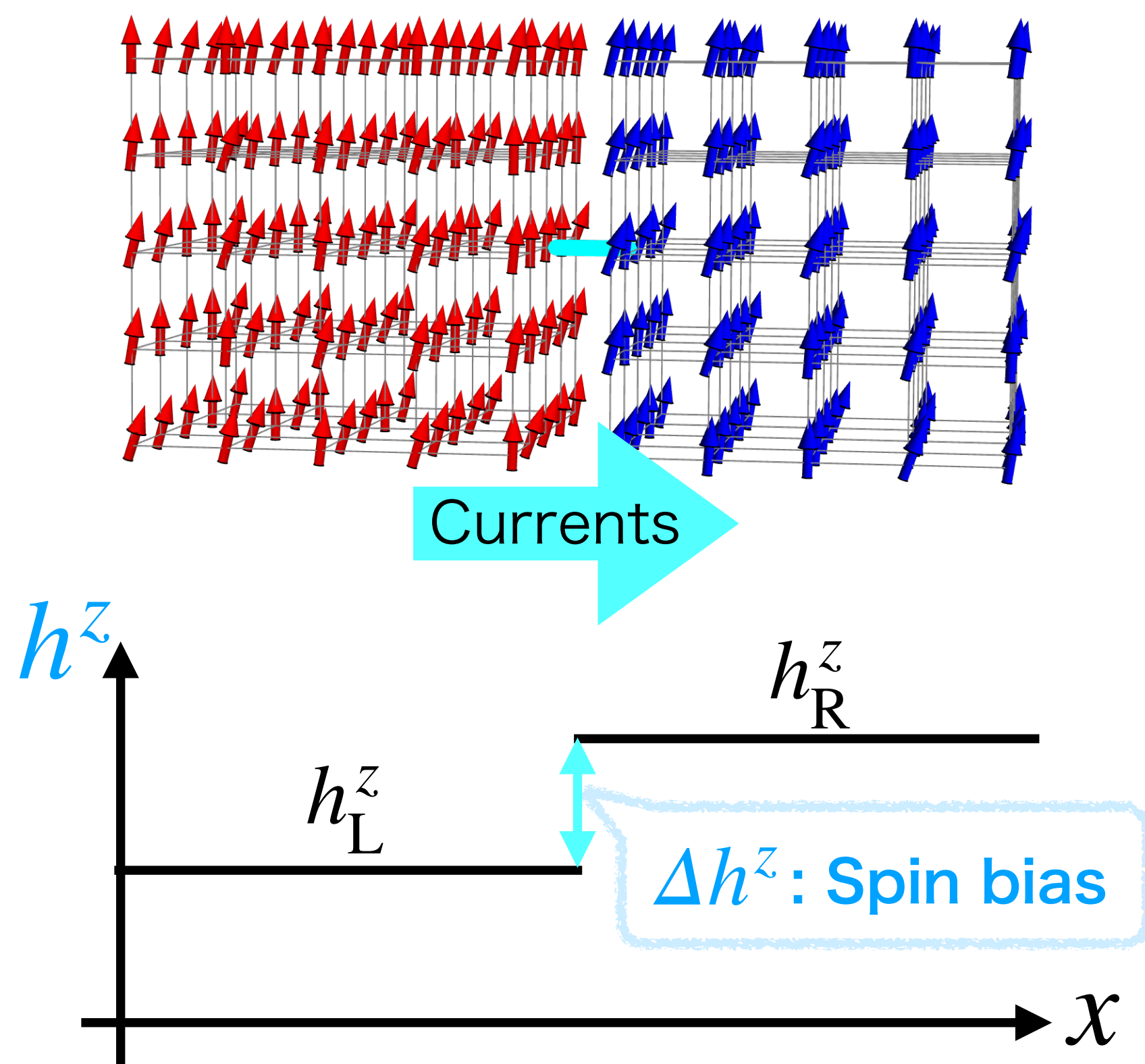
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2. Controllability of **effective Zeeman field** $\vec{h}(\vec{r})$ (= chemical potential of spin)

$$H_{\text{Zeeman}} = - \int d\vec{r} \vec{h}(\vec{r}) \cdot \vec{s}(\vec{r})$$



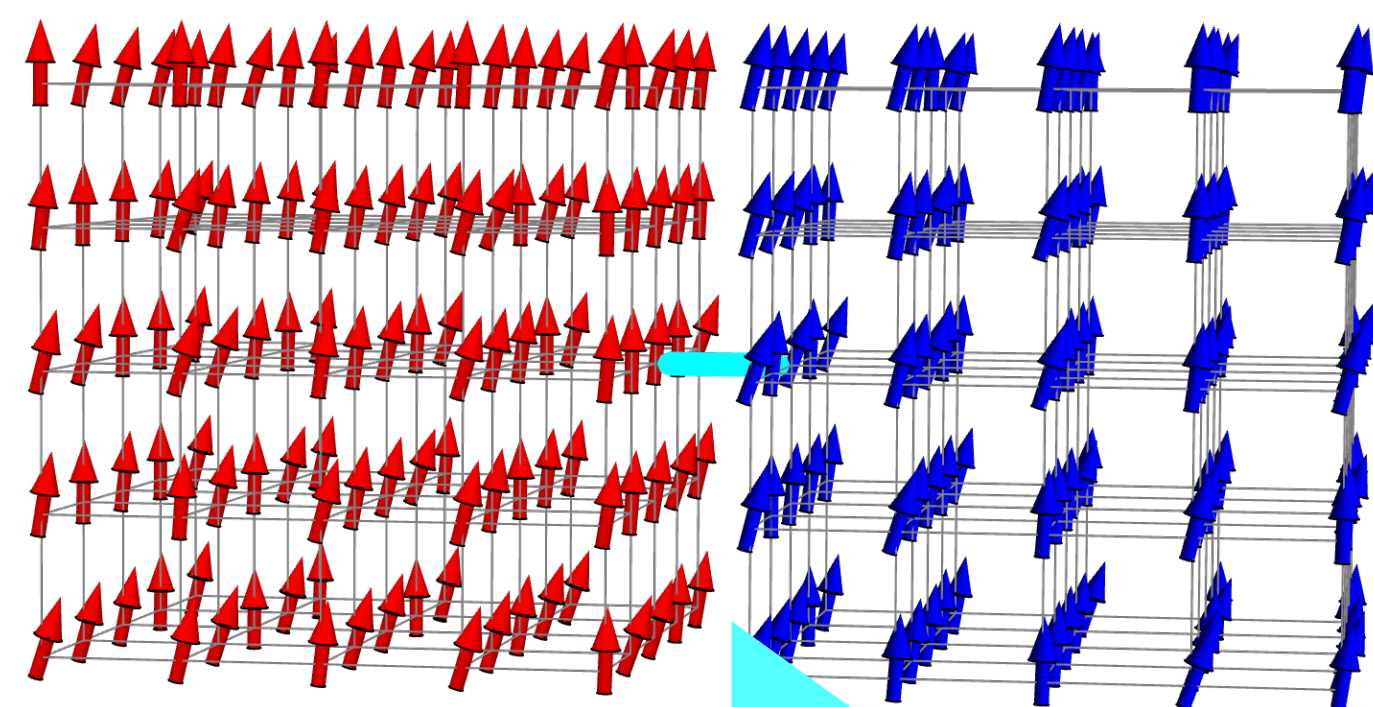
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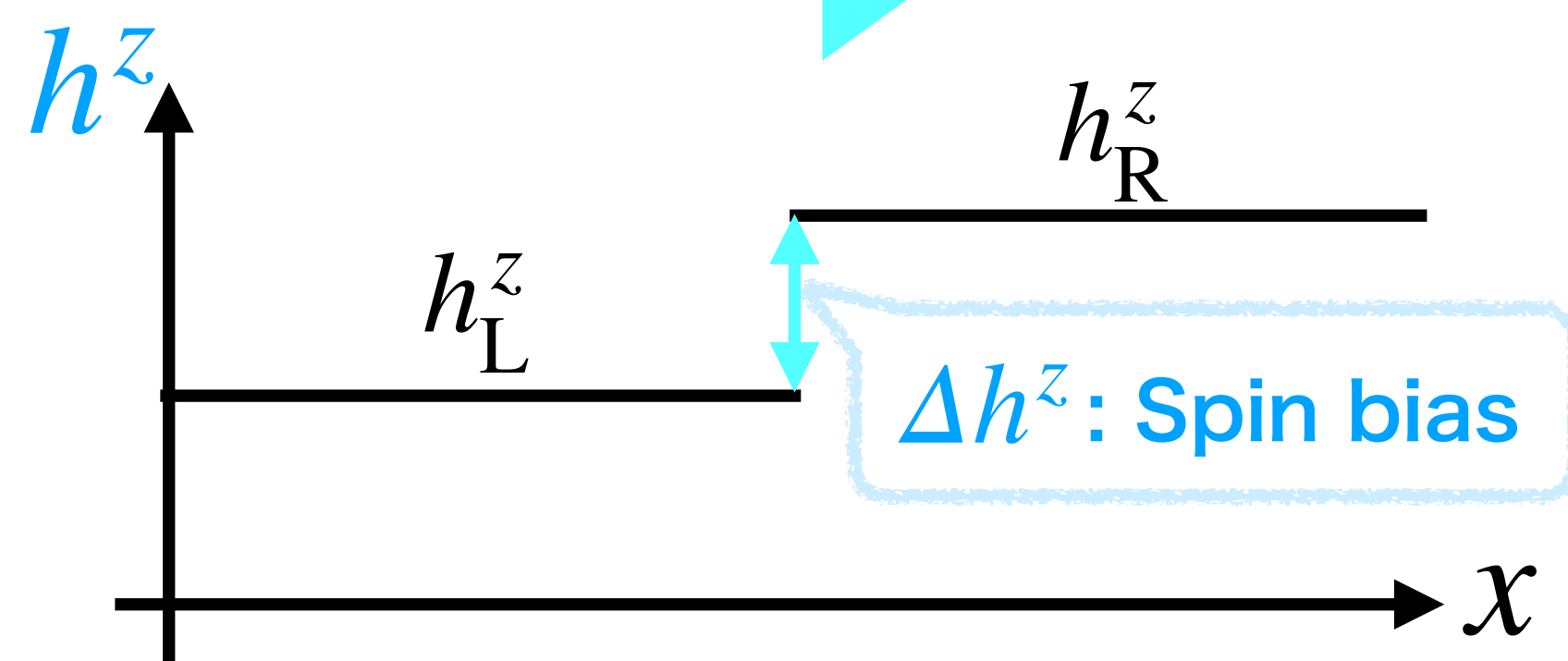
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Solid-state experiments:

Generation of Δh is **challenging**

w/ spatially modulated magnetic fields

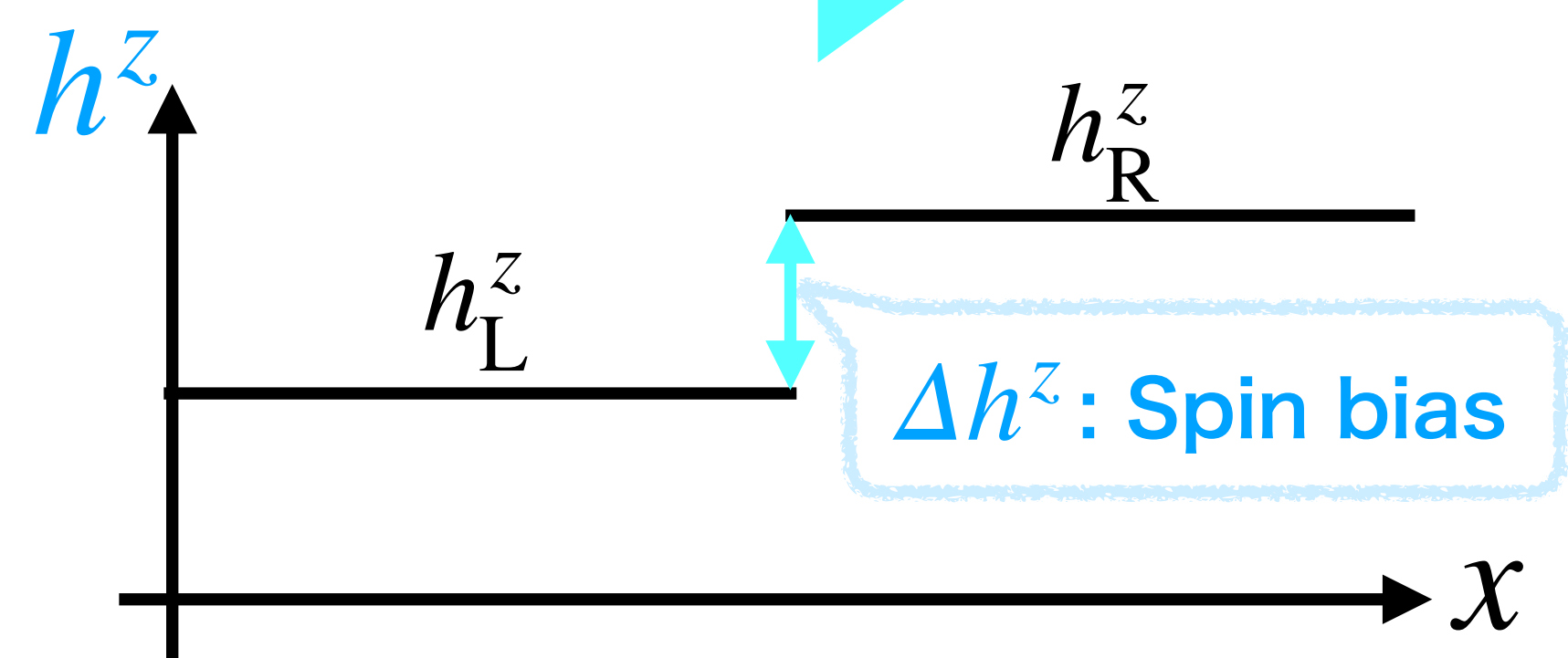
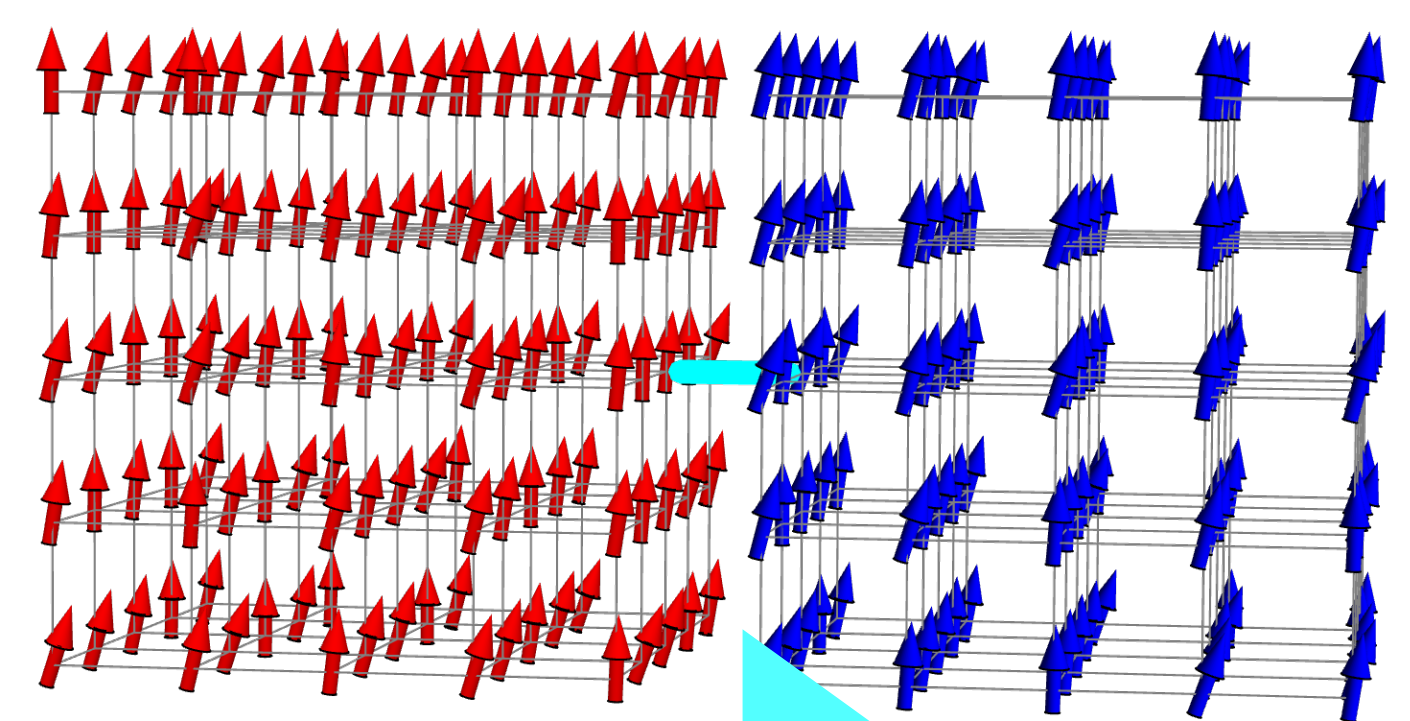
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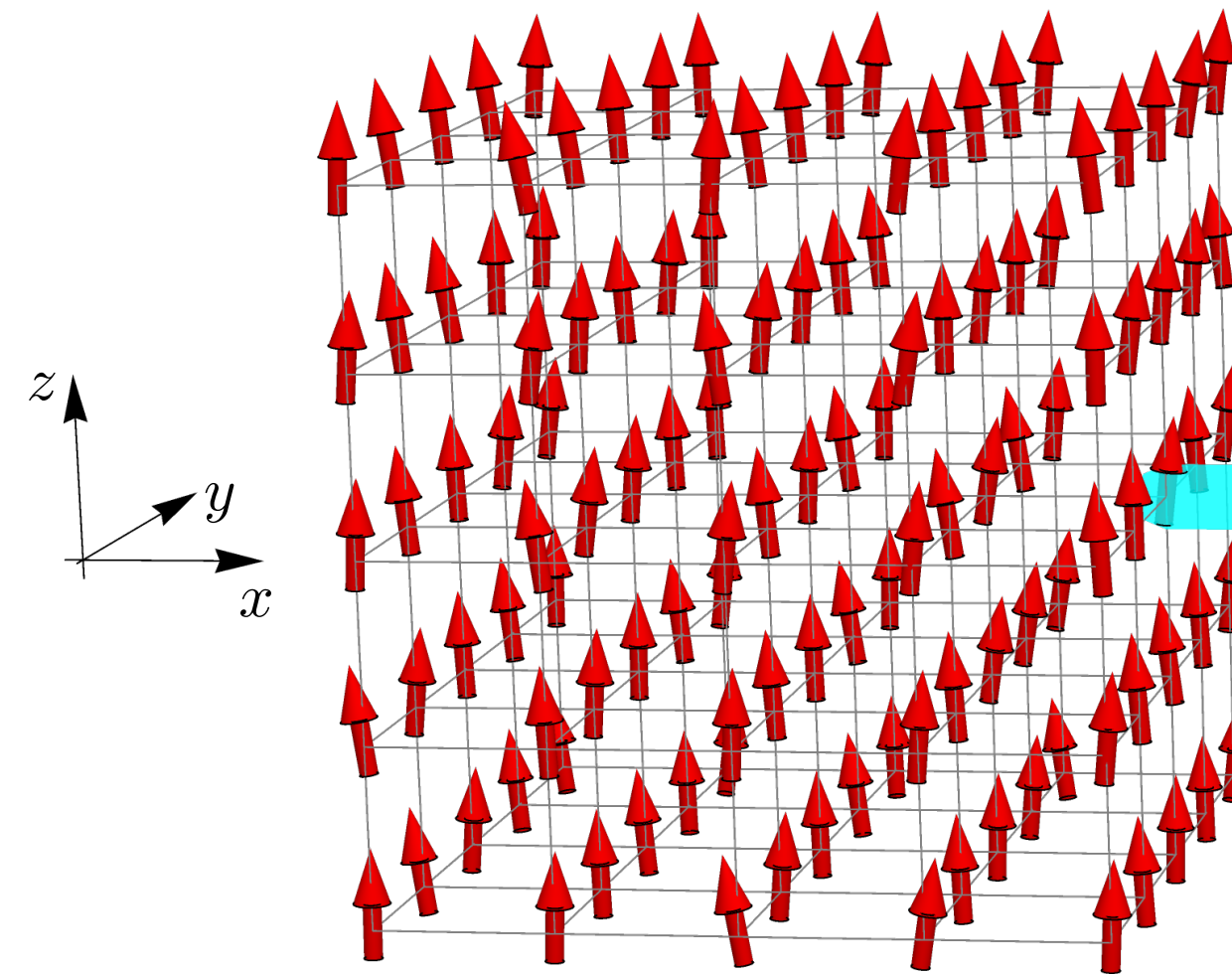
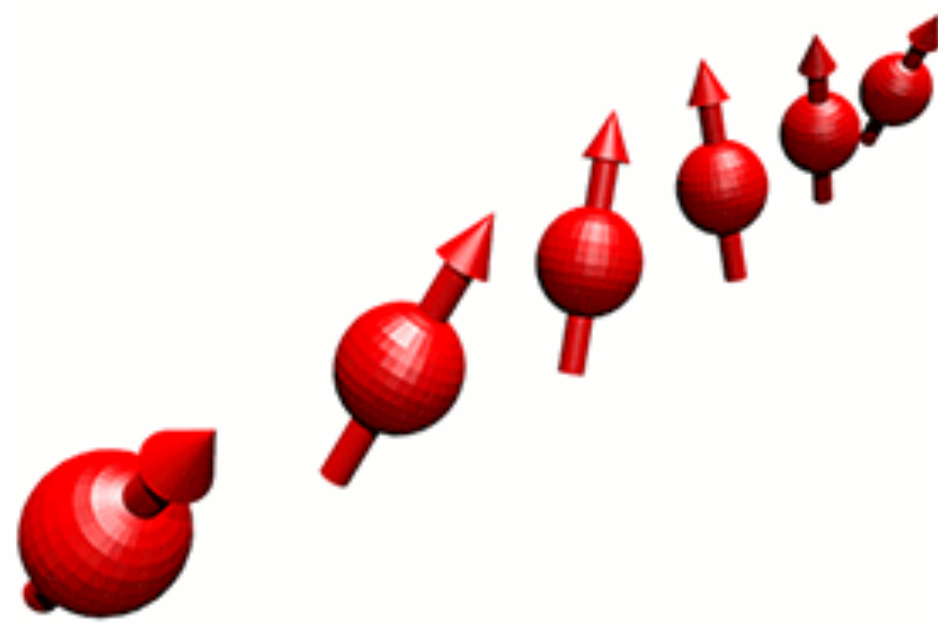
Generation of Δh is **challenging**
w/ spatially modulated magnetic fields

Cold-atom experiments:

Controllable, effective Δh has been **achieved** by directly manipulating magnetization M_L^z, M_R^z of each reservoir

For Fermi gases
[Kriner et al., PNAS, **113** (29) 8144-8149 (2016)]

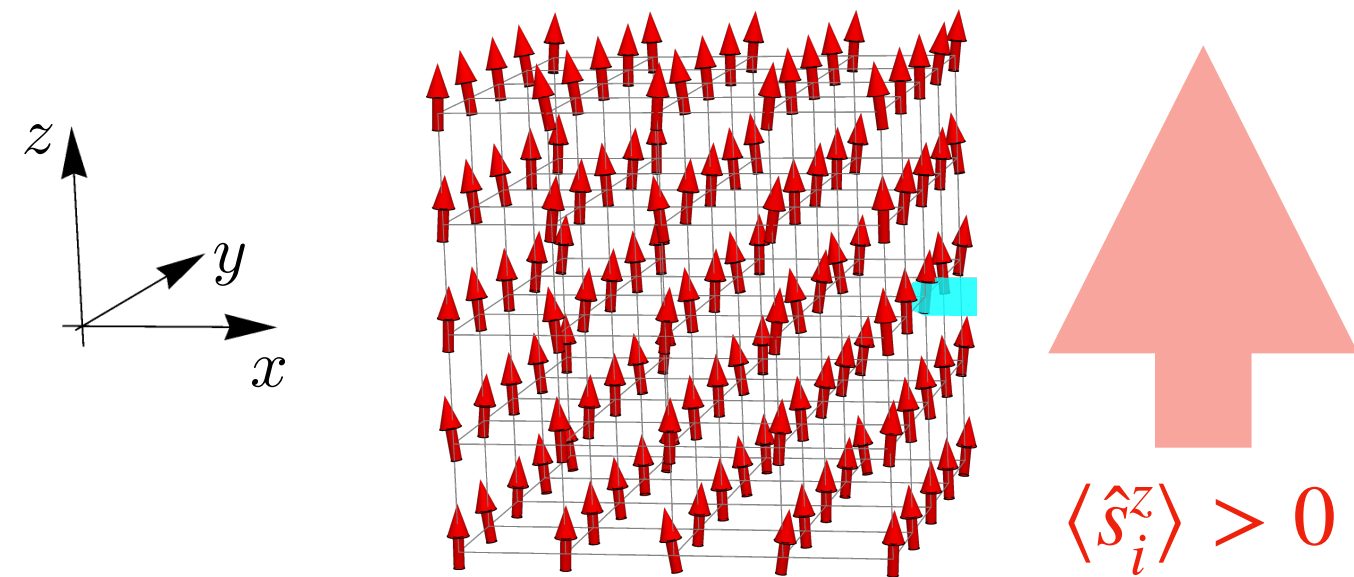
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Magnons as quasiparticle in ferromagnets

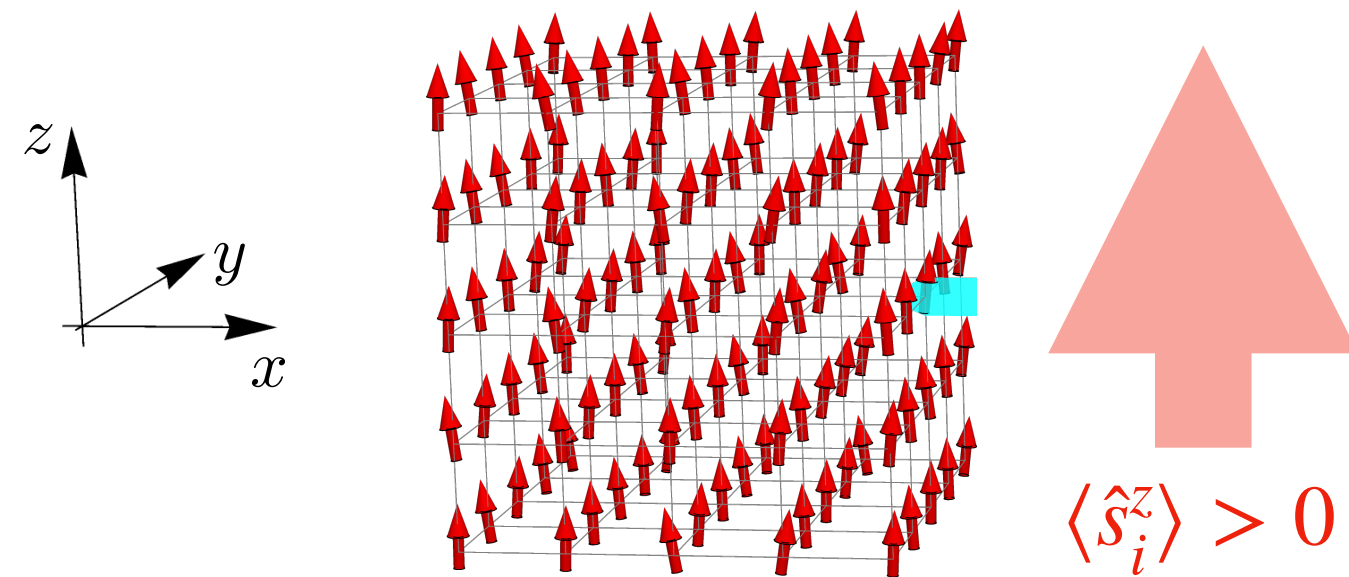
Ferromagnetic Heisenberg model



$$\hat{H}^{\text{Hei}} = -J \sum_{\langle i,j \rangle} \vec{\hat{s}}_i \cdot \vec{\hat{s}}_j - h \sum_i \hat{s}_i^z$$

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Holstein-Primakov trans.

$$\hat{s}_i^z = S - \hat{b}_i^\dagger \hat{b}_i$$

$$\hat{s}_i^- = \sqrt{2S - \hat{b}_i^\dagger \hat{b}_i} \hat{b}_i^\dagger$$

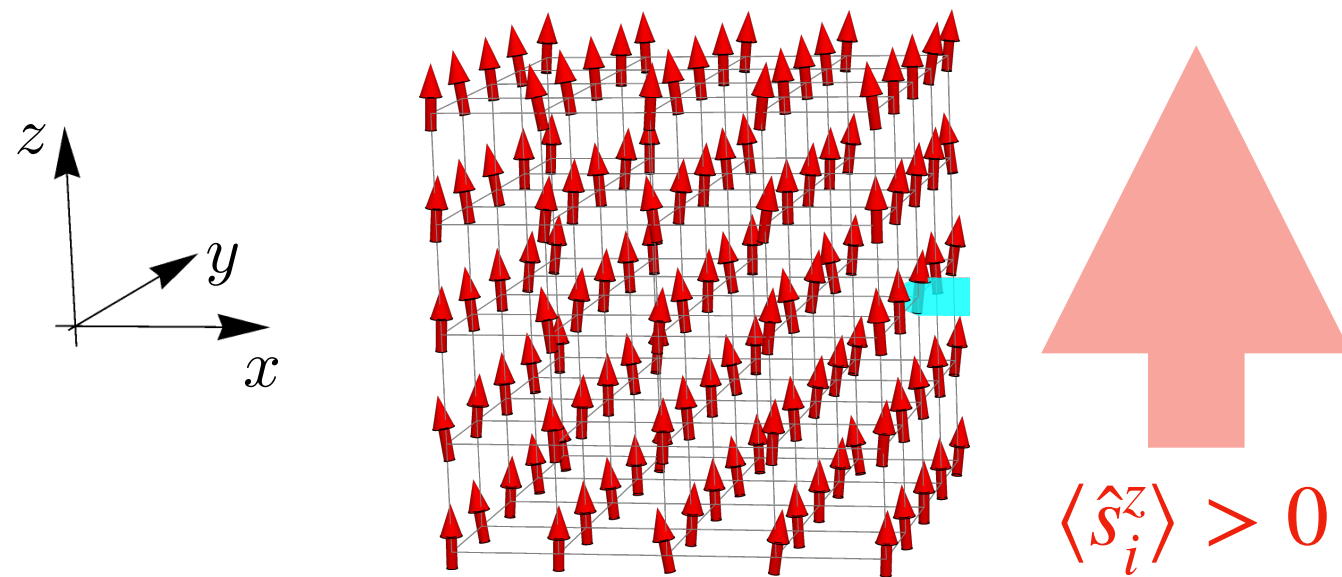
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b_i : Bosonic field

$S = 1/2, 1, 3/2, \dots$

Magnons as quasiparticle in ferromagnets

Ferromagnetic Heisenberg model



High polarization

$$\langle \hat{s}_i^z \rangle \approx S \text{ or } \langle \hat{b}_i^\dagger \hat{b}_i \rangle \ll 1$$

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Spin-wave approx.

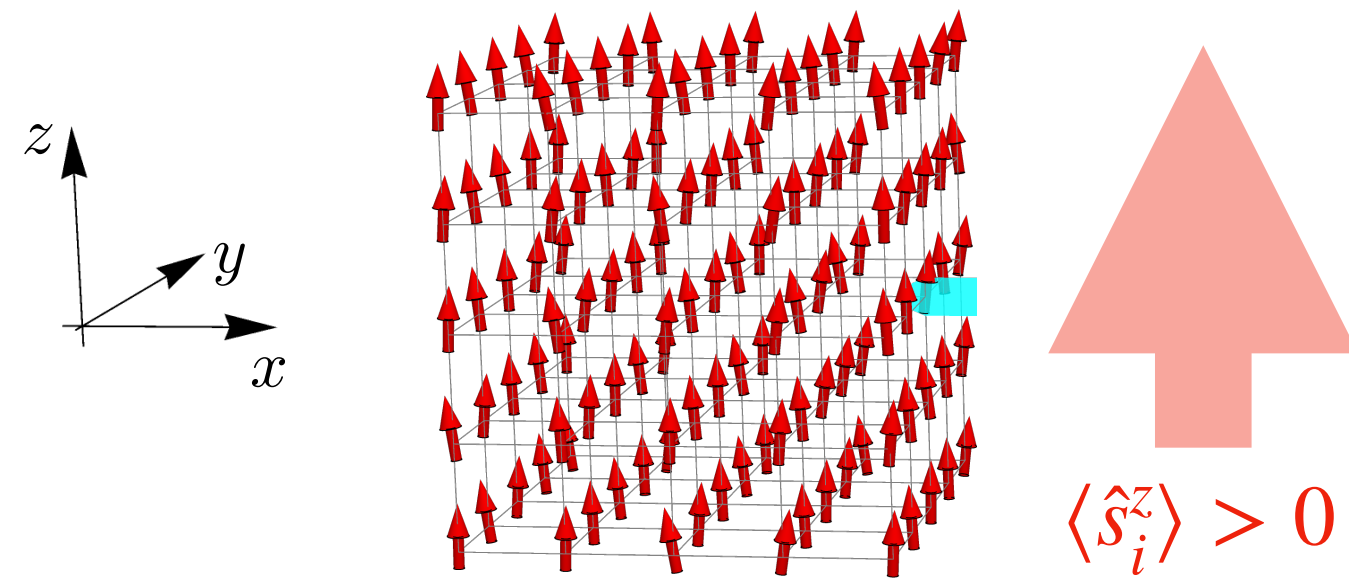
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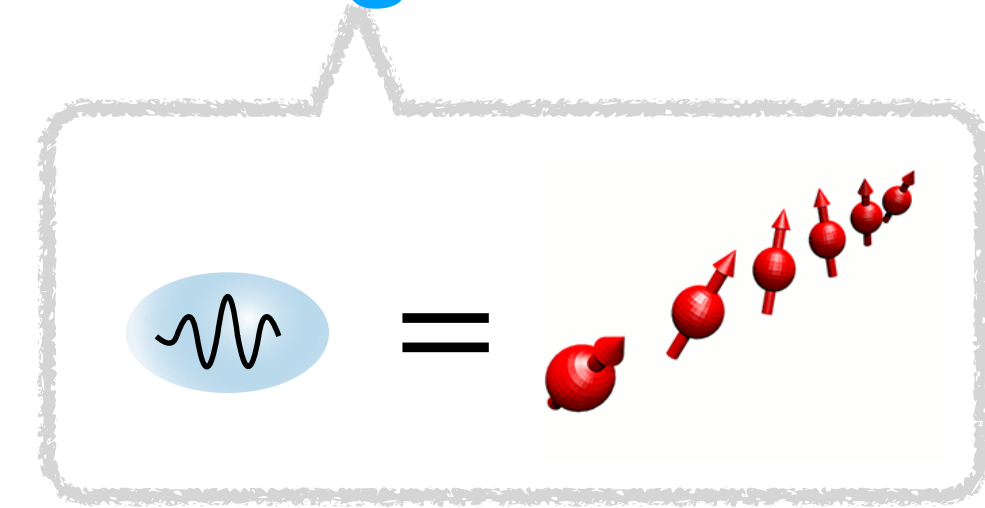
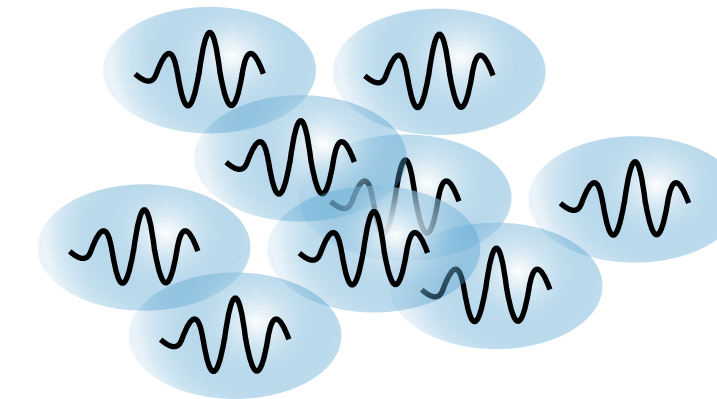
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Gas of thermally excited magnons



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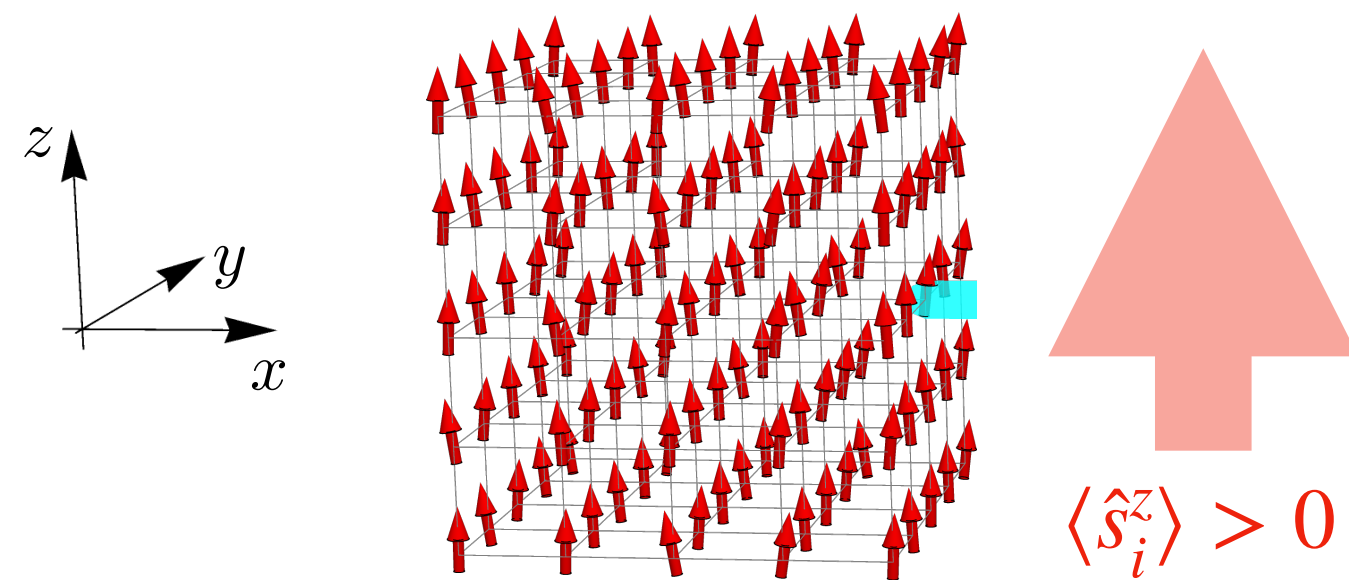
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$$H^{\text{Hei}} \approx \sum_{\vec{k}} E_{\vec{k}} b_{\vec{k}}^\dagger b_{\vec{k}}$$

$b_{\vec{k}}$: field of magnon

Magnons as quasiparticle in ferromagnets

Ferromagnetic Heisenberg model



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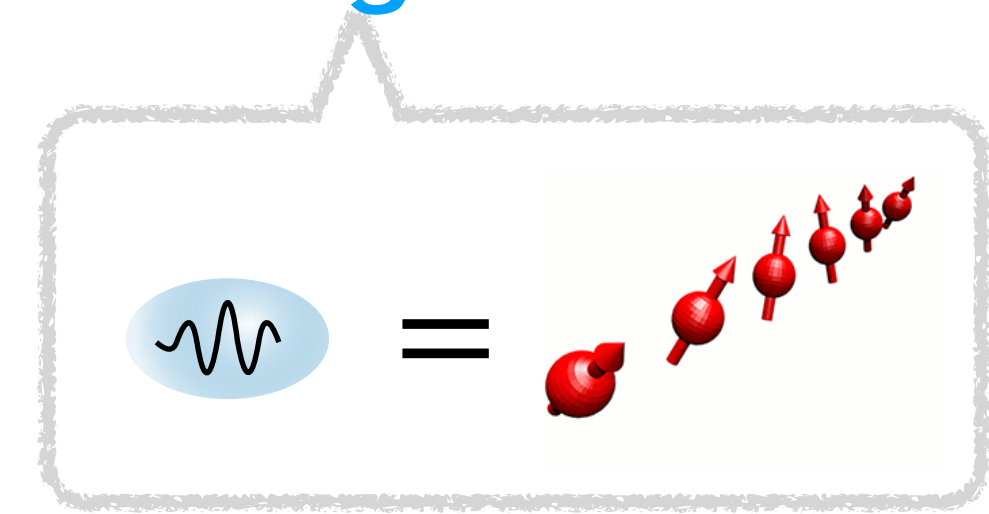
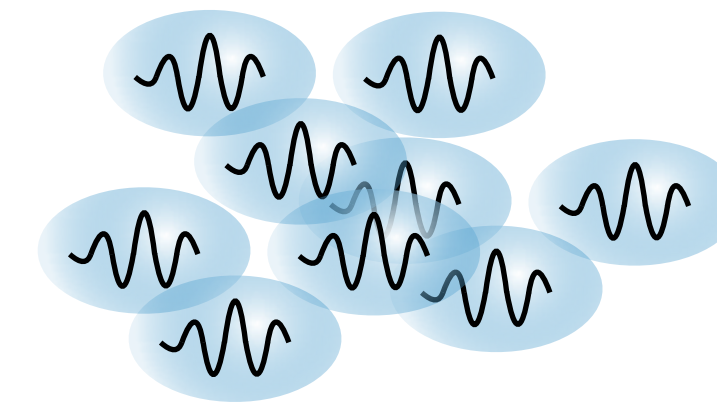
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Gas of thermally excited magnons

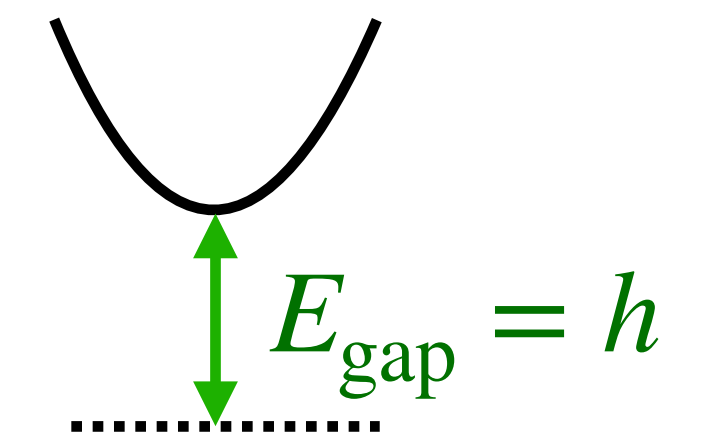


$$H^{\text{Hei}} \approx \sum_{\vec{k}} E_{\vec{k}} b_{\vec{k}}^\dagger b_{\vec{k}}$$

$b_{\vec{k}}$: field of magnon

Magnon energy

$$E_k = 2JSk^2 + h$$

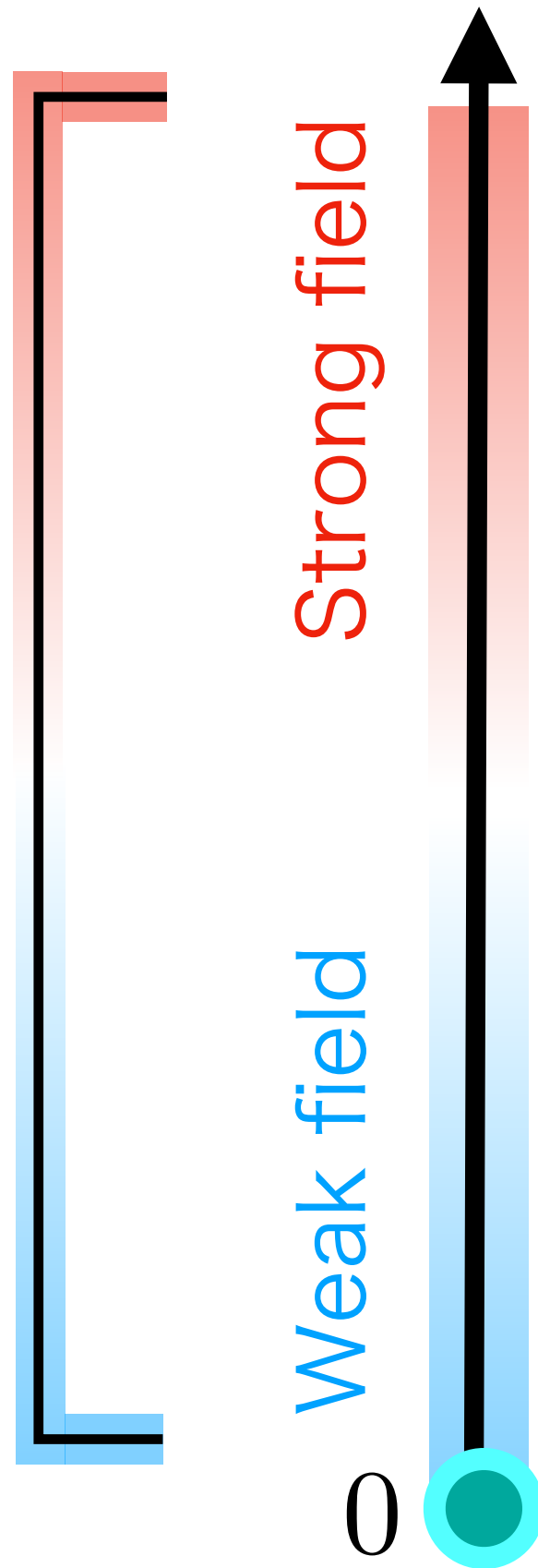


Magnon gap by Zeeman field h

Quantum regime of magnons near critical point

Zeeman field h ($\propto E_{\text{gap}}$)

Cold atoms



Strong field

Weak field

0

$\sim T \ll T_c$

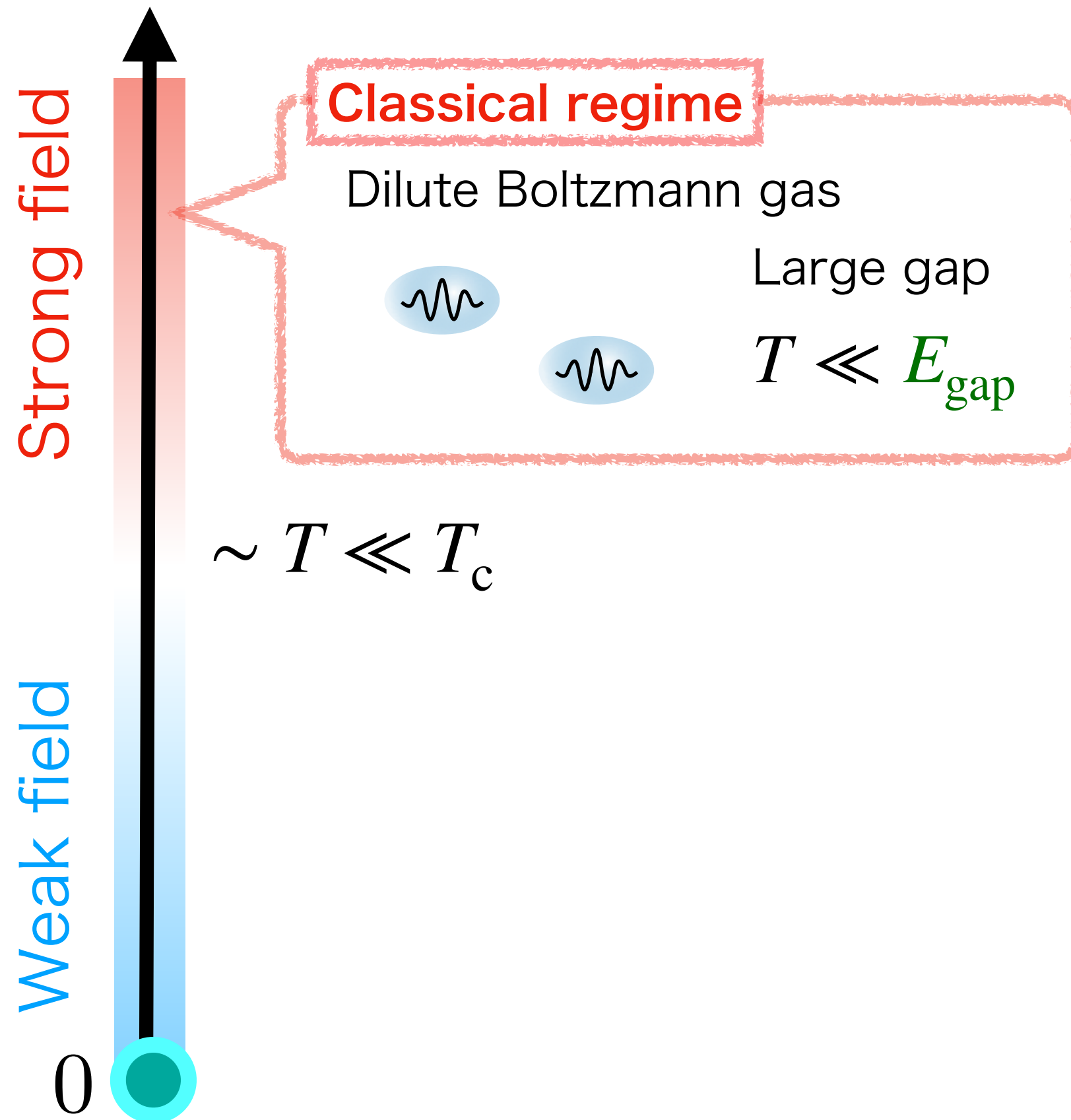
Controllable h

Krinner et al., PNAS
113, 8144 (2016)

Quantum regime of magnons near critical point

Zeeman field h ($\propto E_{\text{gap}}$)

Cold atoms



Spintronics w/ solid FI

Tserkovnyak et al., Rev. Mod. Phys. **77**, 1375 (2005)

W. Bauer et al., Nat. Mater. **11**, 391 (2012)

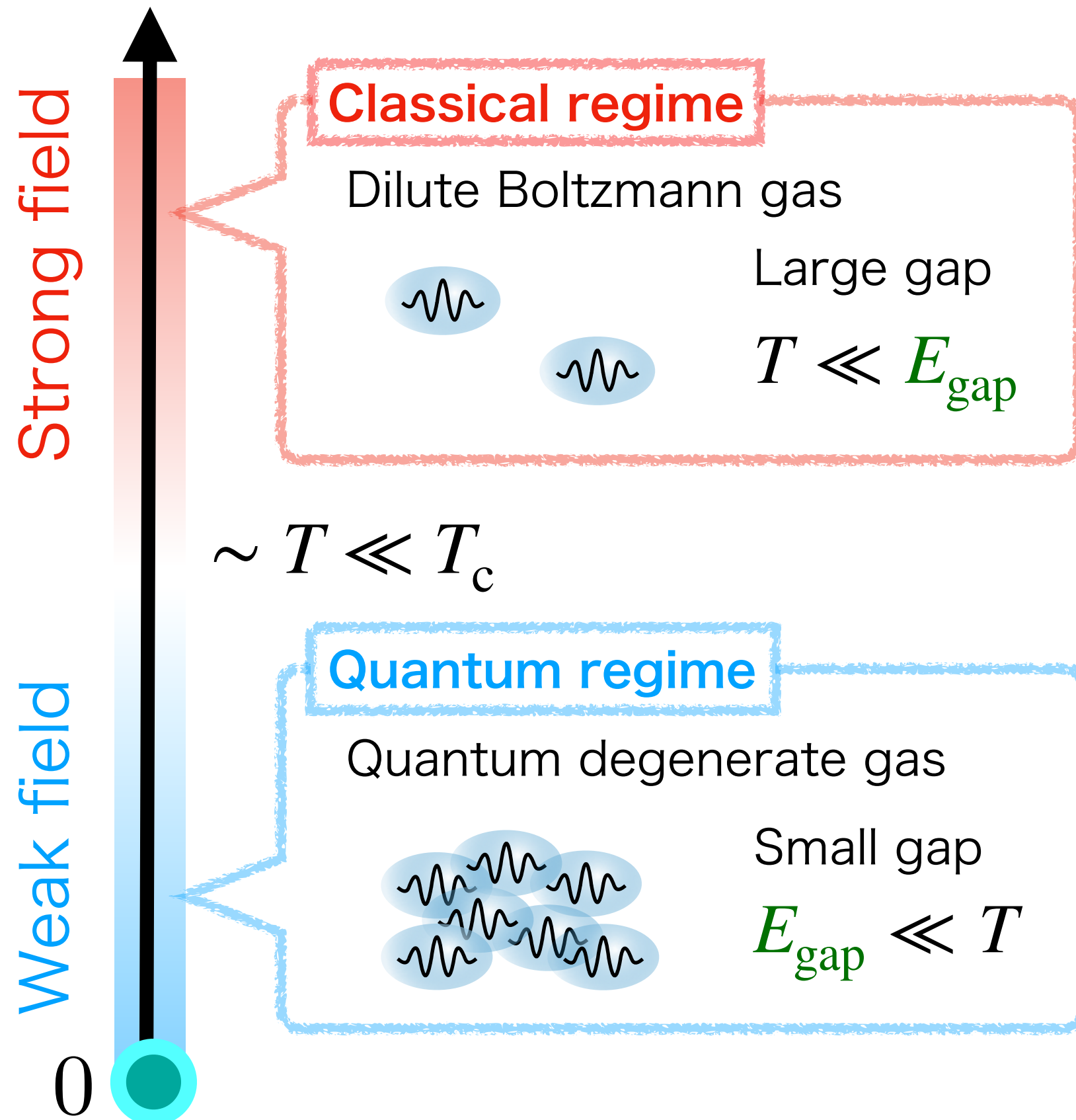
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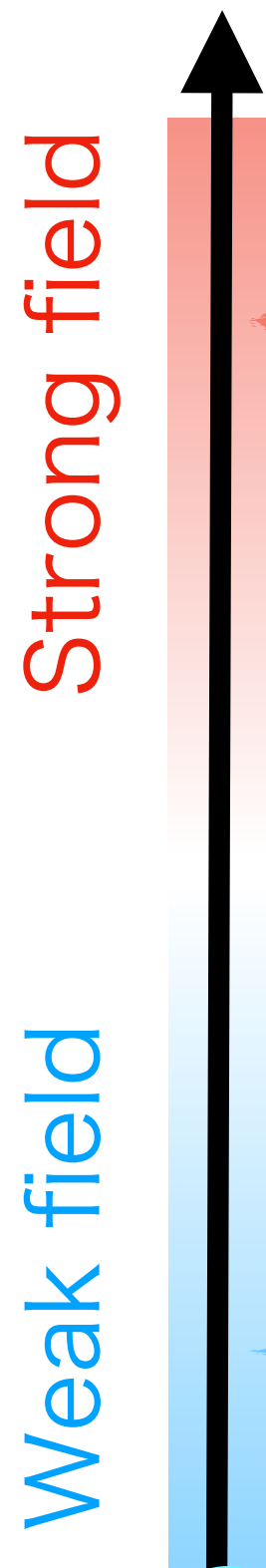
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Quantum regime of magnons near critical point

Zeeman field h ($\propto E_{\text{gap}}$)

Cold atoms



Classical regime
 Dilute Boltzmann gas
 Large gap
 $T \ll E_{\text{gap}}$

Spintronics w/ solid FI

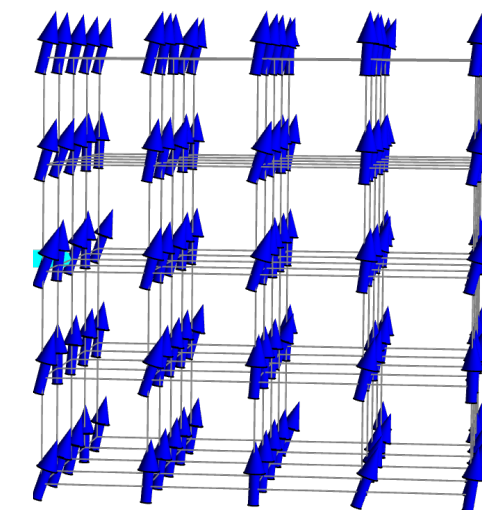
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$\sim T \ll T_c$

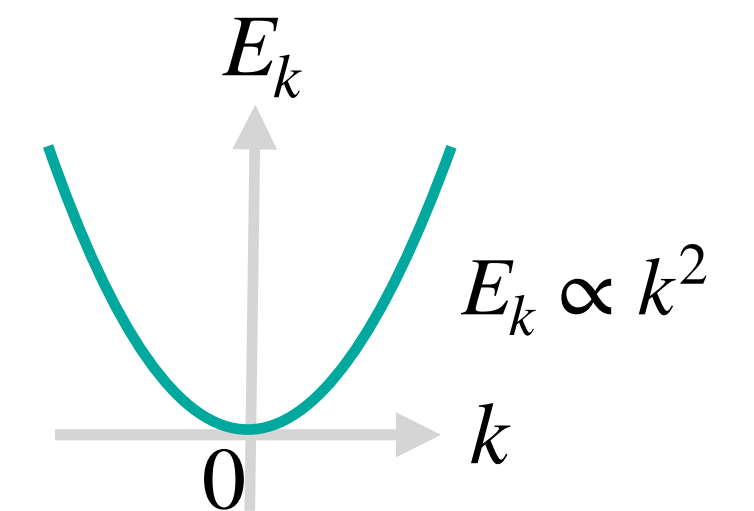
Quantum regime
 Quantum degenerate gas
 Small gap
 $E_{\text{gap}} \ll T$

Spontaneous breaking of O(3) symmetry in FIs

Gapless magnon as Nambu-Goldstone mode



Spontaneous Magnetization



0 **Magnonic critical point**
 $E_{\text{gap}} = h = 0$

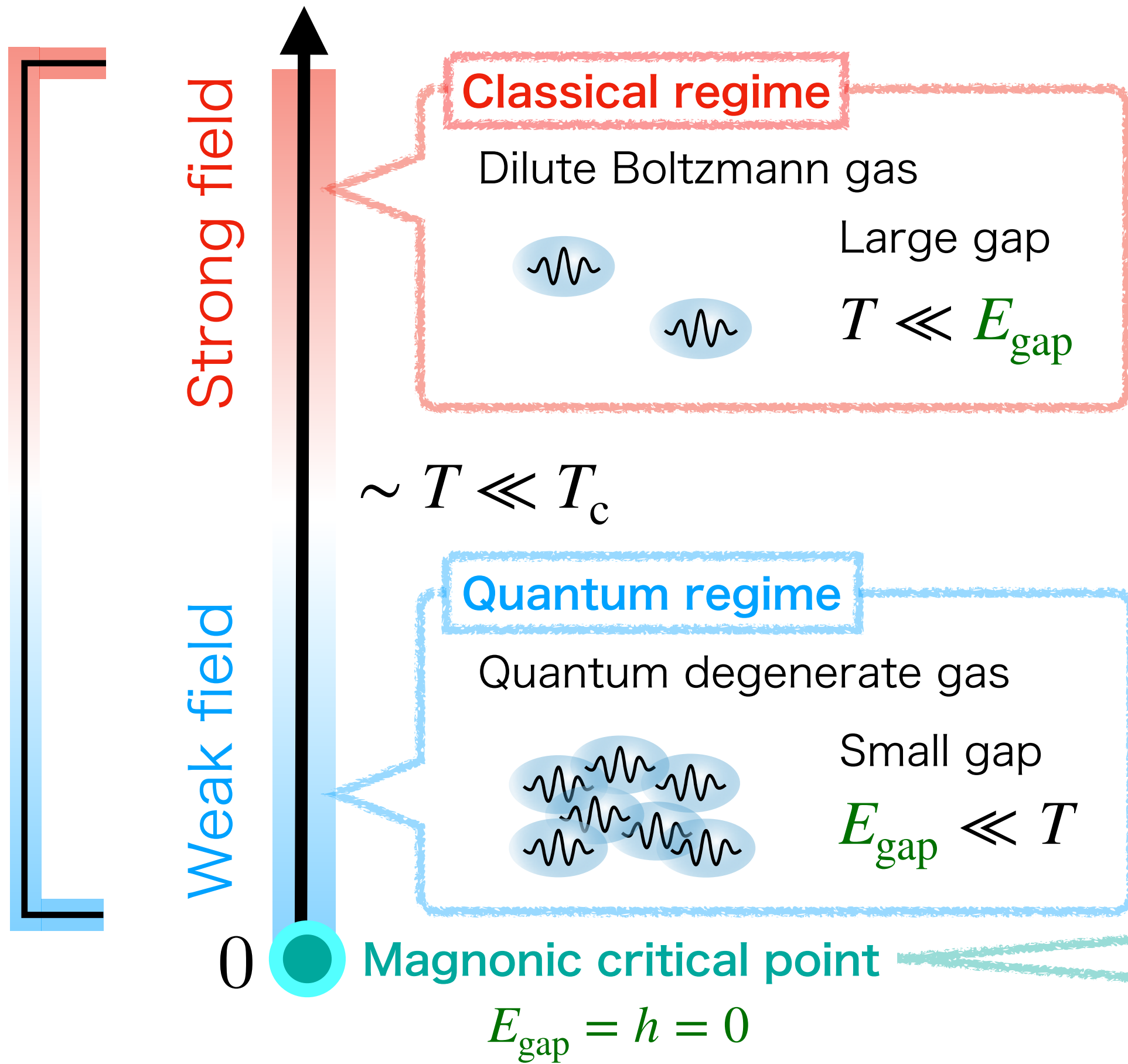
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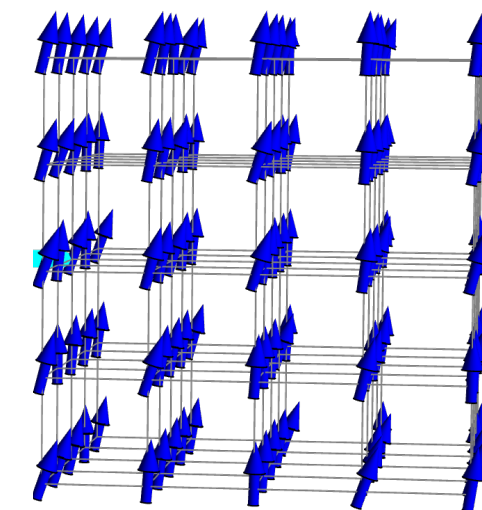


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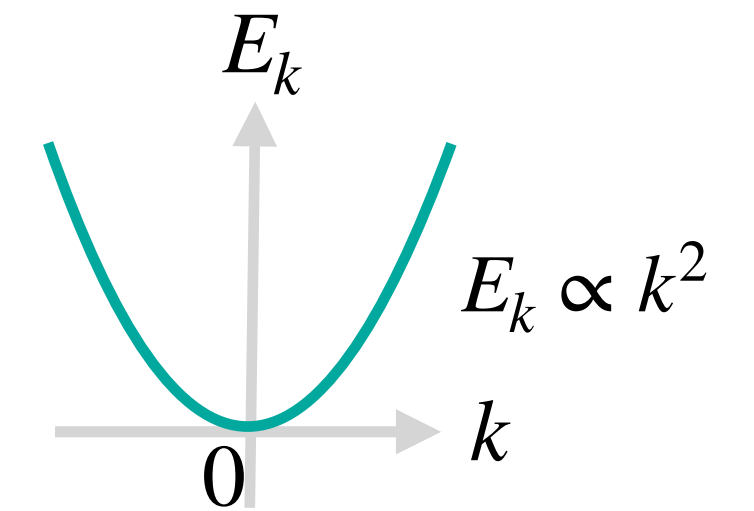
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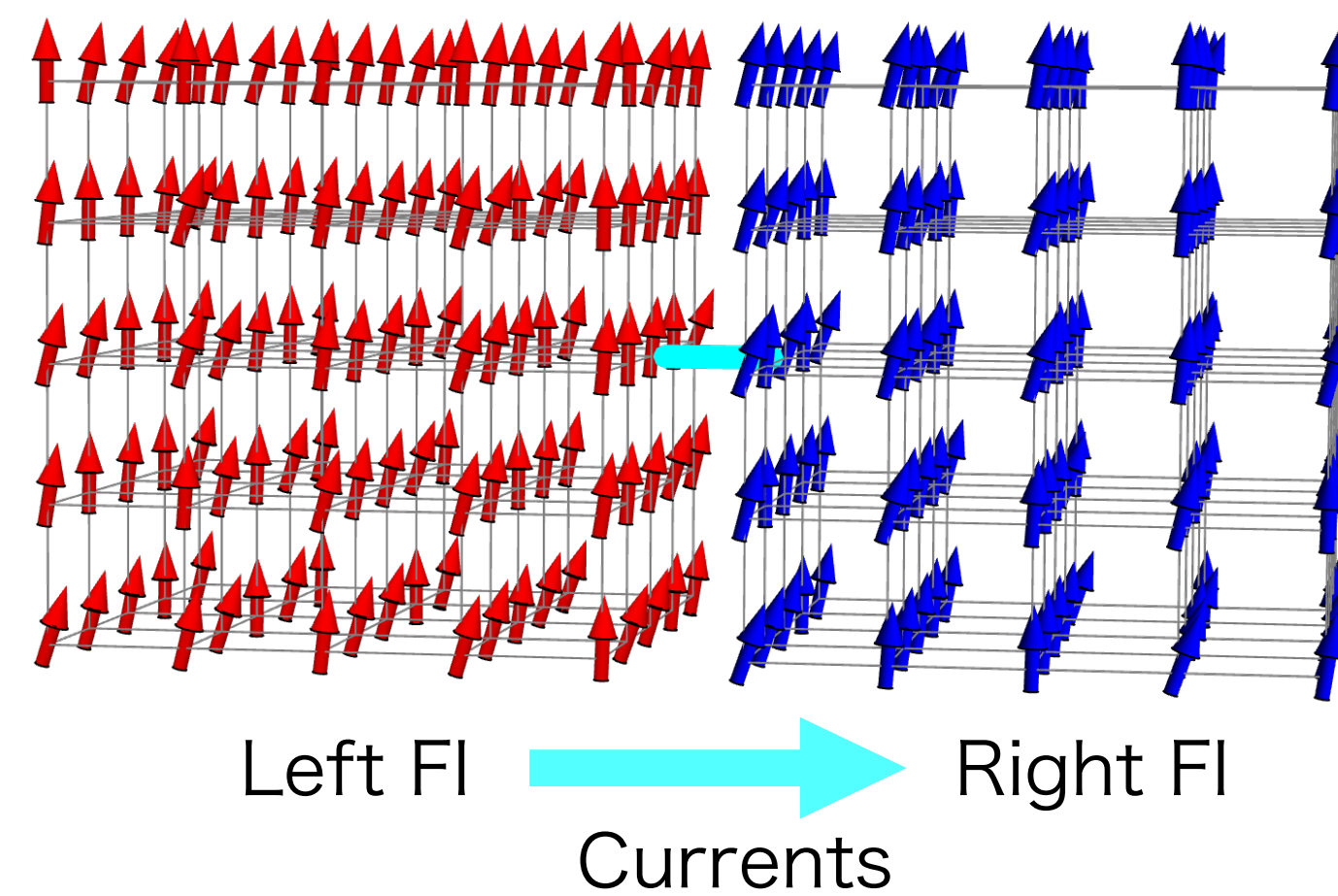
Cold atoms allow us to explore **quantum transport** near **the magnonic critical point**

Controllable h

Krinner et al., PNAS **113**, 8144 (2016)

Anomalous tunneling transport of magnons

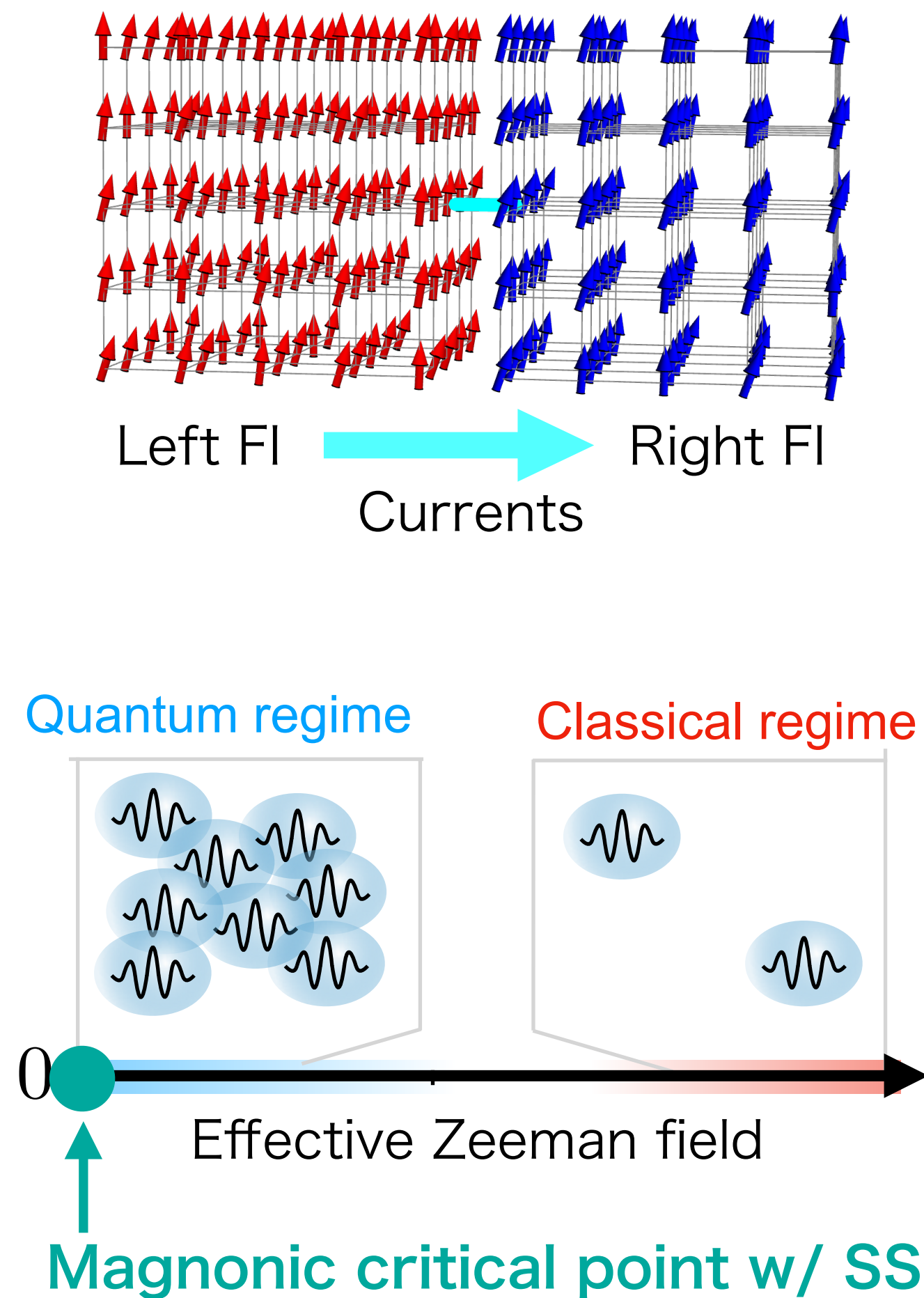
YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280



Summary of setup and results

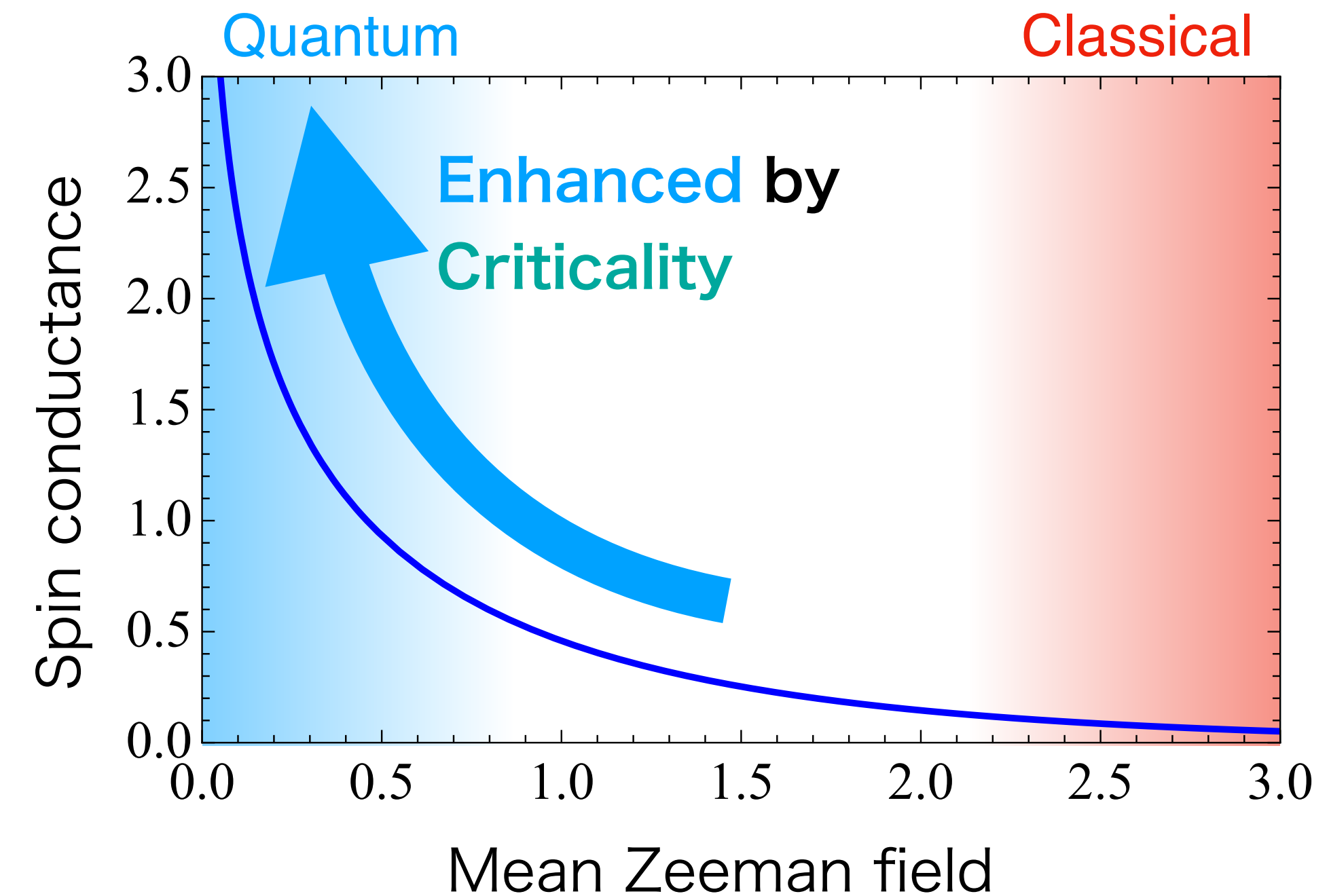
YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

Highly spin-polarized FIs connected with a magnetic quantum point contact



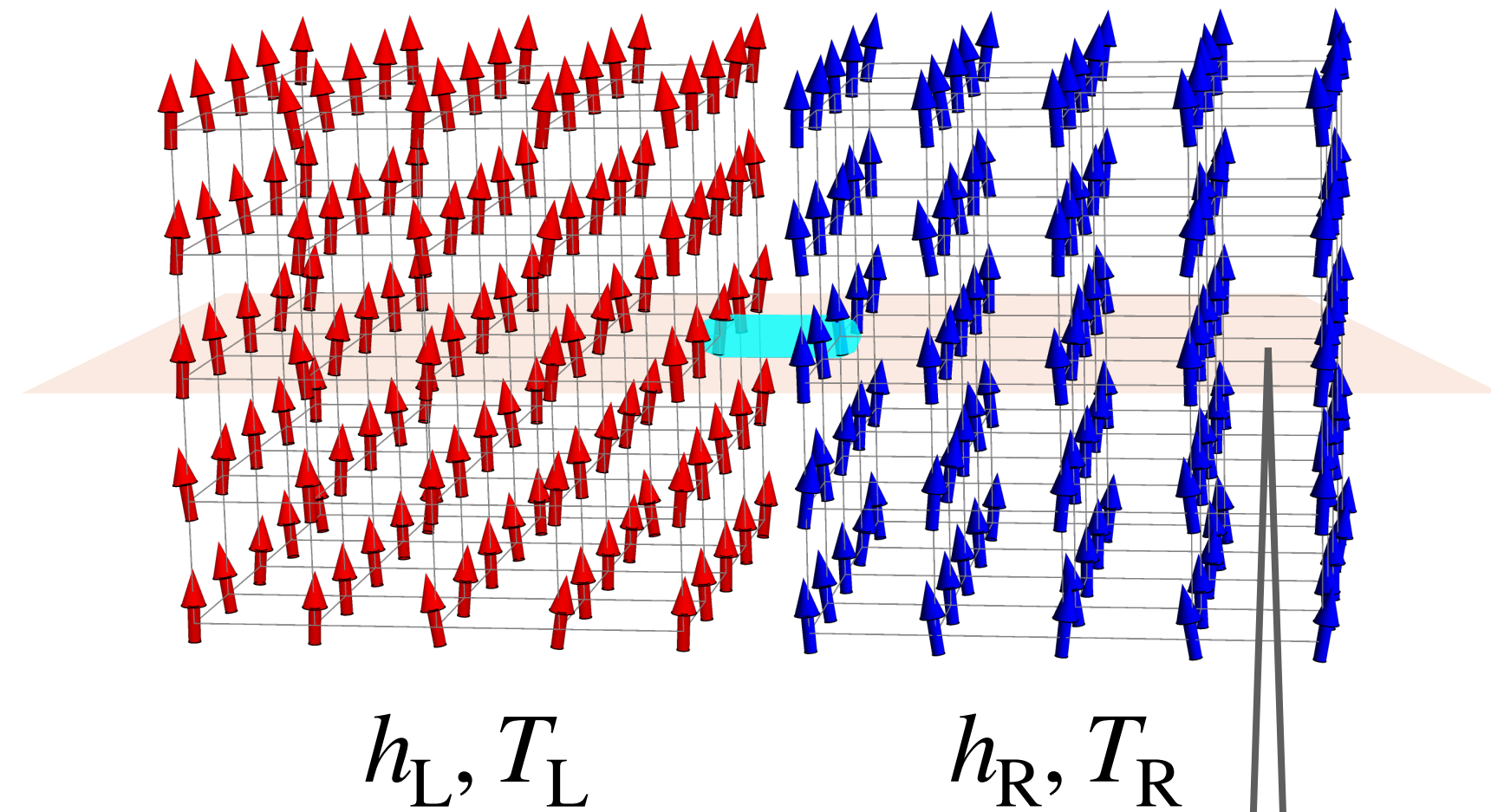
Anomalous thermomagnetic transport by **magnonic criticality**

e.g. **Anomalous enhancement** in spin conductance



Model for magnonic tunneling transport

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280



Hamiltonian:

$$H = H_L + H_R + H_T$$

$$H_{\alpha=L,R} = -J \sum_{\langle \vec{r}_\alpha, \vec{r}'_\alpha \rangle} \vec{s}_{\vec{r}_\alpha} \cdot \vec{s}_{\vec{r}'_\alpha} - h_\alpha \sum_{\vec{r}_\alpha} s_{\vec{r}_\alpha}^z$$

$$H_T = -J_T \vec{s}_{\vec{R}_L} \cdot \vec{s}_{\vec{R}_R}$$

Effective magnetic field

1. Weak tunneling coupling

$$J_T \ll J$$

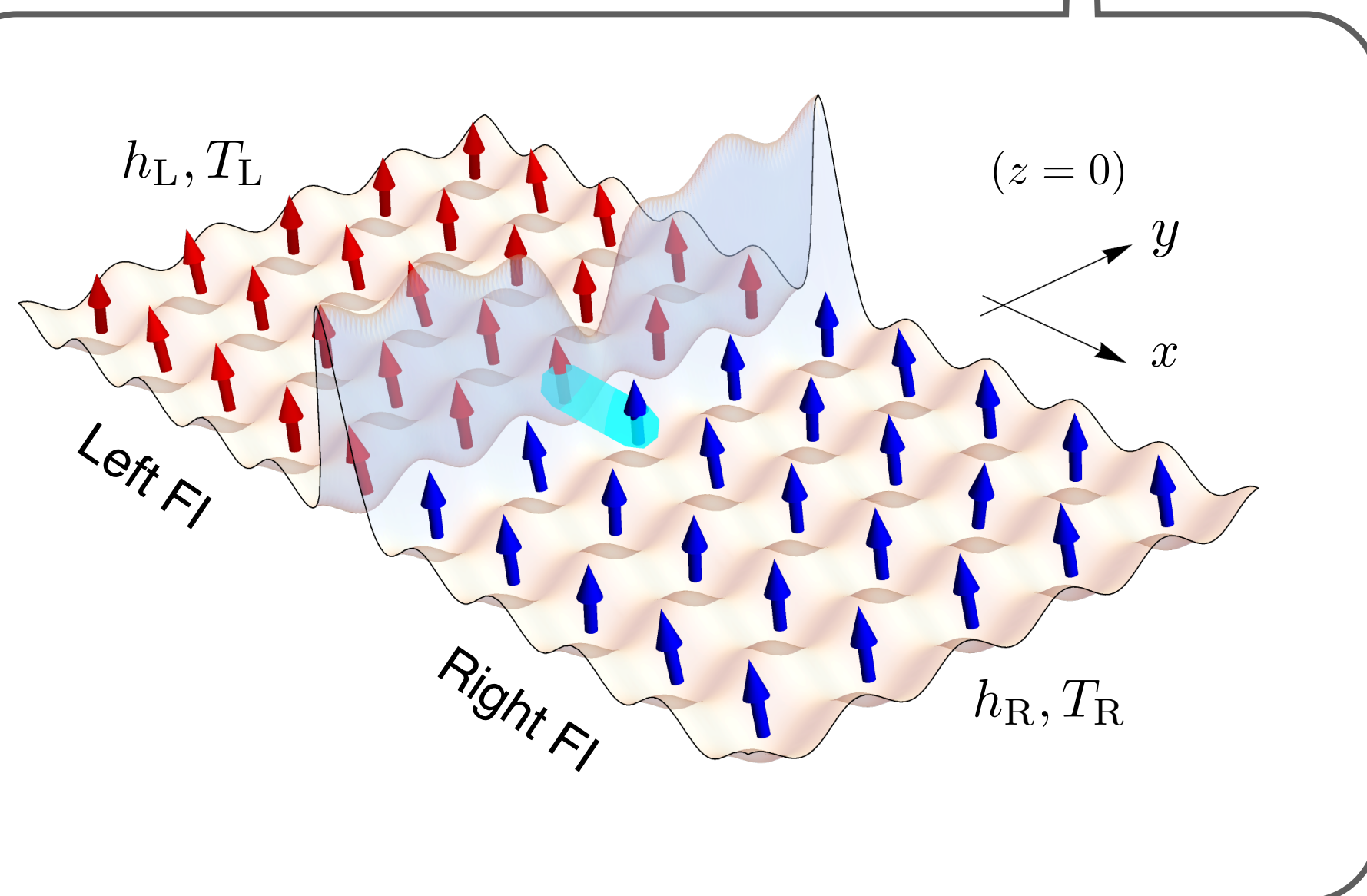
2. Spin-wave theory to analyze the highly spin-polarized case

Magnon:

$$s_{\vec{k}\alpha}^- \approx b_{\vec{k}\alpha}^\dagger$$

$$E_{\vec{k}\alpha} = h_\alpha + (J/2)k^2$$

Magnon energy gap

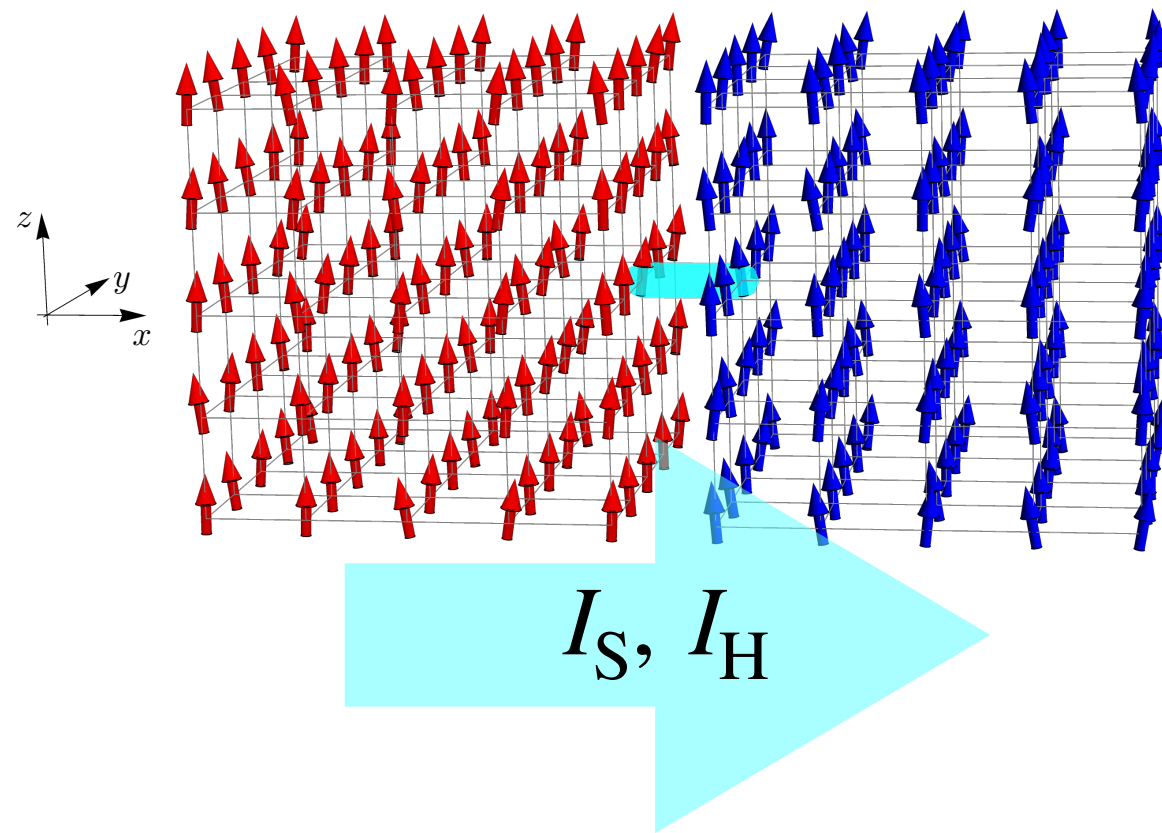


Tunneling Hamiltonian formalism

Evaluating the spin and heat currents I_S, I_H up to $O(J_T^2)$ w/ Schwinger-Keldysh formalism

Electron systems : Meir & Wingreen PRL **68**, 2512 (1992)

Bosonic atoms : Meier and Zwerger, PRA **64**, 033610 (2001)



$$H = \underline{H_L} + \underline{H_R} + \underline{H_T}$$

$$\underline{H_T} = -J_T \vec{s}_{R_L} \cdot \vec{s}_{R_R}$$

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

Spin current: $I_S = \left\langle \frac{d\hat{M}_L^z}{dt} \right\rangle \sim (J_T)^2 \int_{-\infty}^{\infty} d\omega \rho_L(\omega) \rho_R(\omega) \Delta n_B(\omega)$

Heat current: $I_H = - \left\langle \frac{d\hat{H}_L}{dt} \right\rangle \sim (J_T)^2 \int_{-\infty}^{\infty} d\omega (\omega + h_L) \rho_L(\omega) \rho_R(\omega) \Delta n_B(\omega)$

Magnetization: $\hat{M}_\alpha^z = \sum_{\vec{r}_\alpha} s_{\vec{r}_\alpha}^z$

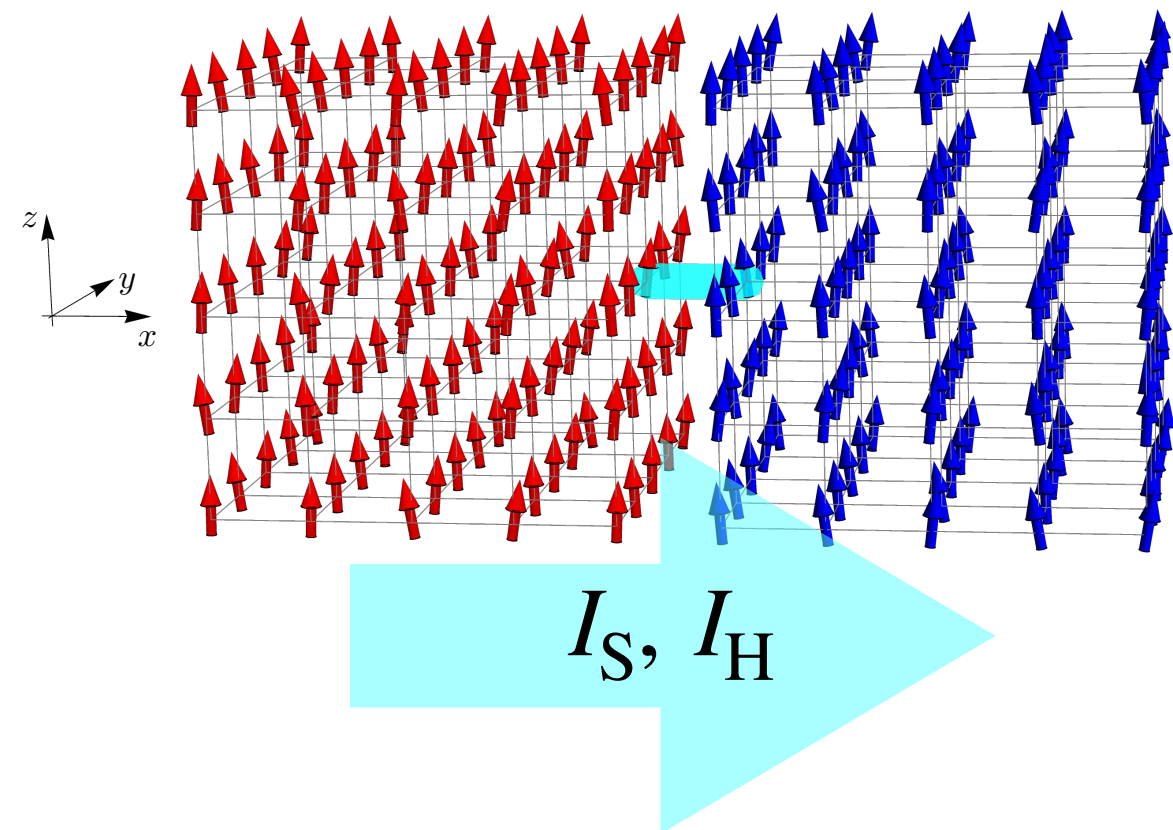
Magnon DoS: $\rho_{L/R}(\omega) \propto \sqrt{\omega} \theta(\omega)$

Difference of magnon distribution:

$$\Delta n_B(\omega) = n_{B,L}(\omega) - n_{B,R}(\omega)$$

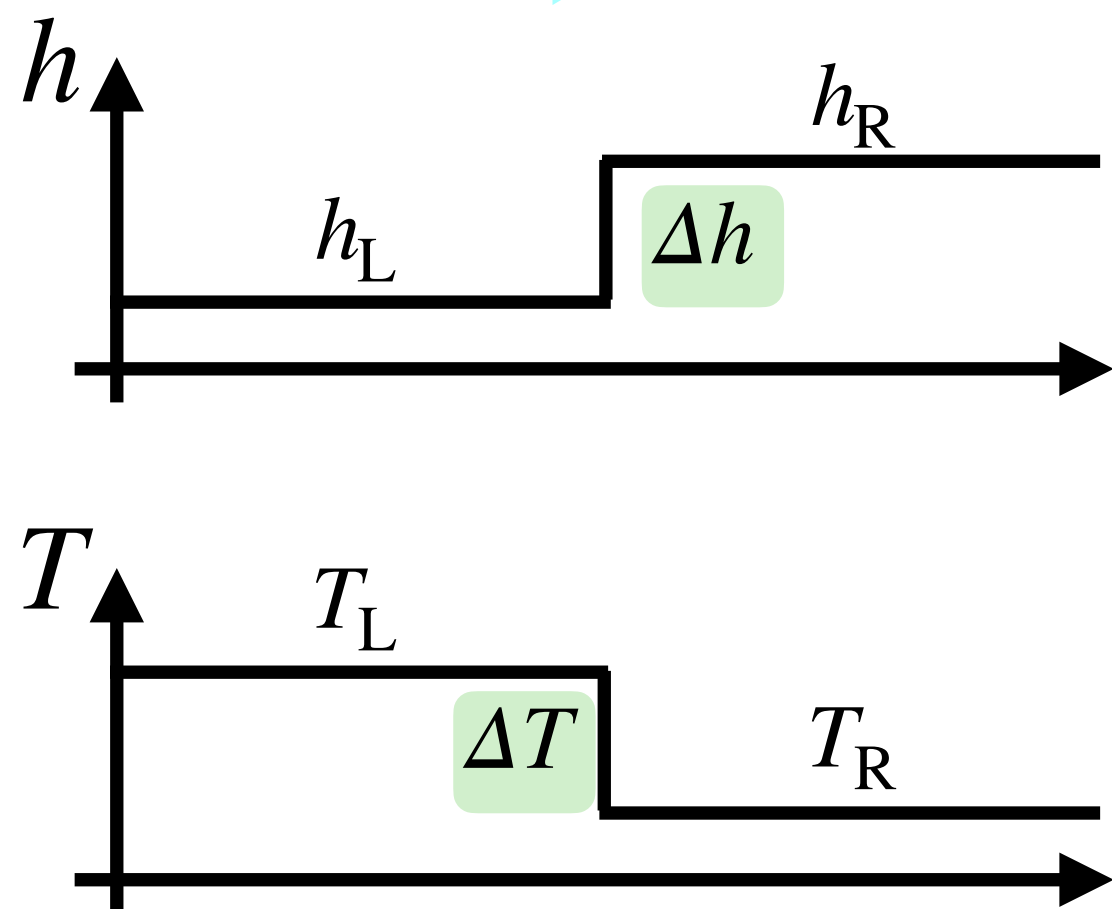
Conductances of spin and heat

Expanding currents to small spin & temperature biases $\Delta h, \Delta T$



$$\begin{pmatrix} I_S \\ I_H \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix} + O((\Delta h)^2, (\Delta T)^2, \Delta h \Delta T)$$

Conductance Bias



$$\frac{L_{11}}{AT} = F_1(x),$$

$$\frac{L_{12}}{AT} = \frac{L_{21}}{AT^2} = 2F_2(x) + xF_1(x),$$

$$\frac{L_{22}}{AT^2} = 6F_3(x) + 4xF_2(x) + x^2F_1(x),$$

$$x = \frac{(h_L + h_R)/2}{(T_L + T_R)/2} = \frac{\text{averaged field}}{\text{averaged temp}}$$

Bose-Einstein integral:

$$F_d(x) \propto \int_0^\infty d\omega \frac{\omega^{d-1}}{e^{\omega+x} - 1}$$

Critical behavior of transport coefficients

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

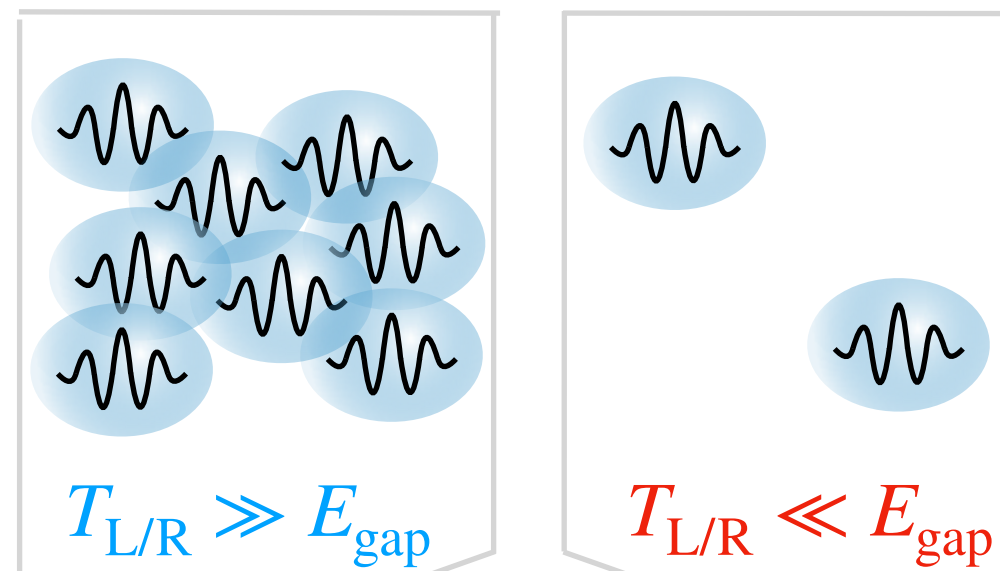
Magnonic criticality enhances conductances L_{ij} in quantum regime

$$\begin{pmatrix} I_S \\ I_H \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

Phase diagram of magnon gas

Quantum regime

Classical regime



0 ↑ Zeeman field $h_{L/R}/T_{L/R}$

Magnonic critical point

Critical behavior of transport coefficients

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

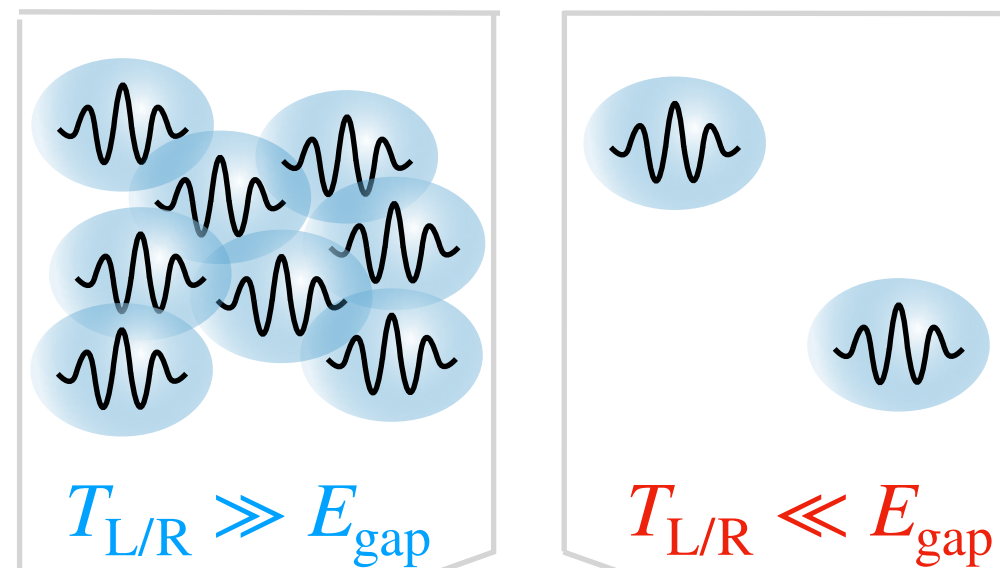
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Phase diagram of magnon gas

Quantum regime

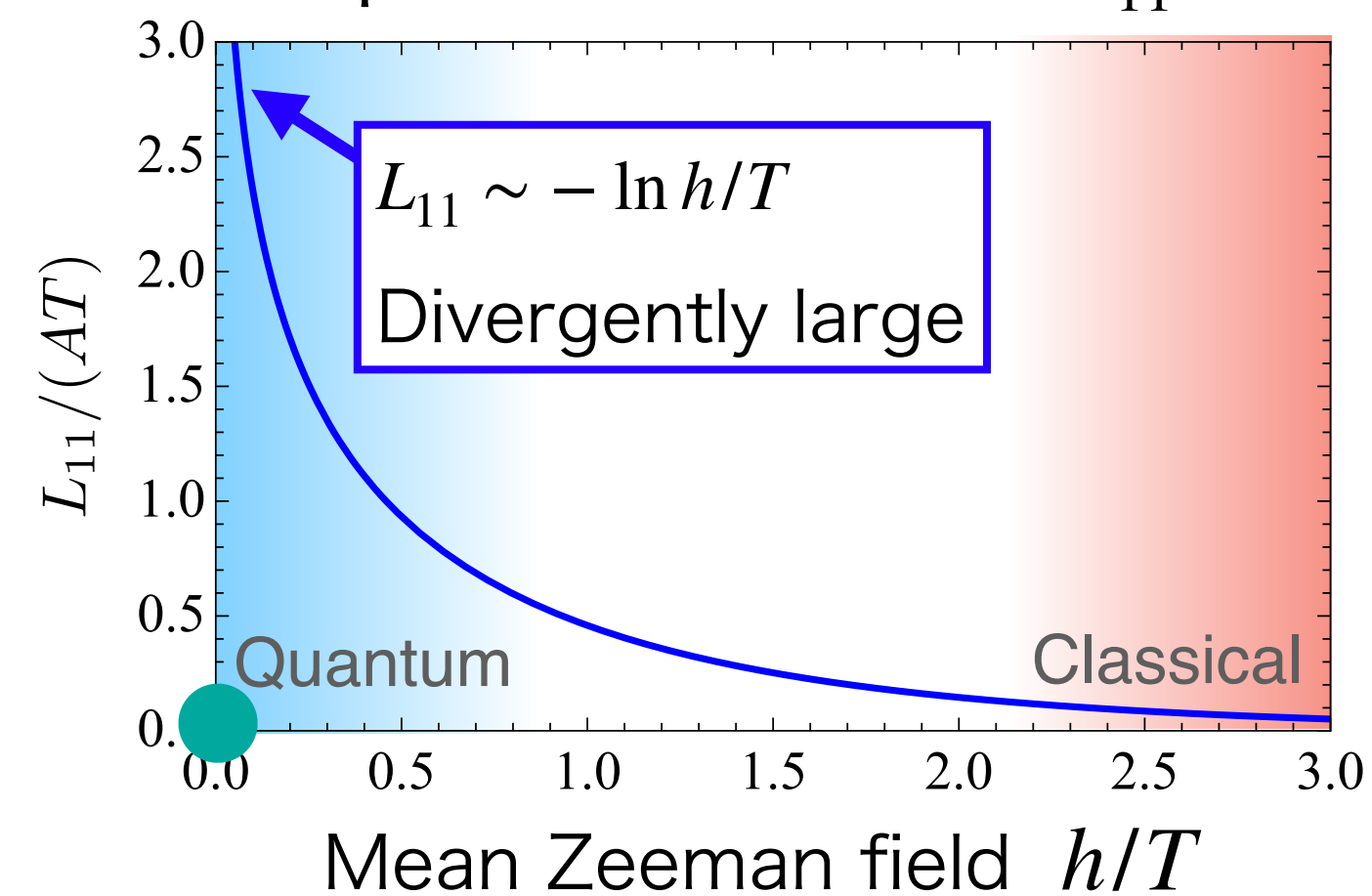
Classical regime



0 ↑ Zeeman field $h_{L/R}/T_{L/R}$

Magnonic critical point

Spin conductance L_{11}



Critical enhancement of conductances L_{ij} in the quantum regime

Critical behavior of transport coefficients

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

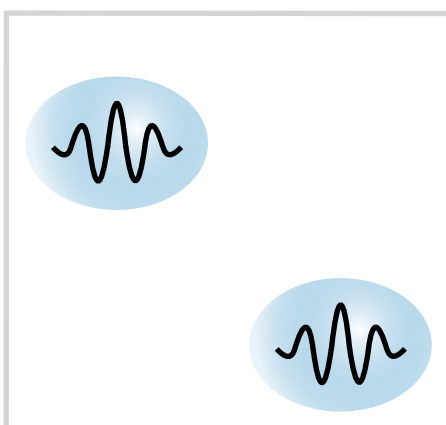
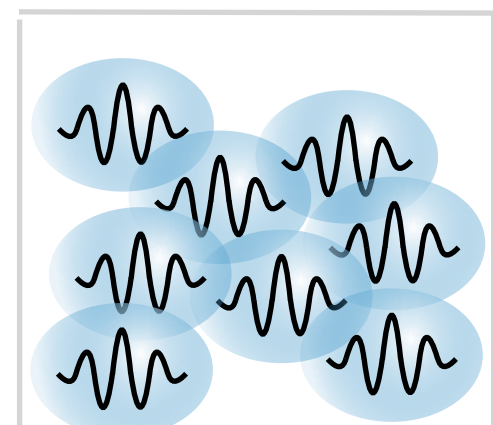
Magnonic criticality enhances conductances L_{ij} in quantum regime

$$\begin{pmatrix} I_S \\ I_H \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

Phase diagram of magnon gas

Quantum regime

Classical regime



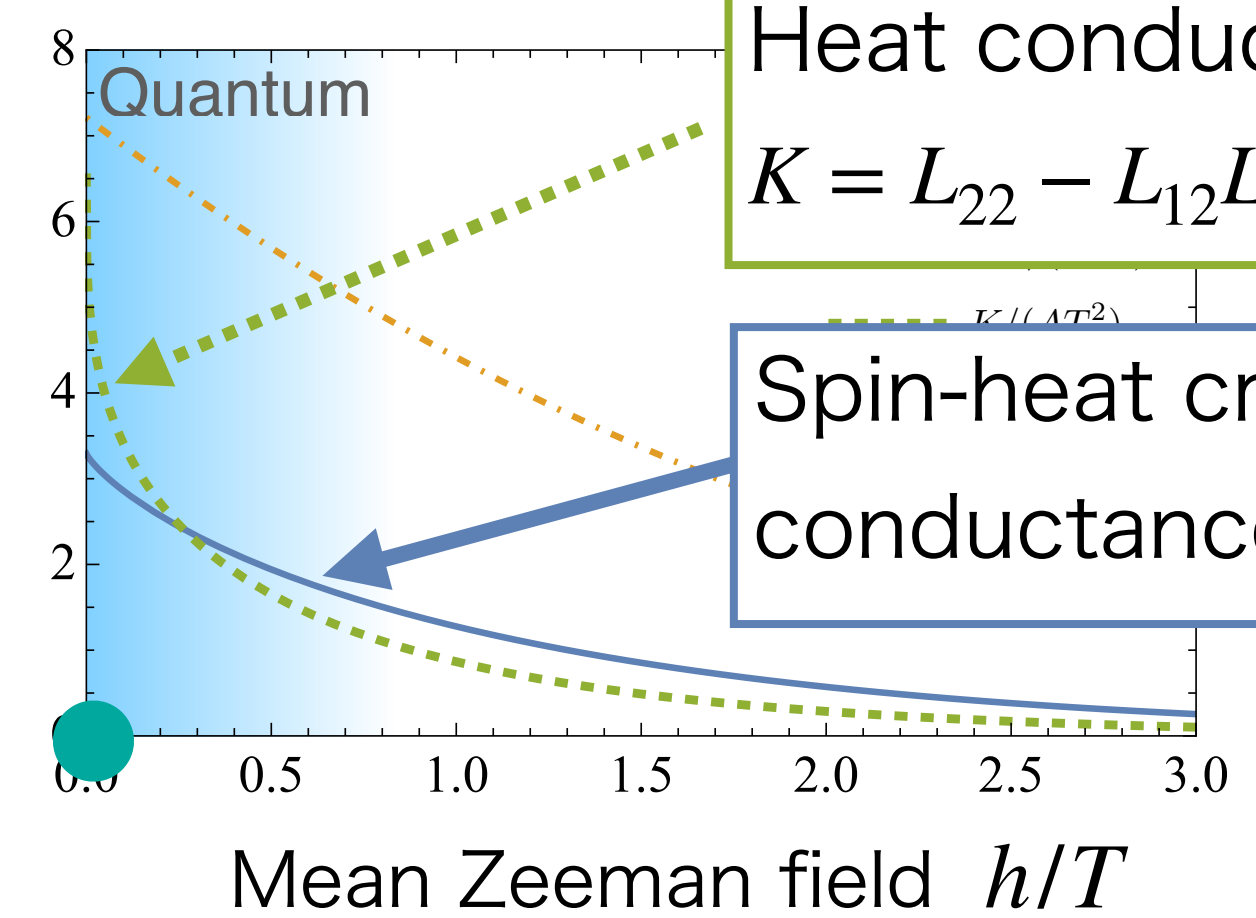
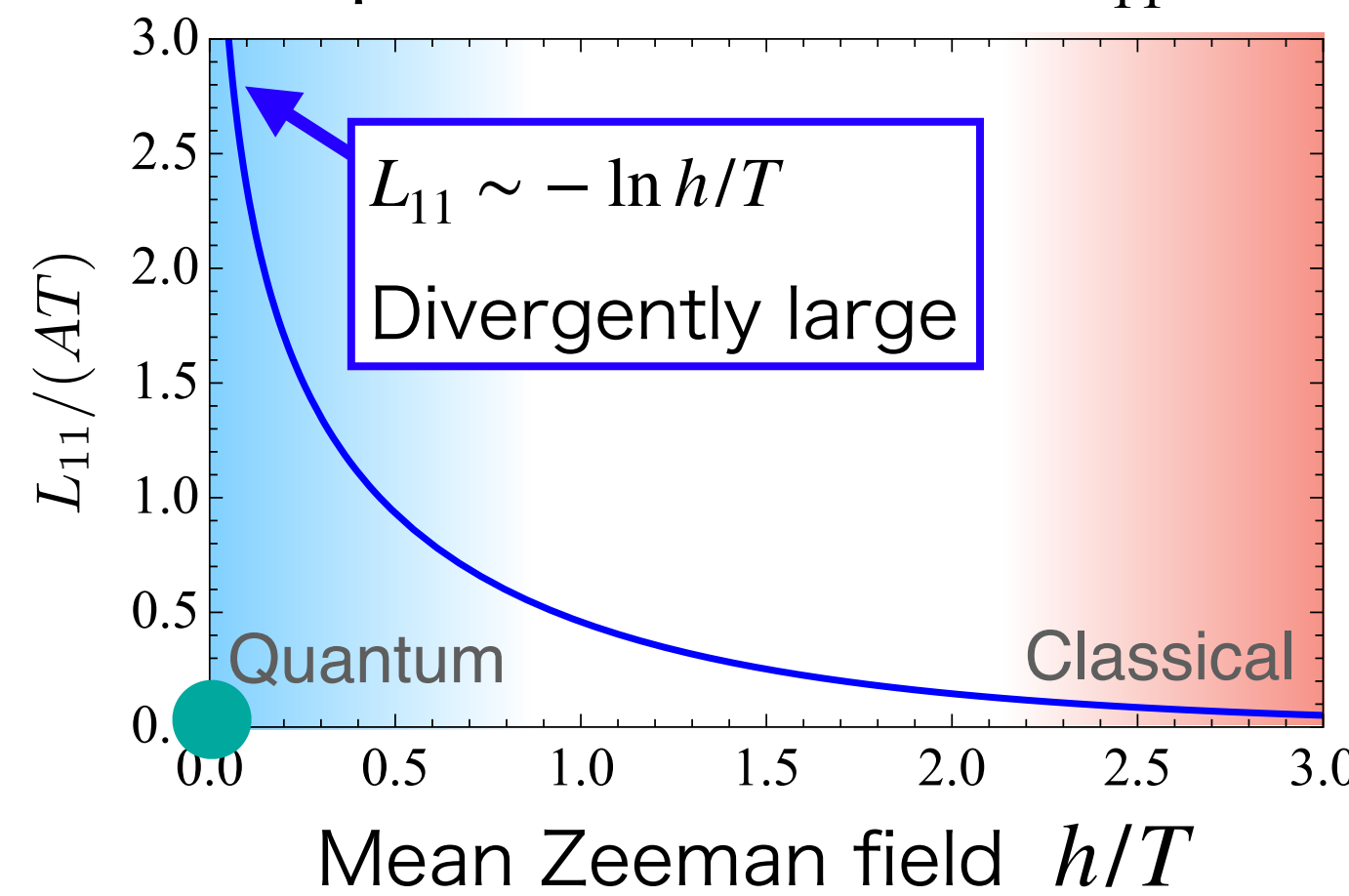
$$T_{L/R} \gg E_{\text{gap}}$$

$$T_{L/R} \ll E_{\text{gap}}$$



Magnonic critical point

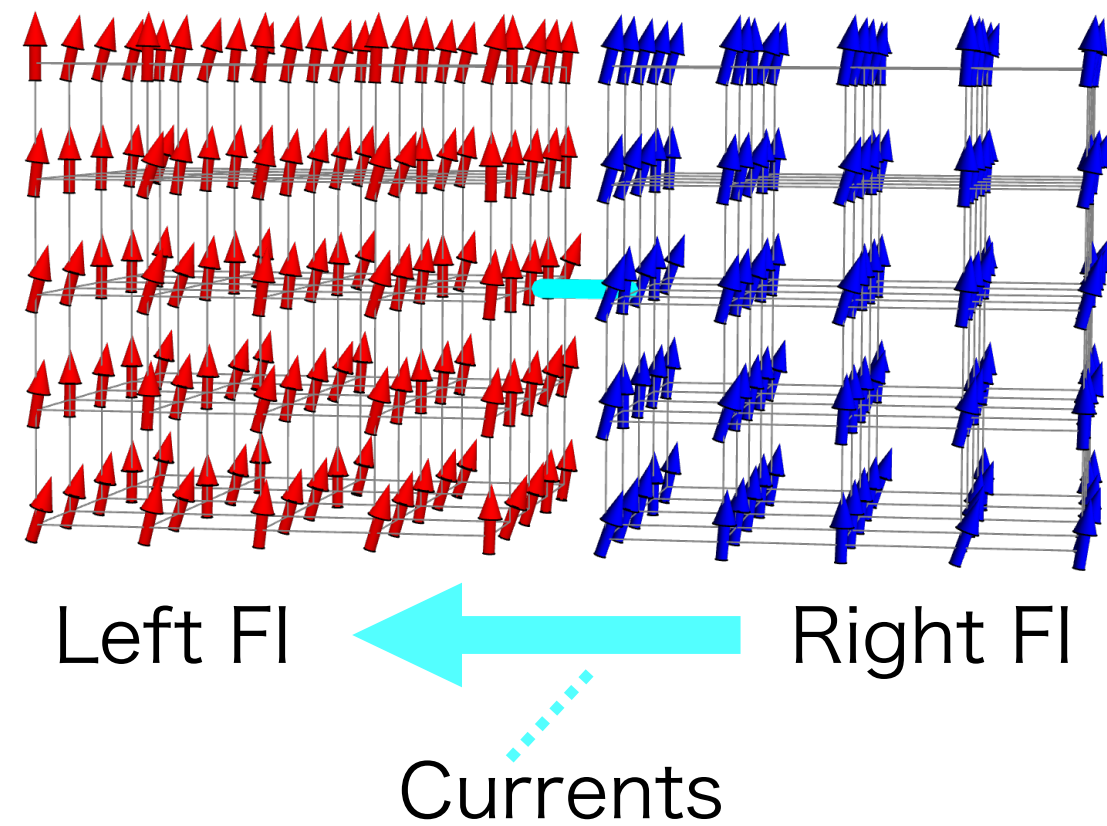
Spin conductance L_{11}



Critical enhancement of conductances L_{ij} in the quantum regime

Summary of this talk

Anomalous tunneling spin and heat transport near the magnonic critical point

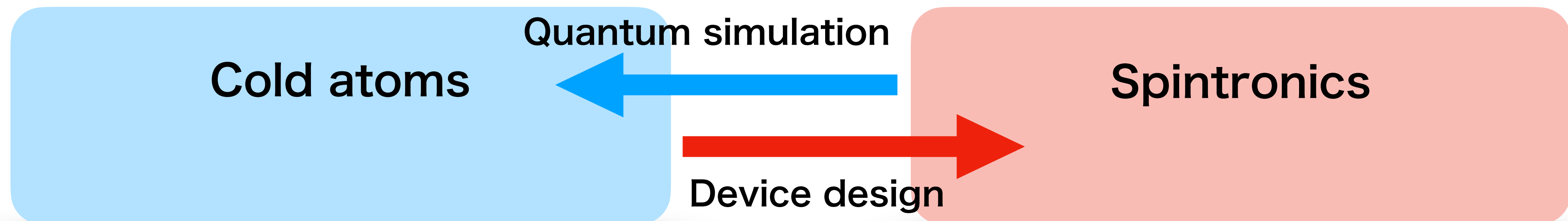


Critical enhancement of spin & heat conductances

Originating from the magnonic critical point corresponding to spontaneous symmetry breaking of $O(3)$ spin rotation

New mechanism for efficient spin & heat transport by criticality

Accelerating interdisciplinary communications b/w Cold atom & Spintronics



Backup slides

Phase diagram of ferromagnetic Heisenberg model

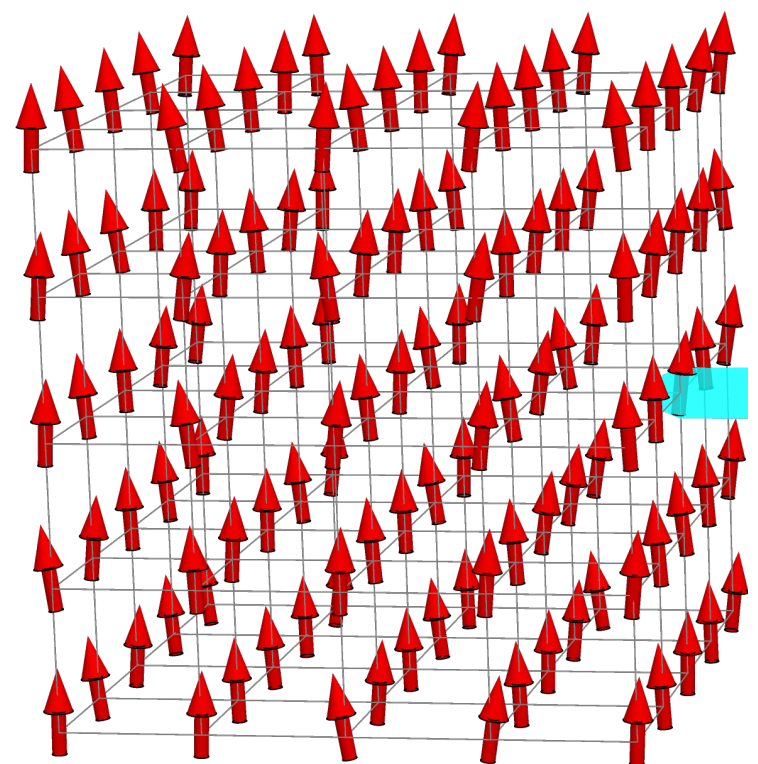
$$\hat{H}^{\text{Hei}} = -J \sum_{\langle i,j \rangle} \vec{\hat{S}}_i \cdot \vec{\hat{S}}_j - h \sum_i \hat{S}_i^z$$

Zeeman field

$$h \geq 0$$

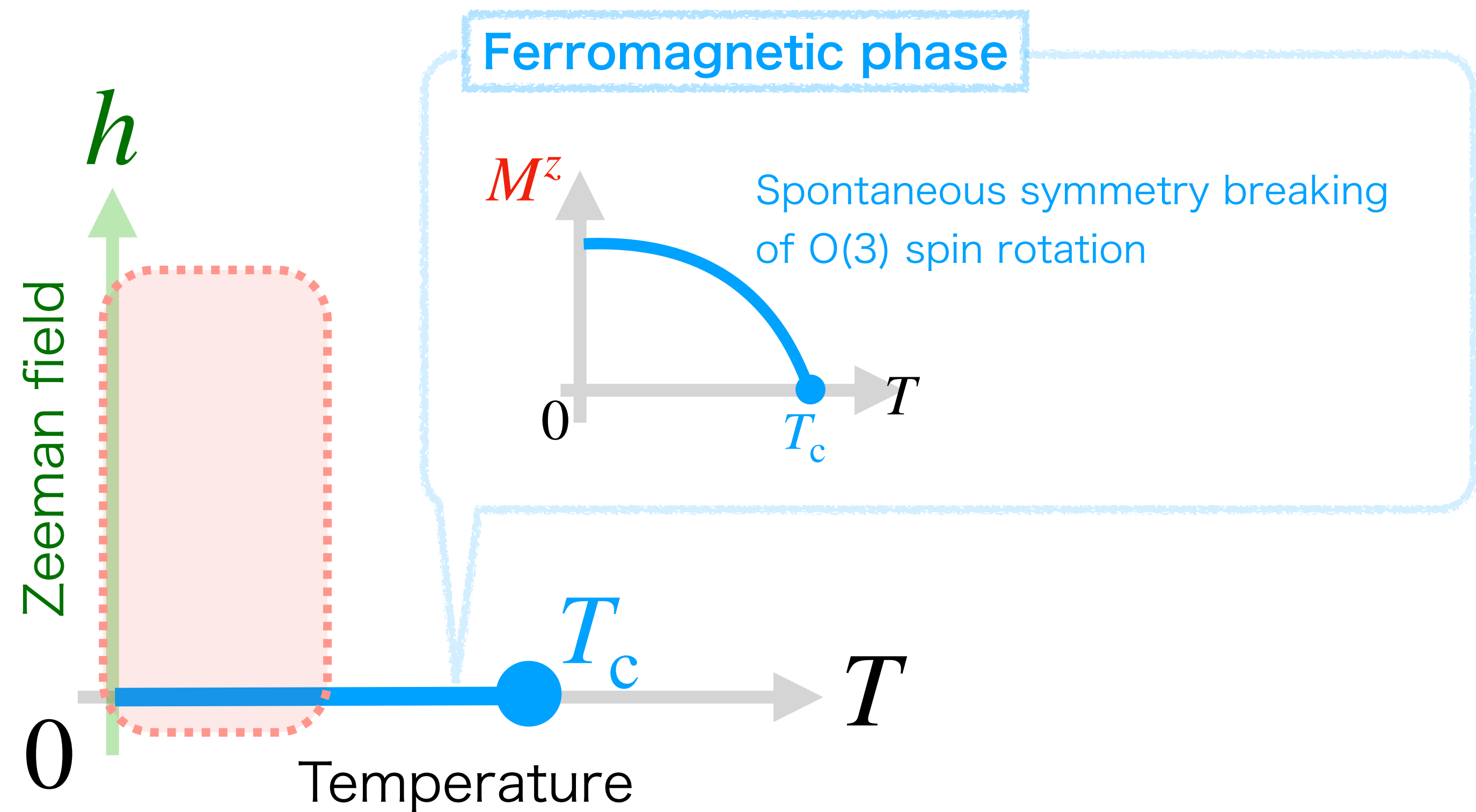
Magnetization

$$M^z = \sum_i \langle \hat{S}_i^z \rangle$$



h

M^z

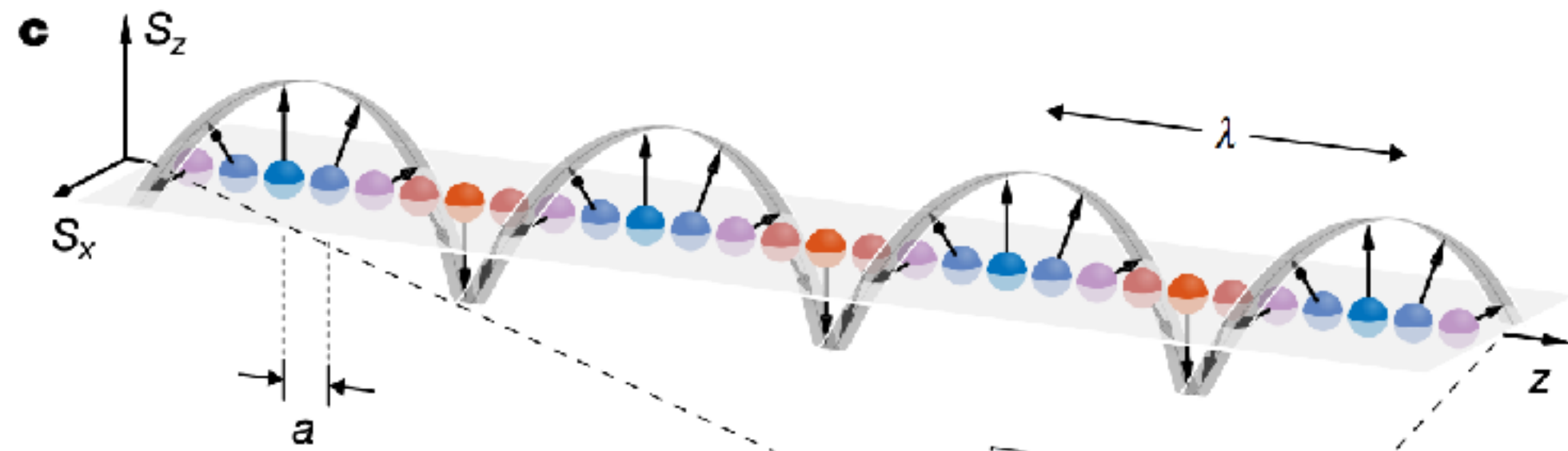


We focus on a low-temp region $T \ll T_c$ with large M^z , where magnons appear

Quantum simulation of ferromagnets: bulk vs tunneling

Cold-atomic experiments on spin dynamics in Heisenberg ferromagnets

Previous experiments



MPI: Fukuhara et al., Nat. Phys (2013); Nature (2013);
 Hild et al., PRL (2014); Wei et al., Science (2022)
 MIT: Jepsen et al., Nature (2020); PRX (2021); Nat. Phys. (2022).

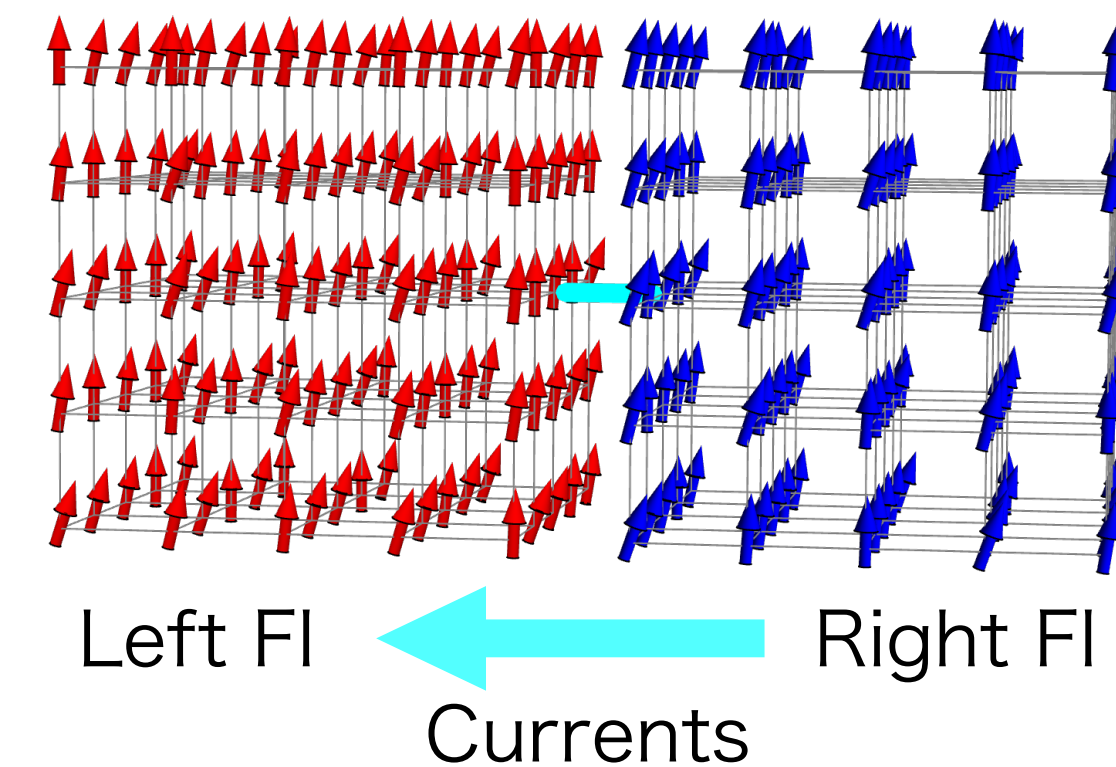
Bulk transport

Mainly 1D Integrable

Far-from equilibrium

Sometimes complex physical picture

Our proposal



Tunneling transport

3D Most relevant to spintronics
 Spontaneous symmetry breaking

Near equilibrium

Clear physical picture from equilibrium states
 Anomalous transport \leftrightarrow Magnonic critical point

Expression of Currents

$$I_S = \int_{-\infty}^{\infty} d\omega \mathcal{T}(\omega) \Delta n_B(\omega)$$

$$I_H = \int_{-\infty}^{\infty} d\omega (\omega + h_L) \mathcal{T}(\omega) \Delta n_B(\omega)$$

$$\begin{pmatrix} I_S \\ I_H \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix} + O((\Delta h)^2, (\Delta T)^2)$$

Transmittance $\mathcal{T}(\omega) \propto (J_T)^2 \rho_L(\omega + h_L) \rho_R(\omega + h_R)$

Magnon DoS $\rho_{\alpha=L,R}(\omega) = \sum_{\vec{k}} \delta(\omega - E_{\vec{k}\alpha}) \propto \sqrt{\omega - h_\alpha}$

Magnon energy $E_{\vec{k}\alpha} = h_\alpha + (J/2)k^2$

Difference of magnon distribution:

$$\Delta n_B(\omega) = n_{B,L}(\omega + h_L) - n_{B,R}(\omega + h_R)$$

$$n_{B,L/R}(\omega) = \frac{1}{1 + e^{\omega/T_{L/R}}}$$

Conductance:

$$\frac{L_{11}}{AT} = F_1(x),$$

$$\frac{L_{12}}{AT} = \frac{L_{21}}{AT^2} = 2F_2(x) + xF_1(x),$$

$$\frac{L_{22}}{AT^2} = 6F_3(x) + 4xF_2(x) + x^2F_1(x),$$

Bose-Einstein integral:

$$F_d(x_\alpha = h_\alpha/T_\alpha) \propto \int_0^\infty d\omega \frac{\omega^{d-1}}{e^{(\omega+h_\alpha)/T_\alpha} - 1}$$

Conductance

Expansion in small bias

$$\Delta h = -(h_L - h_R)$$

$$\Delta T = T_L - T_R$$

$$\begin{pmatrix} I_S \\ I_H \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

Conductance:

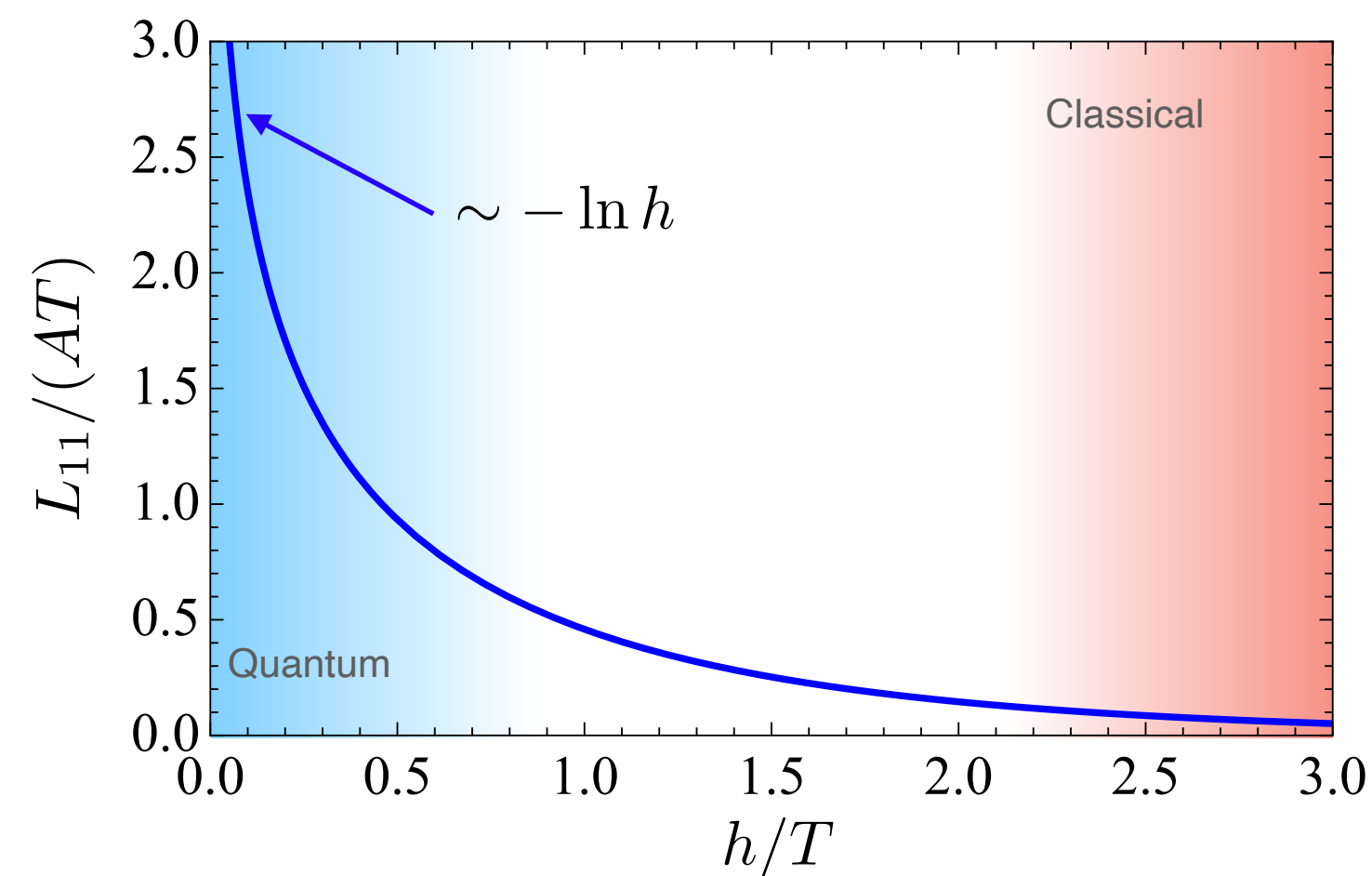
$$\frac{L_{11}}{AT} = F_1(x),$$

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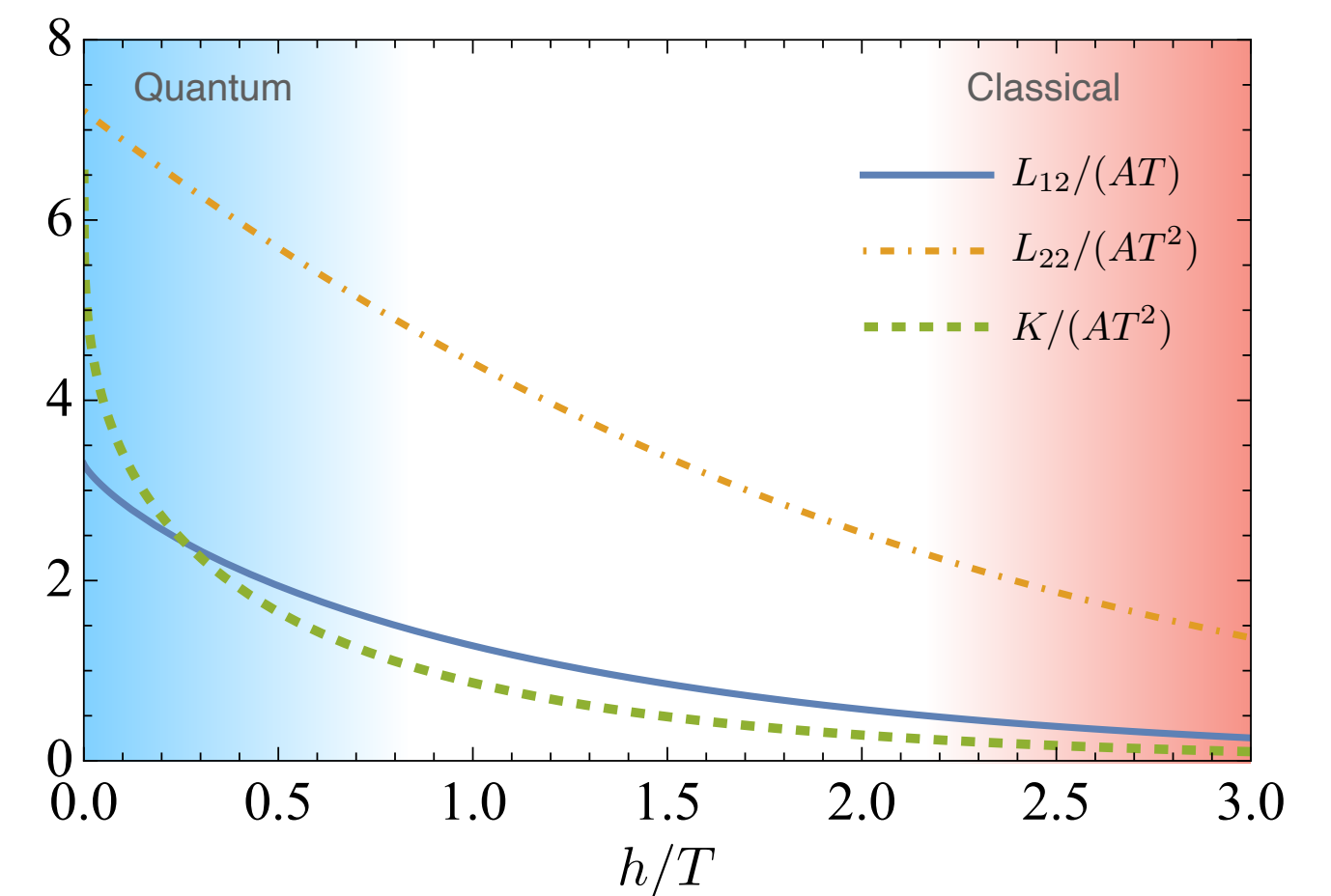
$$\frac{L_{22}}{AT^2} = 6F_3(x) + 4xF_2(x) + x^2F_1(x),$$

$$x = h/T, \quad h = (h_L + h_R)/2, \quad T = (T_L + T_R)/2$$

Divergent L_{11}



Convergent L_{12}, L_{21}, L_{22}



$$F_d(x_\alpha = h_\alpha/T_\alpha) \propto \int_0^\infty d\omega \frac{\omega^{d-1}}{e^{(\omega+h_\alpha)/T_\alpha} - 1}$$

How to experimentally determine conductances?

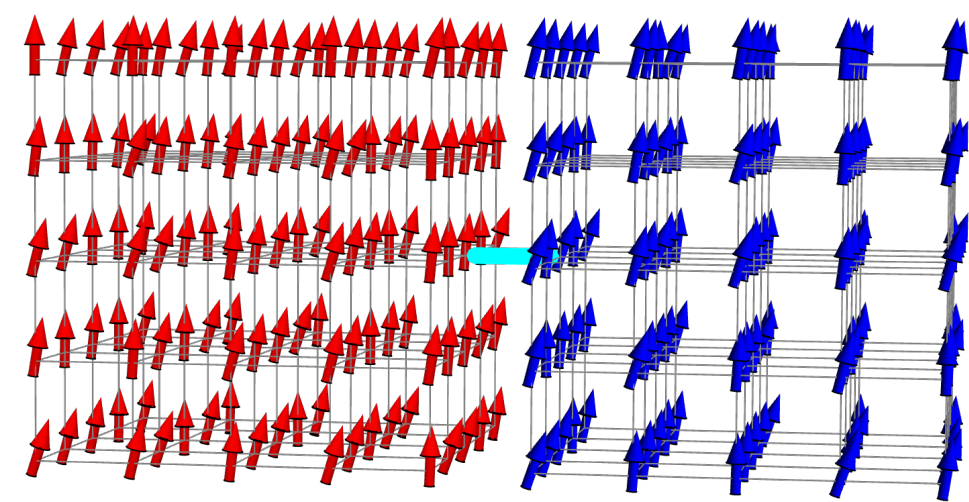
YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

Following the scheme proposed for Fermi-gas cases [ETH: Brantut et al., Science (2013)],

we can experimentally determine **conductances** L_{ij} by measuring

A. Near-equilibrium relaxation dynamics of $\Delta M(t) = M_L(t) - M_R(t)$, $\Delta T(t) = T_L(t) - T_R(t)$ b/w FIs from a thermal initial state

Fit observed $\Delta M(t), \Delta T(t)$ with Quasi-stationary model



$$M_L(t) \longleftrightarrow M_R(t)$$

$$T_L(t) \longleftrightarrow T_R(t)$$

$$\begin{pmatrix} \Delta M(t) \\ \Delta T(t) \end{pmatrix} = \Lambda(t; \mathbf{K}, \mathbf{L}) \begin{pmatrix} \Delta M(0) \\ \Delta T(0) \end{pmatrix}$$

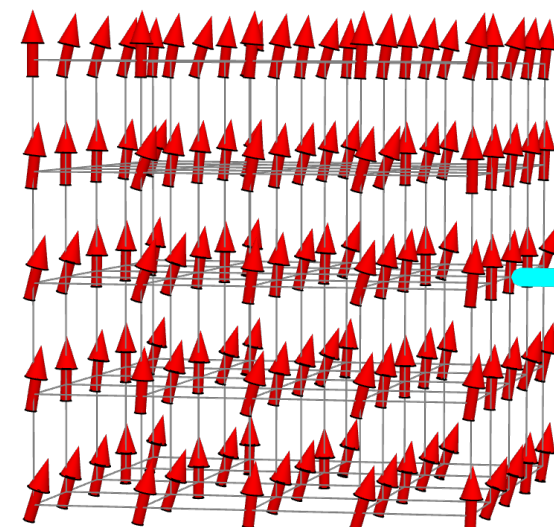
$$\mathbf{K} = \begin{pmatrix} \left(\frac{\partial M}{\partial h}\right)_T & -\left(\frac{\partial M}{\partial T}\right)_h \\ -\left(\frac{\partial S}{\partial h}\right)_T & \left(\frac{\partial S}{\partial T}\right)_h \end{pmatrix}$$

Thermodynamic quantities

$$\mathbf{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}$$

Conductances

B. Thermodynamic quantities of one FI at equilibrium



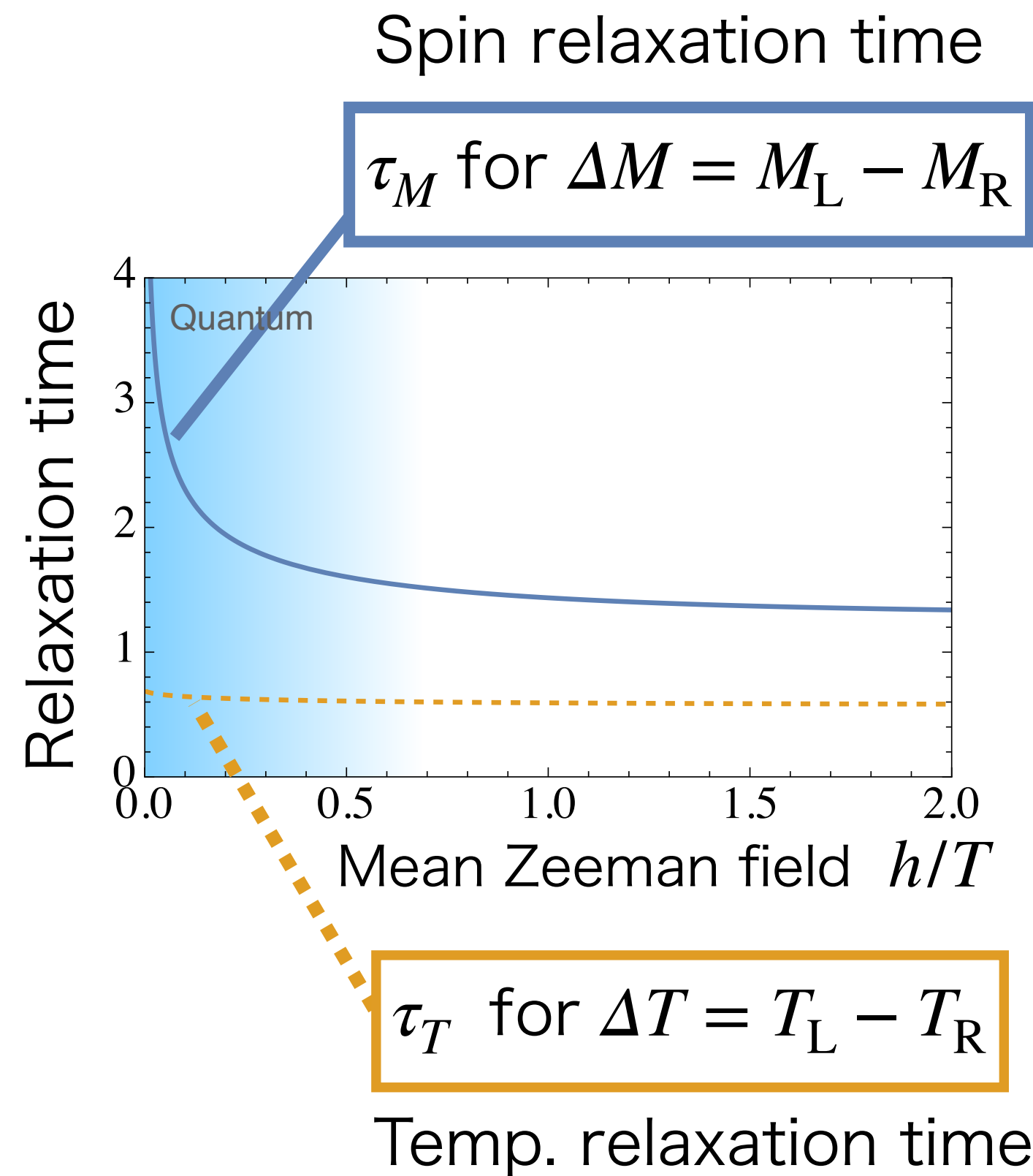
$$h = [h_L(0) + h_R(0)]/2$$

$$T = [T_L(0) + T_R(0)]/2$$

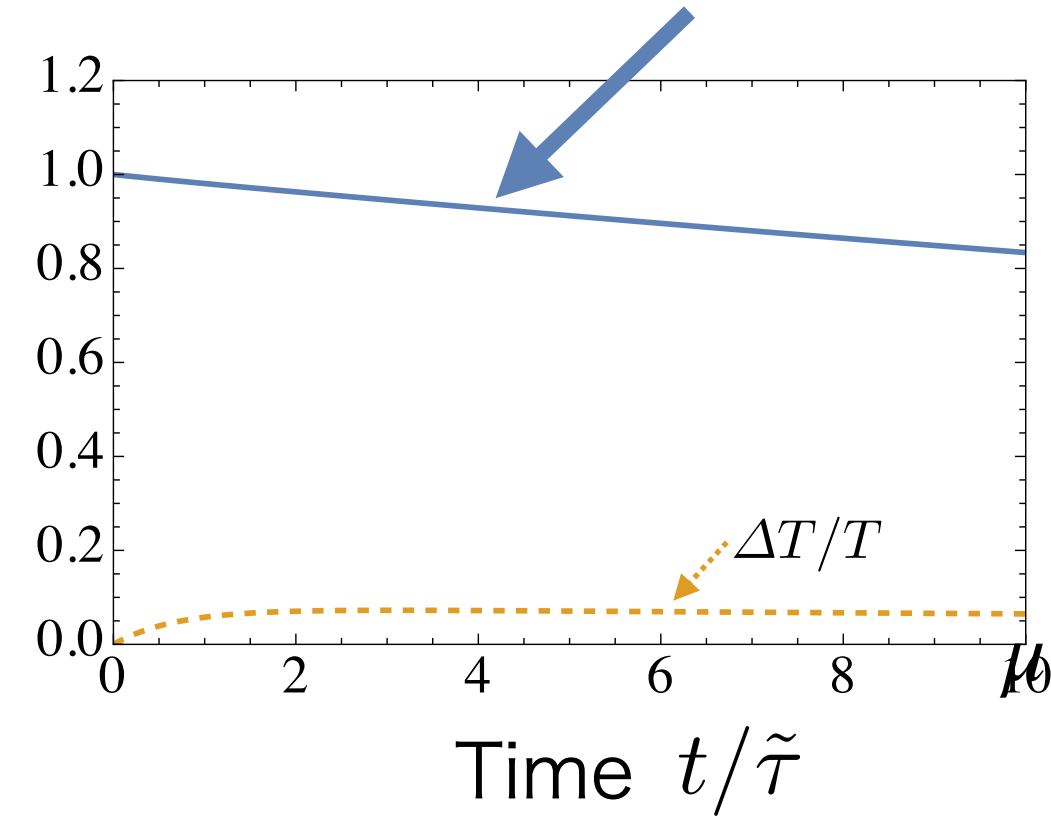
Conductances is determined!!

Slowing down of magnetization relaxation

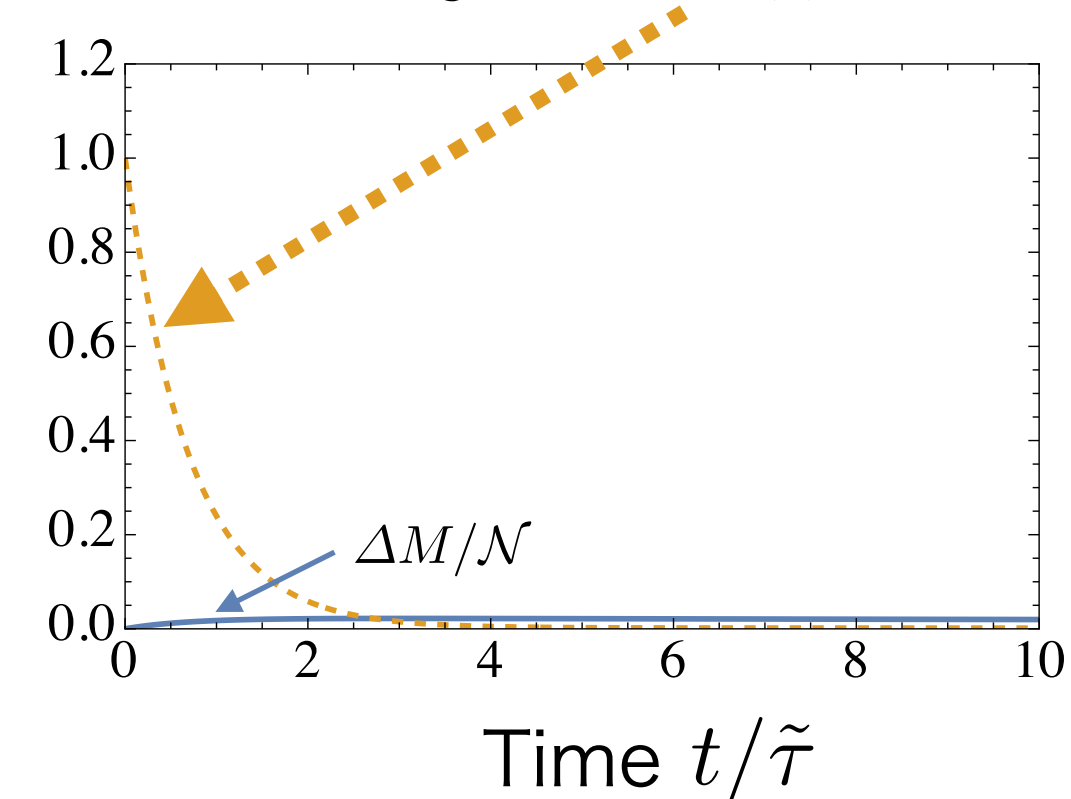
Extremely slow spin relaxation by critical behavior of magnon compressibility



Slow decay of $\Delta M(t) \sim \exp(-t/\tau_M)$



Fast decay of $\Delta T(t) \sim e^{-t/\tau_T}$



$$\tau_M \sim \frac{\kappa}{L_{11}} \sim \frac{1/\sqrt{h}}{-\log h} \rightarrow \infty \quad (h \rightarrow +0)$$

L_{11} : Spin conductance

Diverging magnon compressibility by criticality

$$\kappa = \left(\frac{\partial M}{\partial h} \right)_T \sim 1/\sqrt{h} \rightarrow \infty \quad (h \rightarrow +0)$$

Magnonic critical point = BEC transition point for magnons

$$h \rightarrow +0$$

$$\mu_{\text{magnon}} \rightarrow -0$$

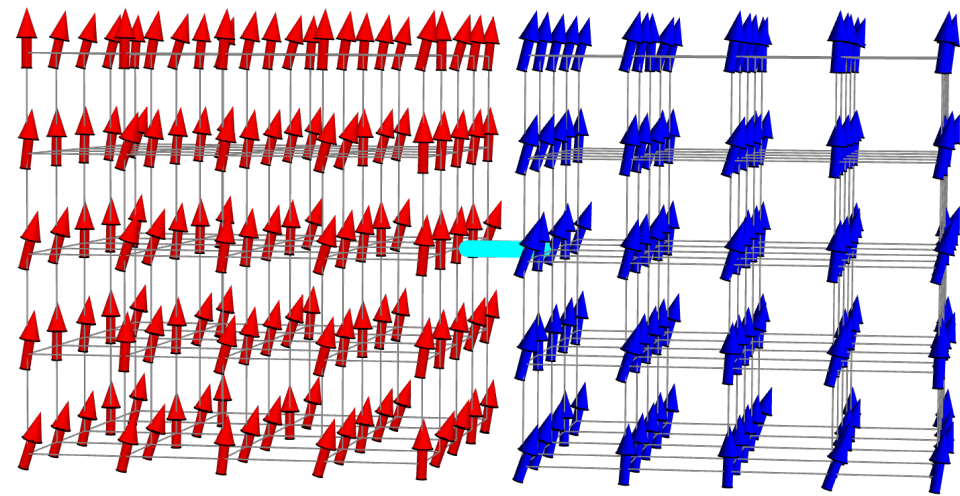
$$-h = \mu_{\text{magnon}}$$

c.f. divergent κ at BEC transition

Relaxation dynamics

Fermi-gas cases [ETH: Brantut et al., Science (2013)],

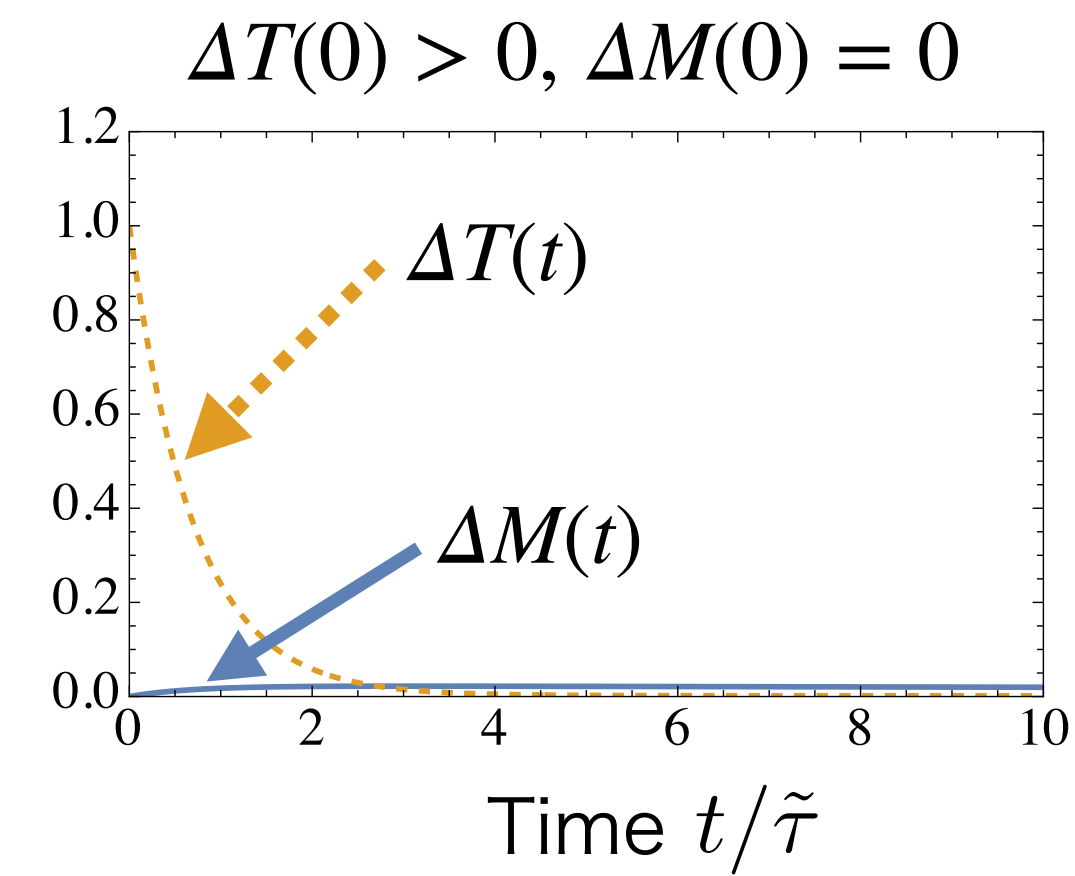
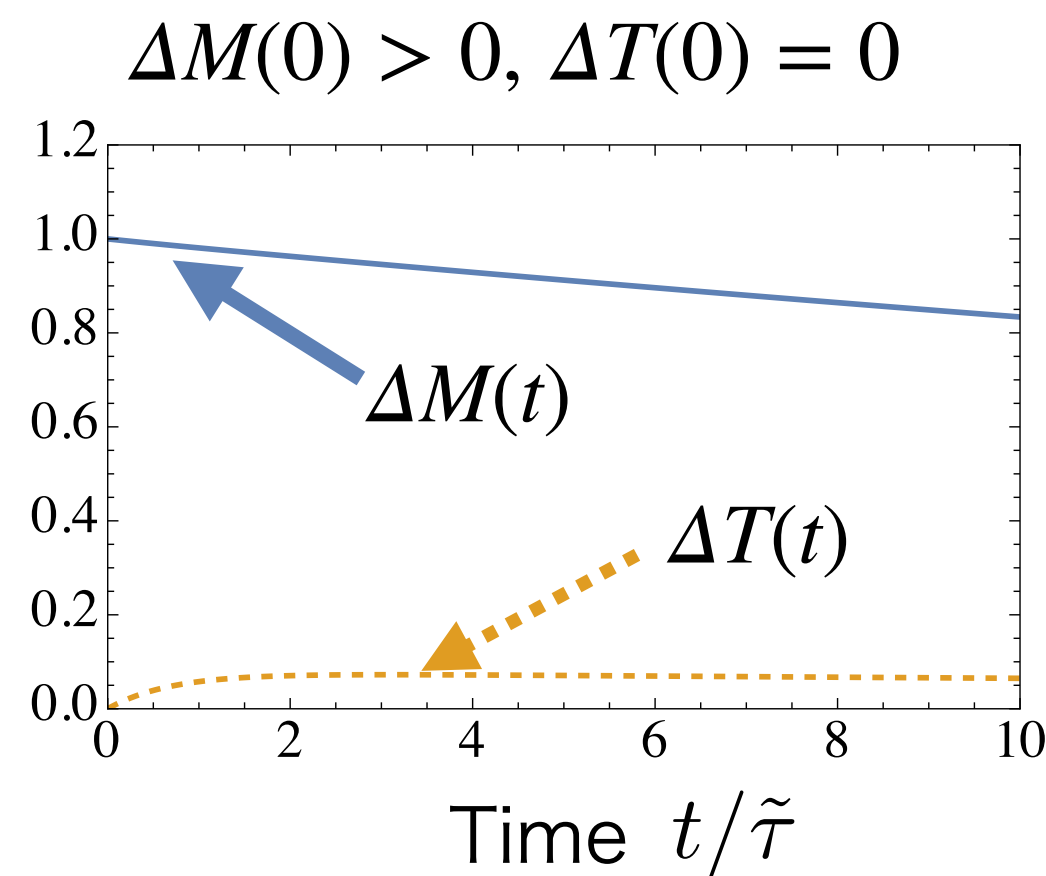
A. Near-equilibrium relaxation dynamics of $\Delta M(t) = M_L(t) - M_R(t)$, $\Delta T(t) = T_L(t) - T_R(t)$ b/w FIs from a thermal initial state



$$M_L(t) \longleftrightarrow M_R(t)$$

$$T_L(t) \longleftrightarrow T_R(t)$$

1. Close the channel and prepare thermal states with $M_{L/R}(0)$, $T_{L/R}(0)$
2. At $t = 0$, open the channel so that $M_{L/R}(t)$, $T_{L/R}(t)$ start time evolution
3. Observe $\Delta M(t) = M_L(t) - M_R(t)$ $\Delta T(t) = T_L(t) - T_R(t)$ at time t

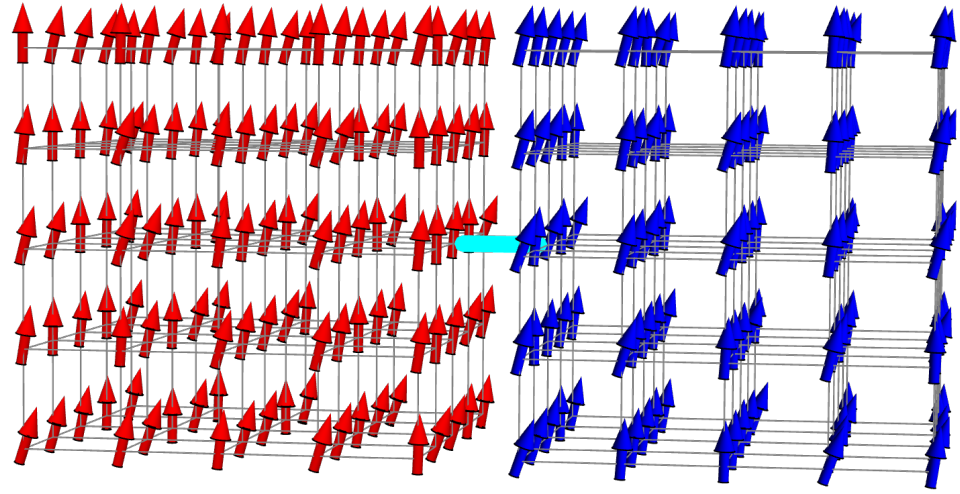


4. Fit obtained $\Delta M(t)$ and $\Delta T(t)$ with solutions of the quasi-stationary model

$$\begin{pmatrix} \Delta M(t)/\mathcal{N} \\ \Delta T(t)/T \end{pmatrix} = \begin{pmatrix} \Lambda_{11}(t) & \Lambda_{12}(t) \\ \Lambda_{21}(t) & \Lambda_{22}(t) \end{pmatrix} \begin{pmatrix} \Delta M(0)/\mathcal{N} \\ \Delta T(0)/T(0) \end{pmatrix}$$

Quasi-stationary model

Fermi-gas cases [ETH: Brantut et al., Science (2013)],



$$\begin{pmatrix} \Delta M(t)/\mathcal{N} \\ \Delta T(t)/T \end{pmatrix} = \Lambda(t; \mathbf{L}, \mathbf{K}) \begin{pmatrix} \Delta M(0)/\mathcal{N} \\ \Delta T(0)/T(0) \end{pmatrix}$$

Matrix $\Lambda(t; \mathbf{L}, \mathbf{K})$ depend on **conductances** and **thermodynamic quantities**

$$\mathbf{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} \left(\frac{\partial M}{\partial h}\right)_T & -\left(\frac{\partial M}{\partial T}\right)_h \\ -\left(\frac{\partial S}{\partial h}\right)_T & \left(\frac{\partial S}{\partial T}\right)_h \end{pmatrix}$$

Because $\begin{pmatrix} \Delta M(t)/\mathcal{N} \\ \Delta T(t)/T \end{pmatrix}$ is the solution of the following equations:

$$\text{Transport relation: } \frac{d}{dt} \begin{pmatrix} -\Delta M \\ T \Delta S \end{pmatrix} = -2 \begin{pmatrix} I_S \\ I_H \end{pmatrix} = -2 \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

$$\text{Thermodynamic relation: } \begin{pmatrix} -\Delta M \\ \Delta S \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial M}{\partial h}\right)_T & -\left(\frac{\partial M}{\partial T}\right)_h \\ -\left(\frac{\partial S}{\partial h}\right)_T & \left(\frac{\partial S}{\partial T}\right)_h \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

Form of $\Lambda(t; L, K)$

$$\Lambda(t; \mathbf{L}, \mathbf{K}) = \begin{pmatrix} \Lambda_{11}(t) & \Lambda_{12}(t) \\ \Lambda_{21}(t) & \Lambda_{22}(t) \end{pmatrix}$$

$$\Lambda_{11/22}(t) = \frac{1}{2}(e^{-t/\tau_-} + e^{-t/\tau_+}) \pm \frac{1}{2} \frac{\frac{L+\alpha^2}{l} - 1}{\lambda_+ - \lambda_-} (e^{-t/\tau_-} - e^{-t/\tau_+})$$

$$\Lambda_{12}(t) = \left(\frac{T\kappa}{\mathcal{N}}\right)^2 l \Lambda_{21}(t) = -\frac{T}{\mathcal{N}} \frac{\alpha\kappa}{\lambda_+ - \lambda_-} (e^{-t/\tau_-} - e^{-t/\tau_+})$$

$$\tau_{\pm} = \tau_0 / \lambda_{\pm} \quad \lambda_{\pm} = \frac{1}{2} \left(1 + \frac{L + \alpha^2}{l} \right) \pm \sqrt{\frac{\alpha^2}{l} + \frac{1}{4} \left(1 - \frac{L + \alpha^2}{l} \right)^2}, \quad \tau_0 = \frac{\kappa}{2L_{11}}, \quad \alpha = \alpha_r - \alpha_{ch}.$$

$$\mathbf{K} = \begin{pmatrix} \left(\frac{\partial M}{\partial h}\right)_T & -\left(\frac{\partial M}{\partial T}\right)_h \\ -\left(\frac{\partial S}{\partial h}\right)_T & \left(\frac{\partial S}{\partial T}\right)_h \end{pmatrix} = \kappa \begin{pmatrix} 1 & -\alpha_r \\ -\alpha_r & l + \alpha_r^2 \end{pmatrix} \quad \mathbf{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} = L_{11} \begin{pmatrix} 1 & \alpha_{ch} \\ \alpha_{ch} & L + \alpha_{ch}^2 \end{pmatrix}$$

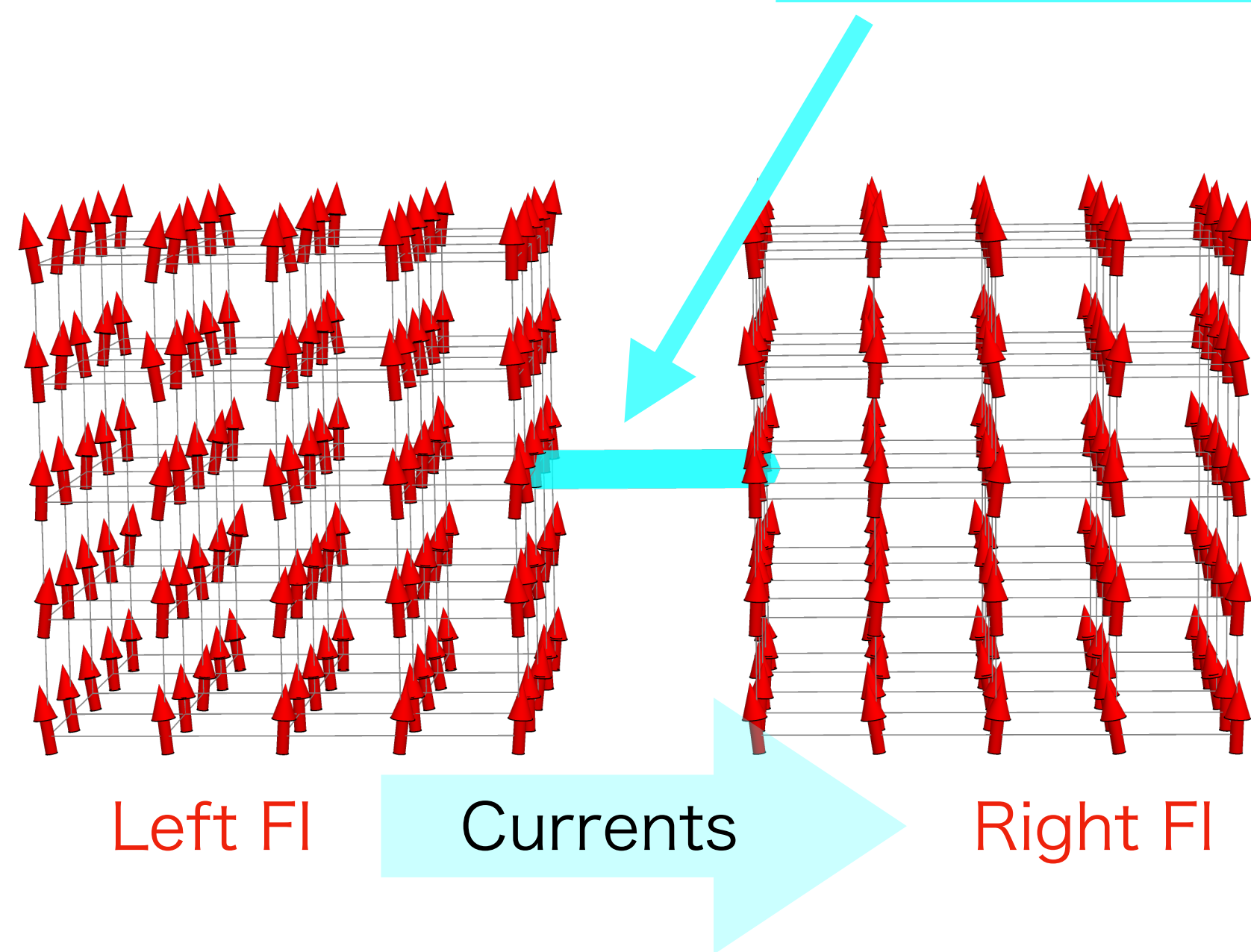
$$\kappa = \left(\frac{\partial N}{\partial \mu}\right)_T, \quad \alpha_r = \left(\frac{\partial S}{\partial N}\right)_T, \quad l = \frac{C_N}{\kappa T} = \frac{1}{\kappa} \left(\frac{\partial S}{\partial T}\right)_N \quad G = 2L_{11}, \quad \alpha_{ch} = \frac{L_{12}}{L_{11}}, \quad L = \frac{L_{22}}{TL_{11}} - \left(\frac{L_{12}}{L_{11}}\right)^2.$$

OLD slides

Takehome message: Tunneling transport by criticality

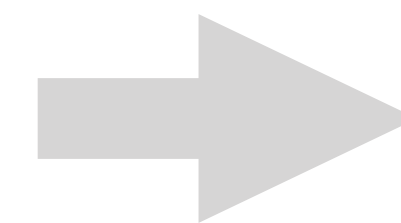
Anomalous tunneling spin and heat transport near **magnonic critical points** of ferromagnets

Two ferromagnetic insulators (FIs) realized with **cold atoms** connected via a **quantum point contact**

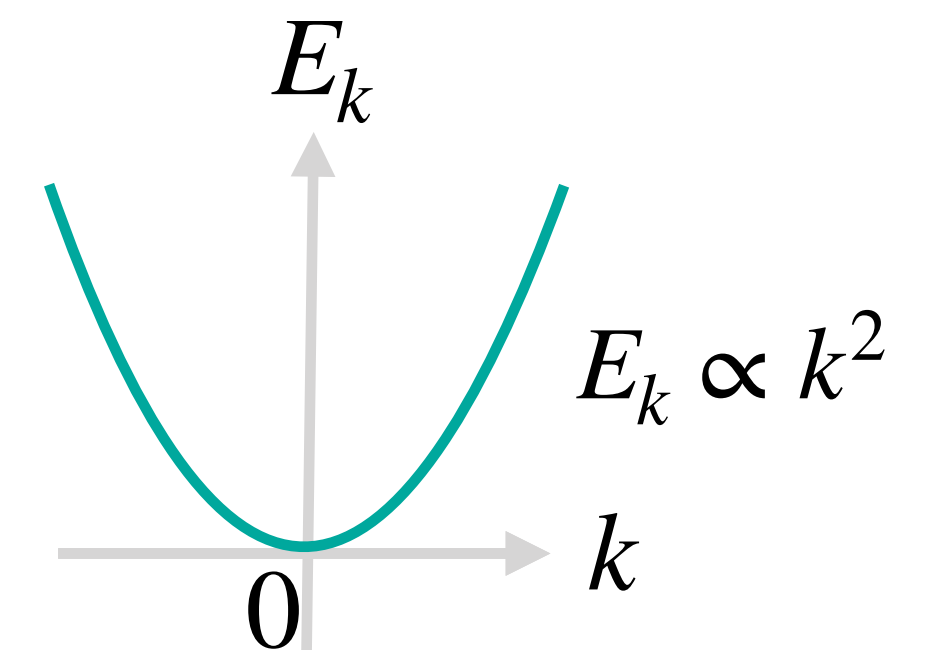
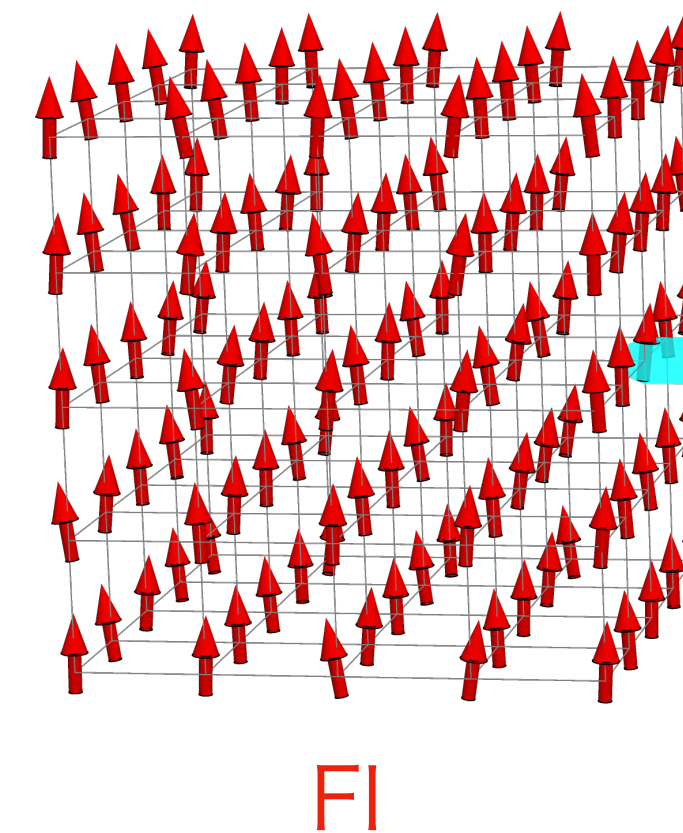


Gapless point of magnons in ferromagnets

Spontaneous breaking of $O(3)$ symmetry in FIs



Magnon as **gapless Nambu-Goldstone mode**

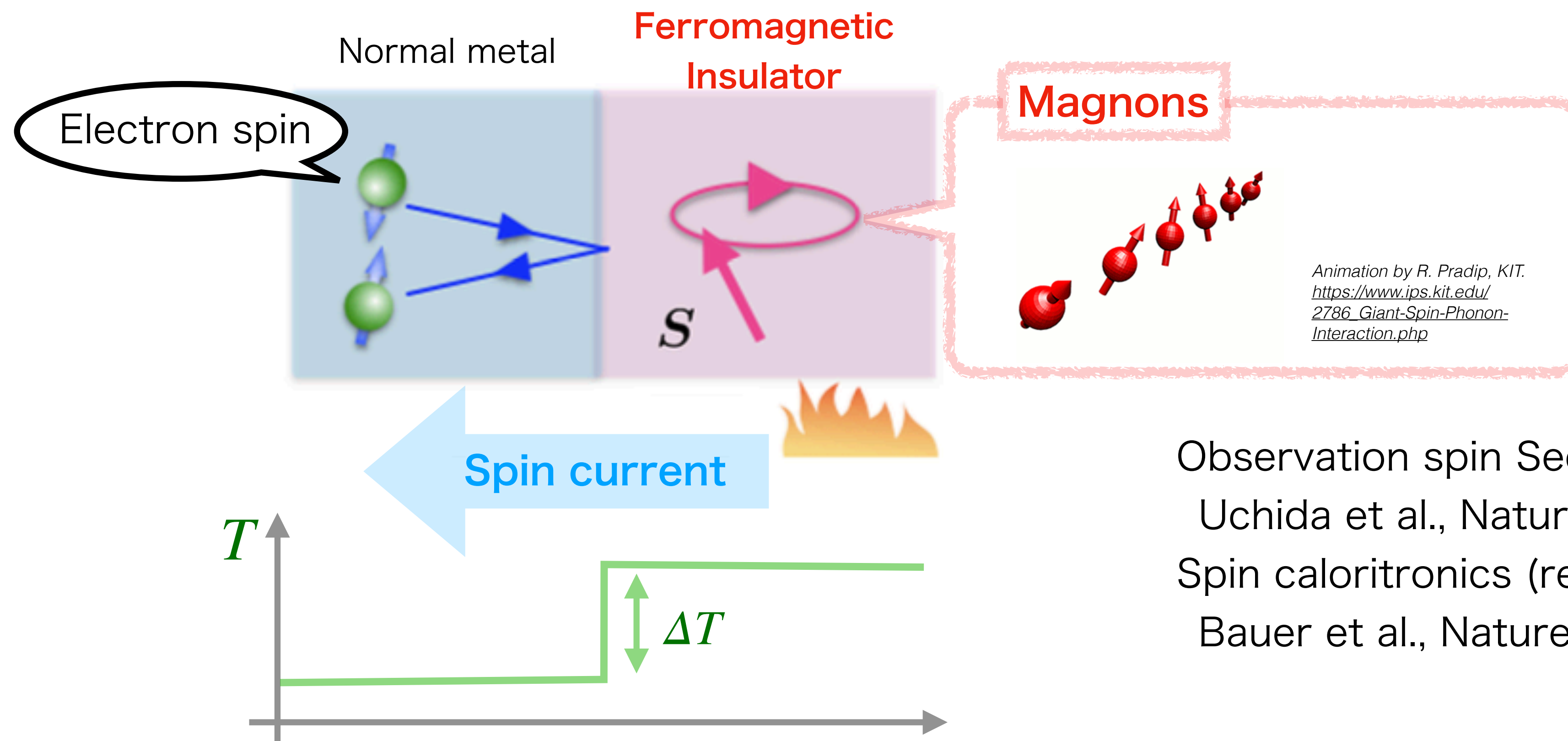


Anomalous enhancement of spin & heat conductances resulting from **the magnonic criticality**

Motivation from solid-state physics

Spin and **heat** tunneling transport with **magnons** is one of the hot topics in spintronics focusing on efficient **spin-heat** conversion for devices applications

► **Spin Seebeck effect**: **spin-current** generation by **temperature bias ΔT**



Observation spin Seebeck effect:

Uchida et al., Nature **455**, 778–781 (2008)

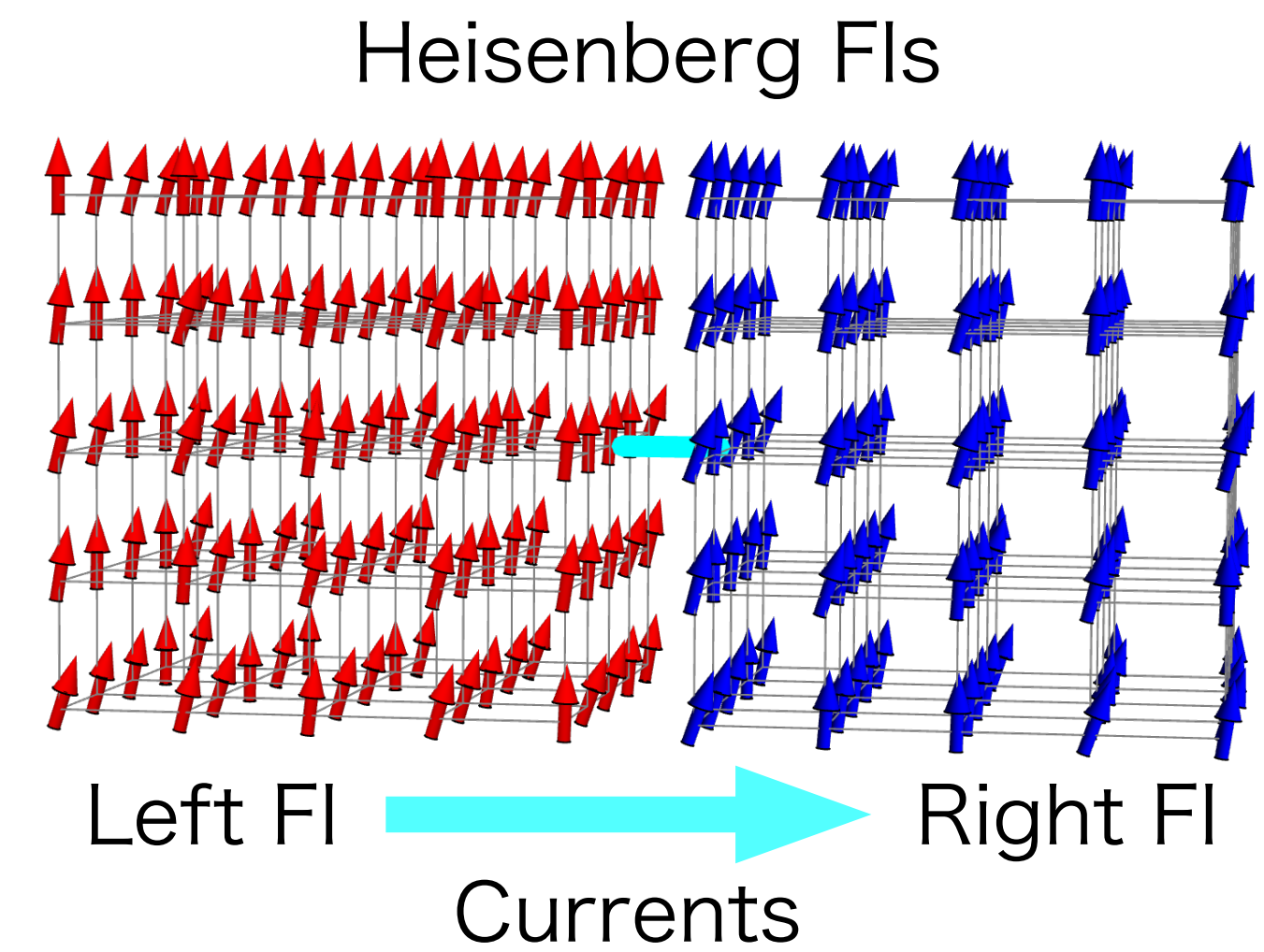
Spin caloritronics (review):

Bauer et al., Nature Materials **11**, 391–399 (2012)

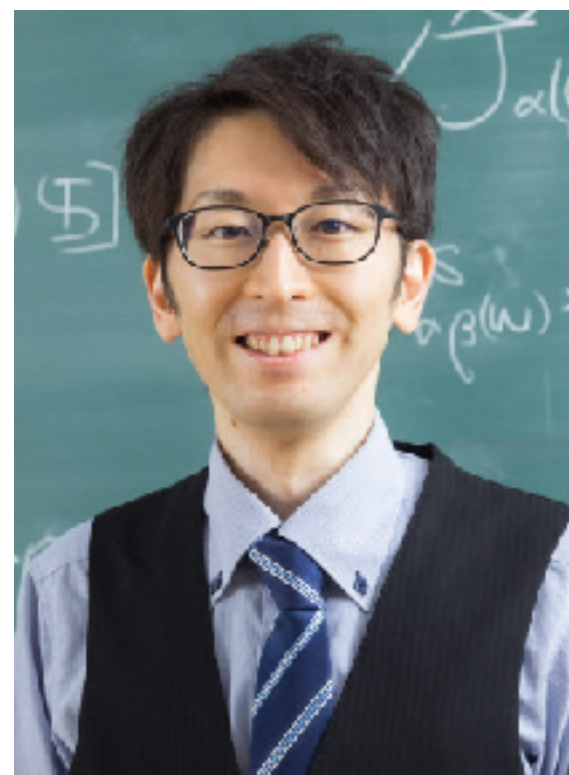
Quantum simulation of magnonic transport

To bridge **cold atoms and spintronics**, we propose a quantum simulation of spin & heat tunneling transport of magnons b/w ferromagnets

YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280



Cold-atomic physics



Y. Sekino
RIKEN



H. Tajima
U. Tokyo



S. Uchino
Waseda Univ.



Solid-state physics



Y. Ominato
Waseda Univ.



M. Matsuo
UCAS, China

Why tunneling spin & heat transport w/ cold atoms?

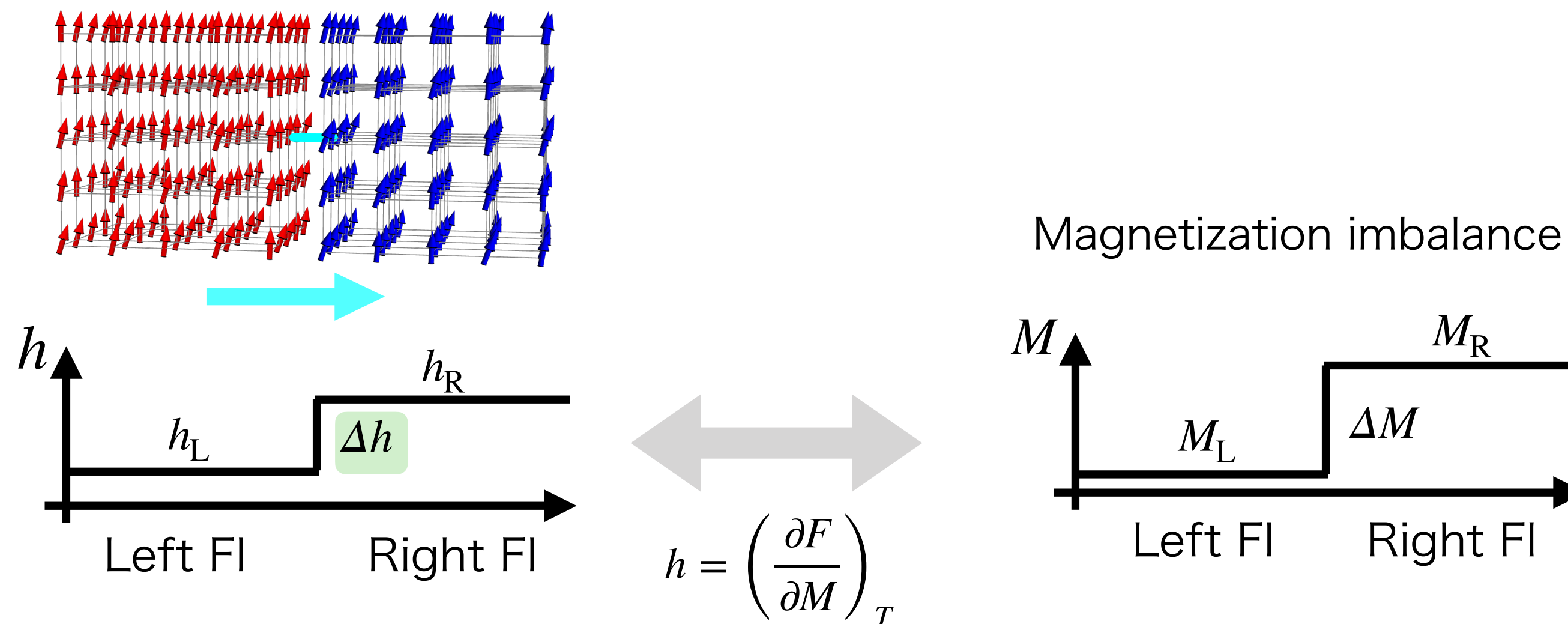
1. Ultraclean systems

No impurity

No roughness & lattice mismatch

2. Quantum controllability of effective Zeeman fields Similarly to Fermi gases [Kriner et al., PNAS, **113** (29) 8144-8149 (2016)]

Control of spin bias Δh to generate spin & heat currents



No solid-state experiment b/w FIs

because inducing Δh by

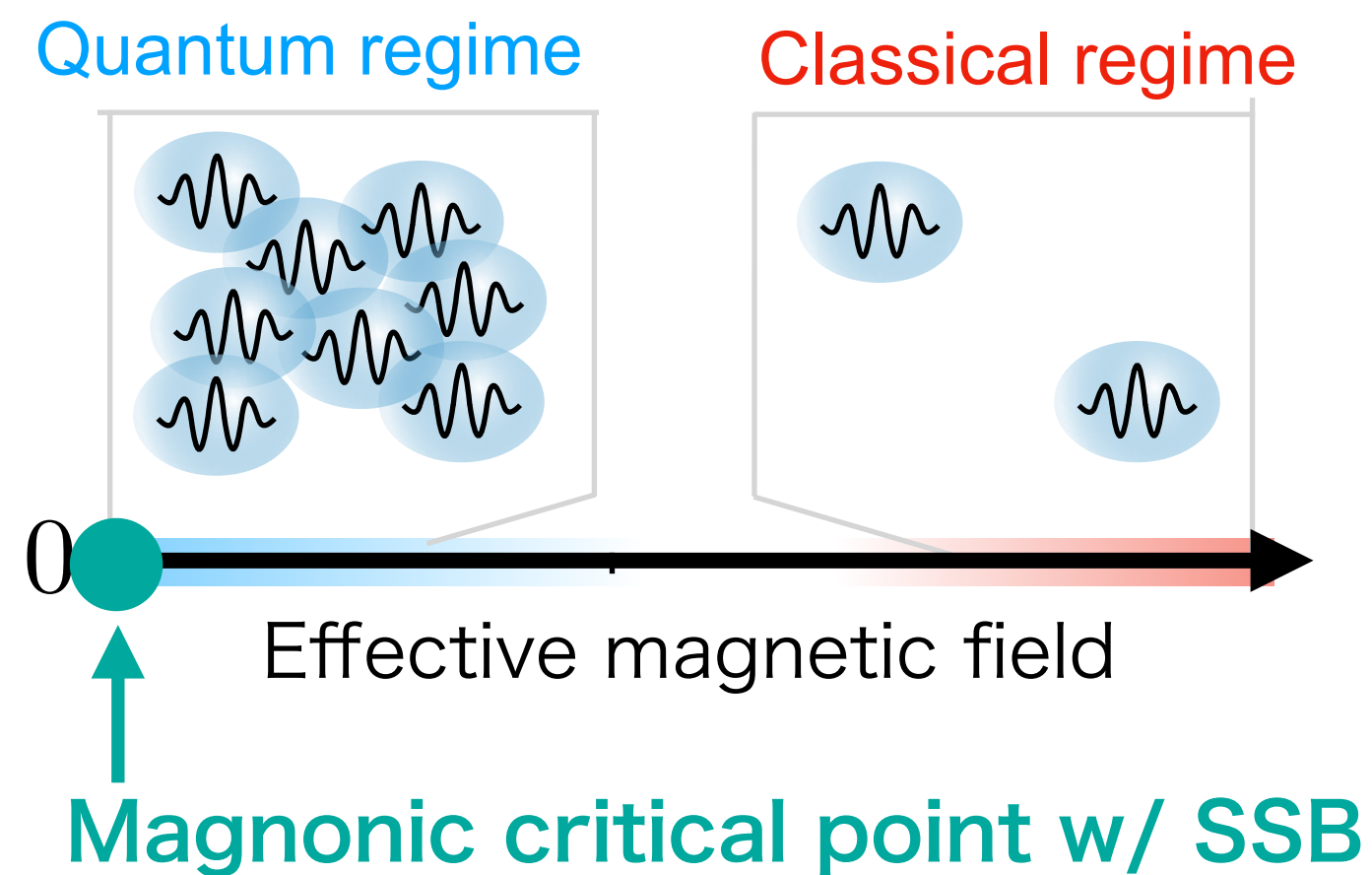
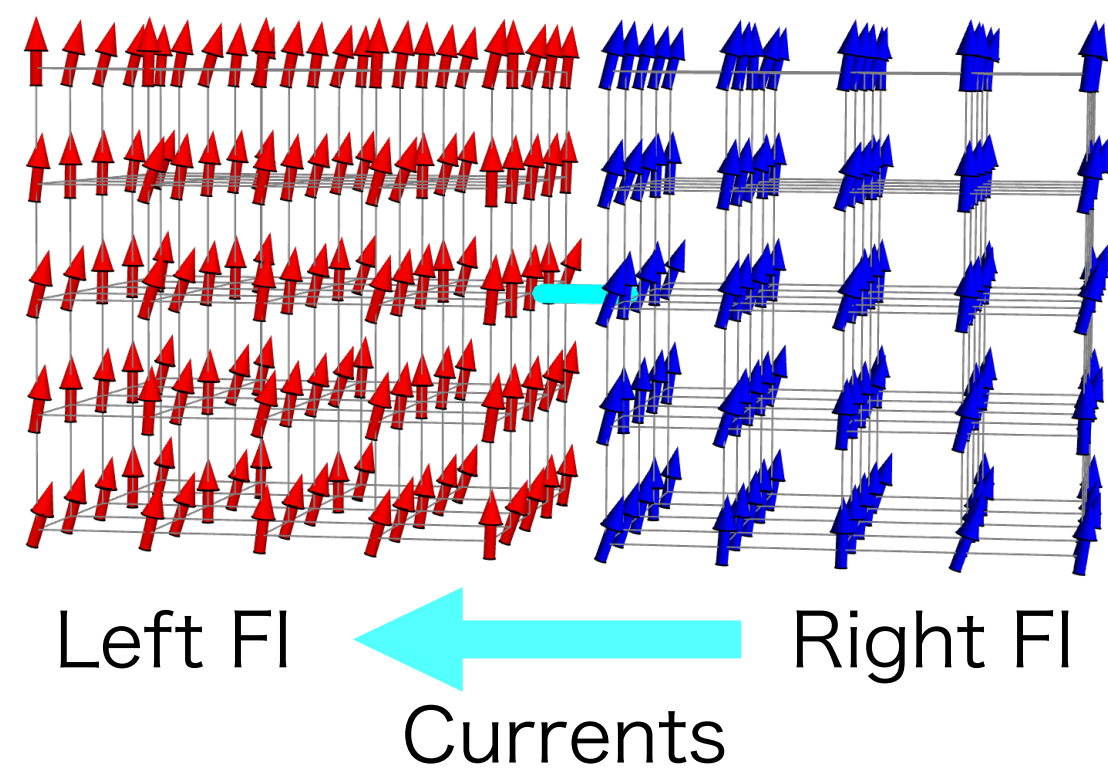
spatially-modulated magnetic field is difficult

Effective Zeeman fields can be tuned by optically controlling $M_{L/R} = (N_{\uparrow}^{L/R} - N_{\downarrow}^{L/R})/2$
 $h_{L/R}$

Summary of setup and results

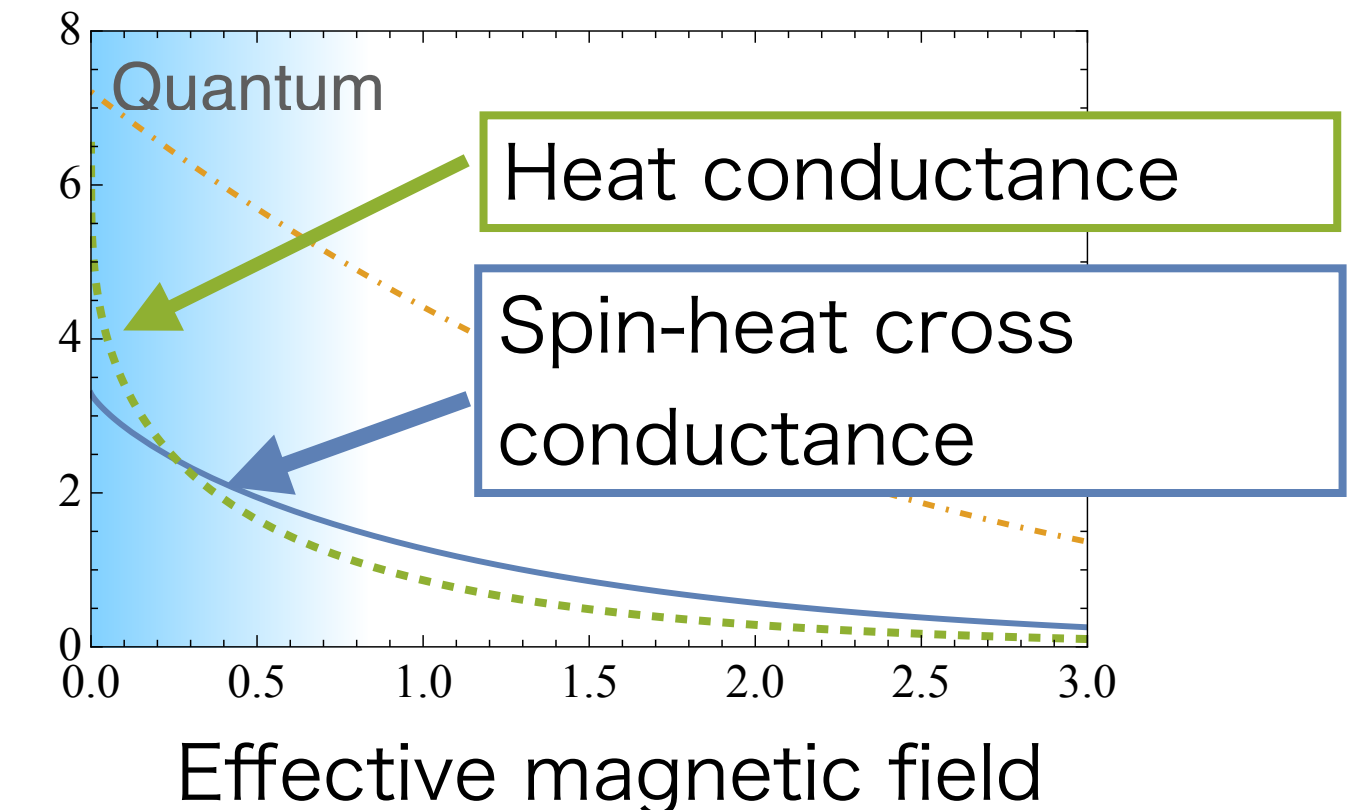
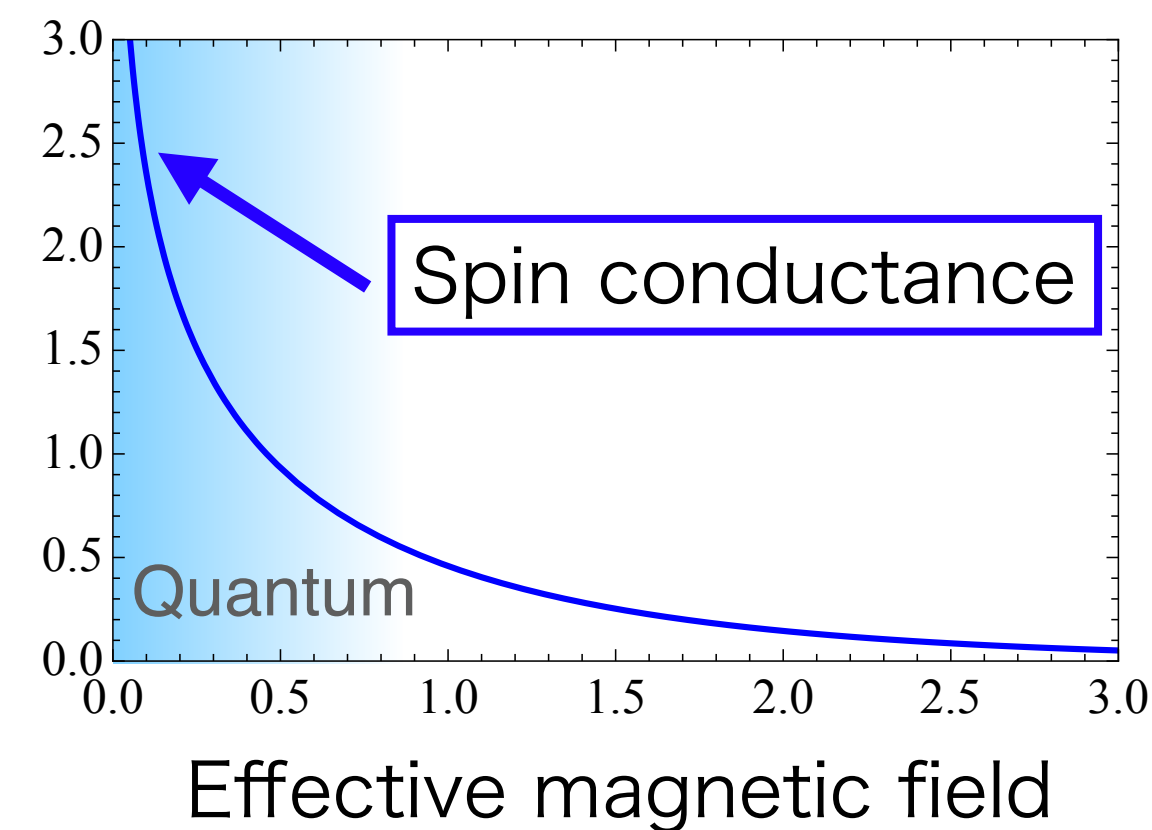
YS, Ominato, Tajima, Uchino, & Matsuo, arXiv:2312.04280

Highly spin-polarized FIs connected with a magnetic quantum point contact



Anomalous thermomagnetic transport by **magnonic criticality**

Enhancement in spin & heat conductances



Large-scale optical lattice

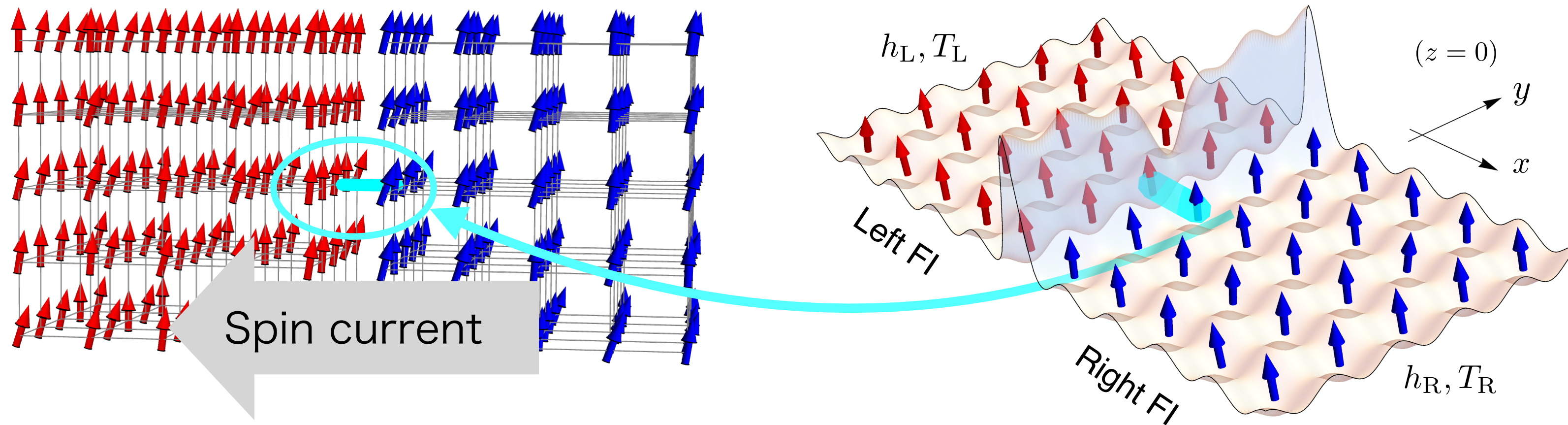
Timeliness:

巨大な系を用意できる => 輸送をやるよね

アボガドロ数での固体系での、スピントロニクス応用できるよね

Takehome message: Tunneling transport by magnonic criticality

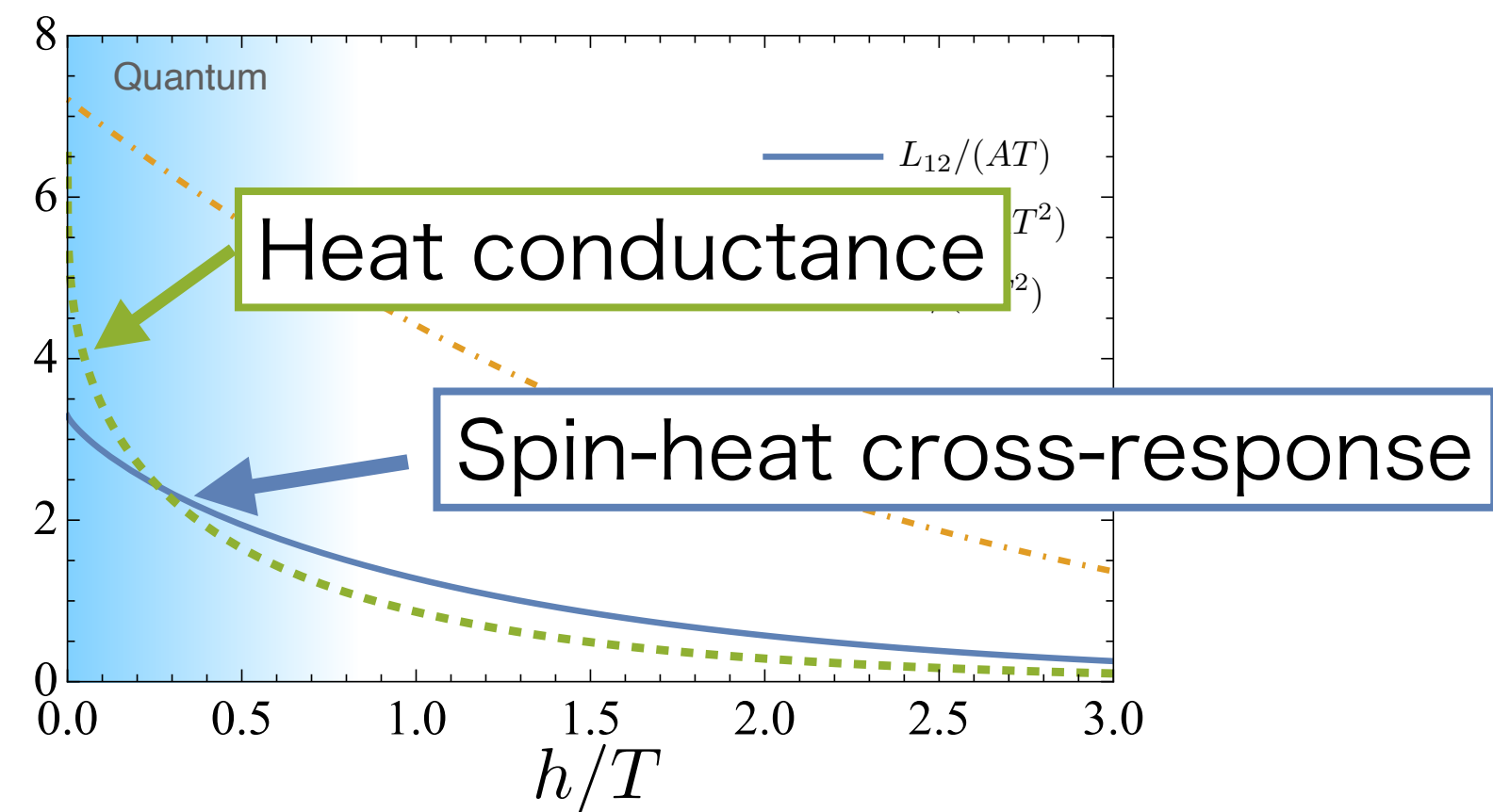
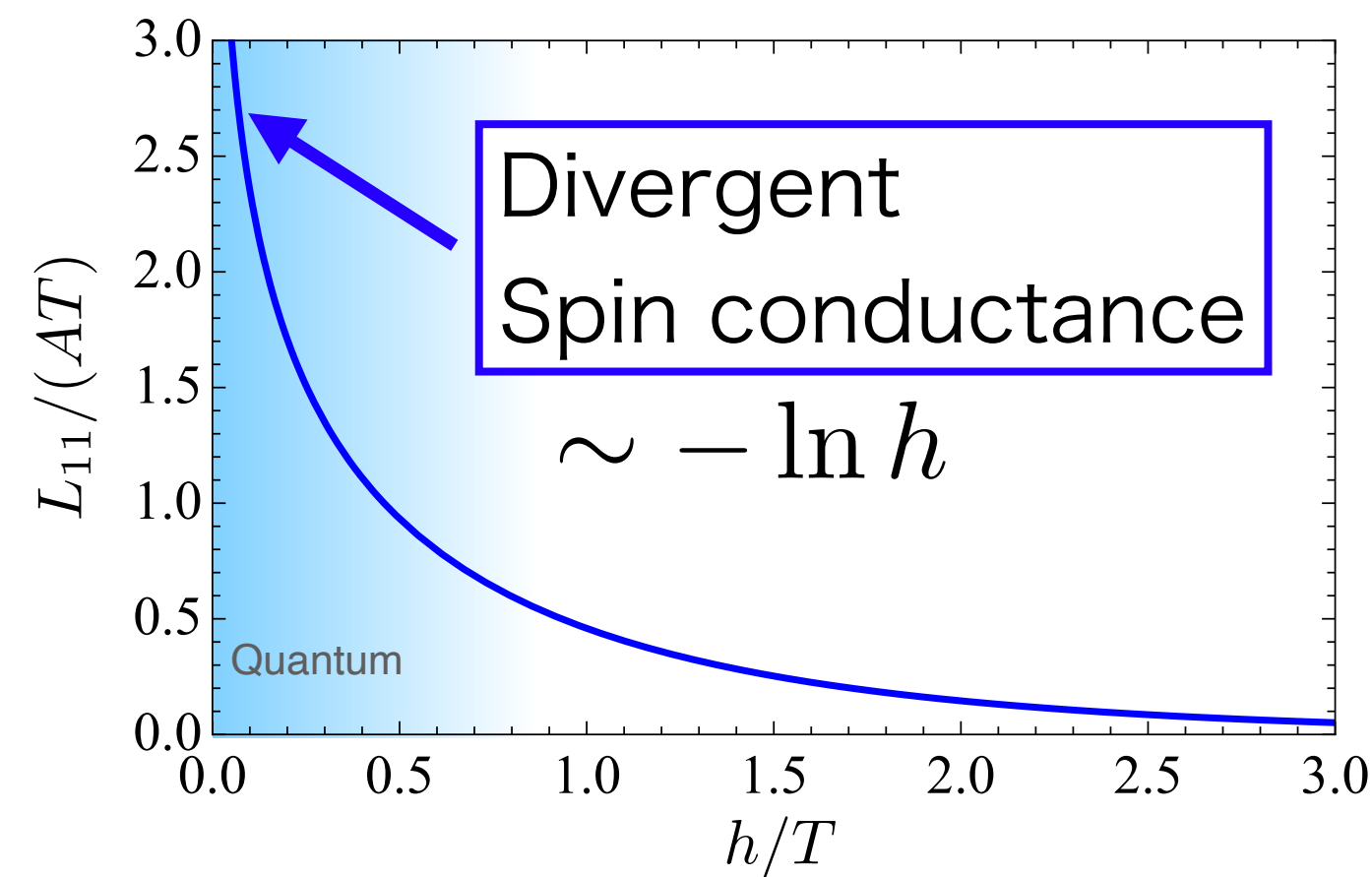
Anomalous tunneling transport due to gapless magnons at the magnonic critical point



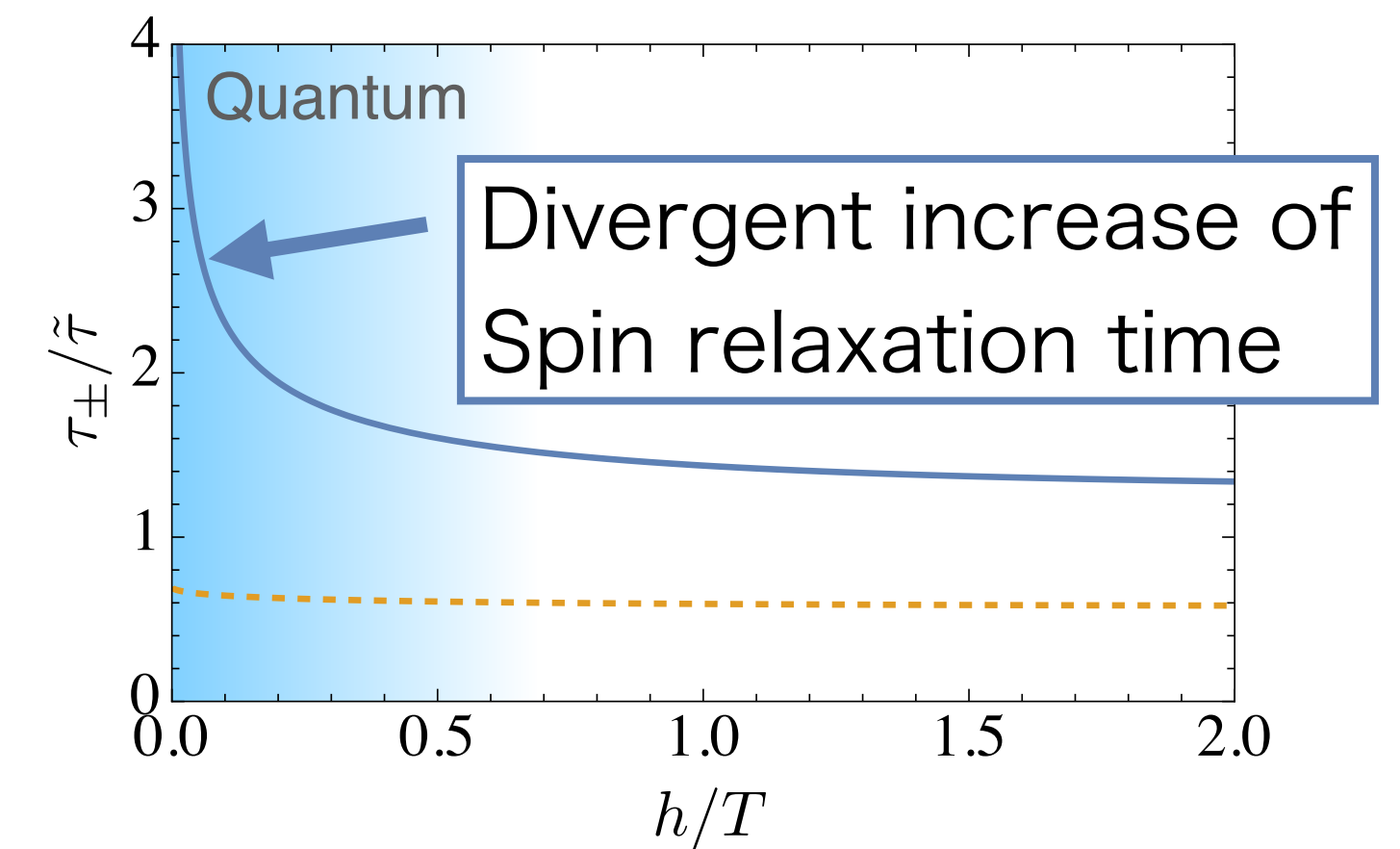
Two Heisenberg ferromagnets connected via a quantum point contact

Anomalous spin & heat transport properties

1. Anomalous enhancement in spin & heat transport coefficients



2. Extremely slow spin relaxation

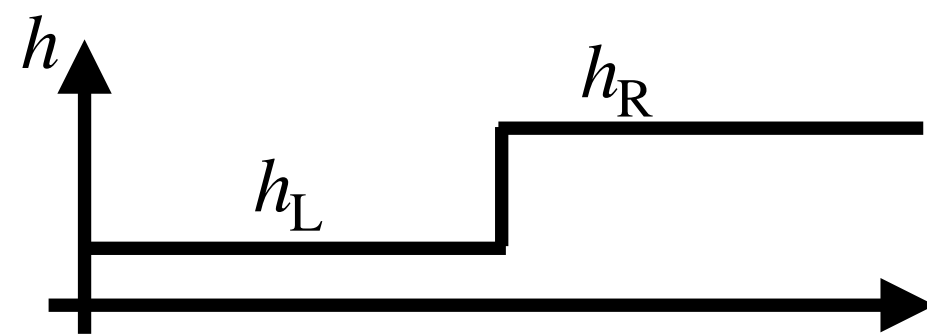


Advantage of cold atoms to investigate tunnel transport

Quantum controllability

冷却原子のメリットをともかくおす（結論を先に言う）
 固体はどうかということと比較はあまりしない
 その一方で、固体では、無理

1. Control of spin bias to induce spin current



2. Ultraclean systems

No impurity, lattice defect, & lattice vibration
 Interfering spin currents（整合性を後で考える）

Roughness, lattice mismatch も追加しても良いかも

3. Control of interface by lasers

Previous proposal for solids

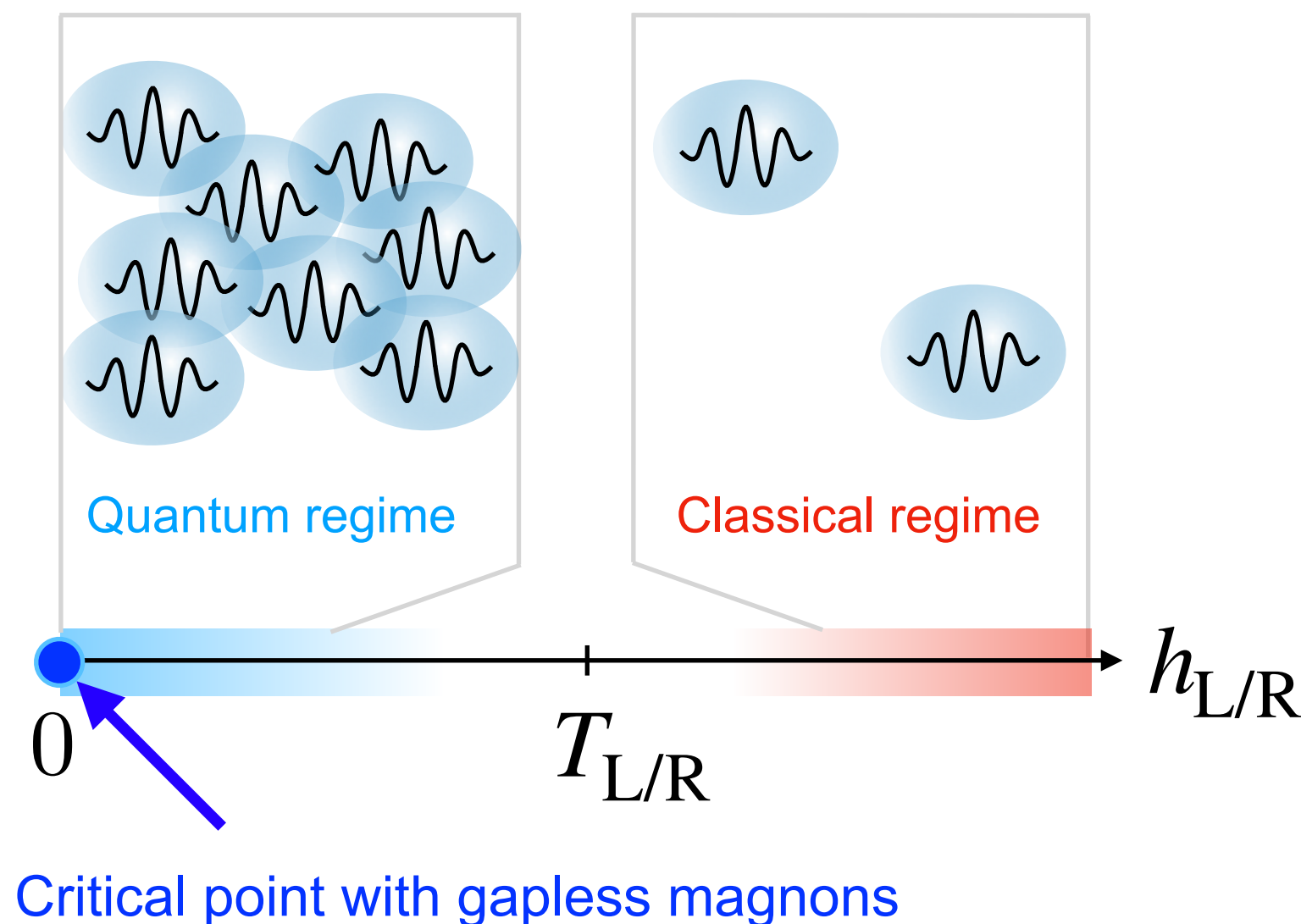
Nakata et al., PRB (2015);
 PRB (2018); ...

To avoid the emergence of magnetic domains

To avoid the emergence of magnetic domains

2つの大きな困難がある：

1. 磁場勾配を定量性が出るように制御するのが困難（古典ですら難しい）
2. 弱磁場での実験が難しい

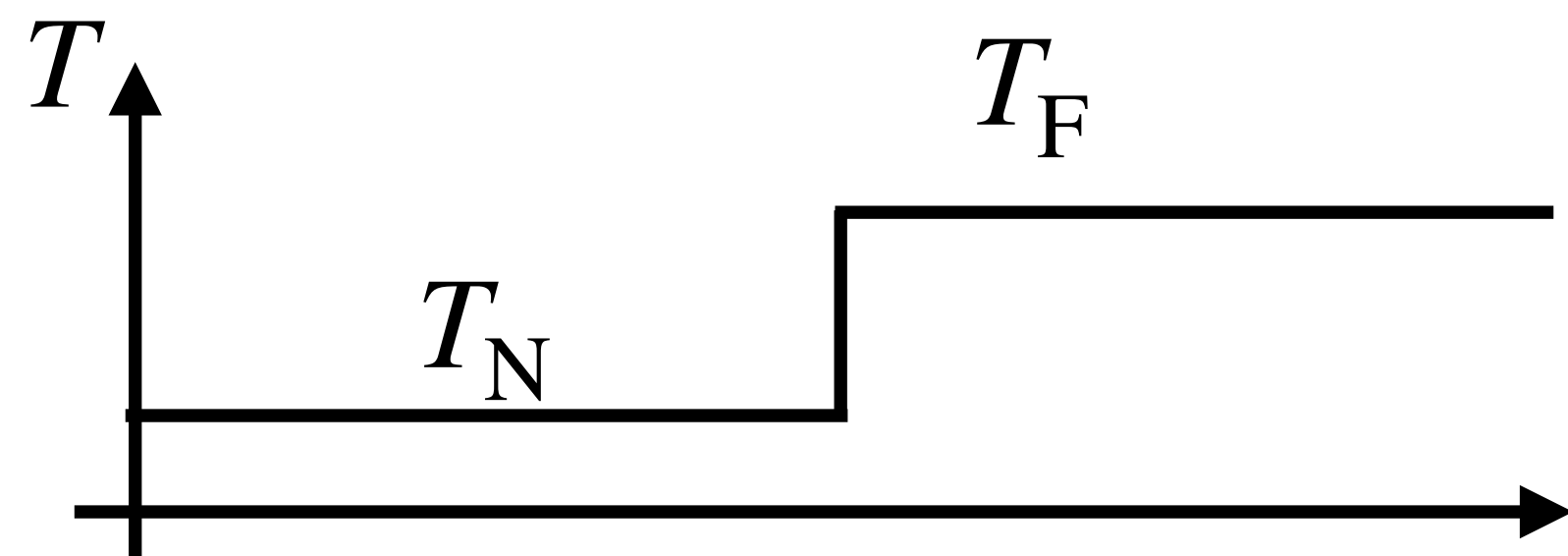


Advantage of cold atoms to investigate tunnel transport

Quantum controllability

Solids

1. Control of magnetic field to induce spin current



2. Uncontrollable elements

Impurity

Lattice defect

Lattice vibration (phonon)

Cold atoms

Effective Zeeman field controlled by spin imbalance

$$h_{\alpha=L/R} = \left(\frac{\partial E_{\alpha}}{\partial M_{\alpha}} \right)_{S_{\alpha}} \quad M = (N_{\uparrow} - N_{\downarrow})/2$$

Total internal energy E_{α}

Equilibrium:

Measured in the single-comp case

Taksasu et al., ...

平衡状態の場合

Clean spin systems

Nothing

Advantage of cold atoms to investigate tunnel transport

Solids

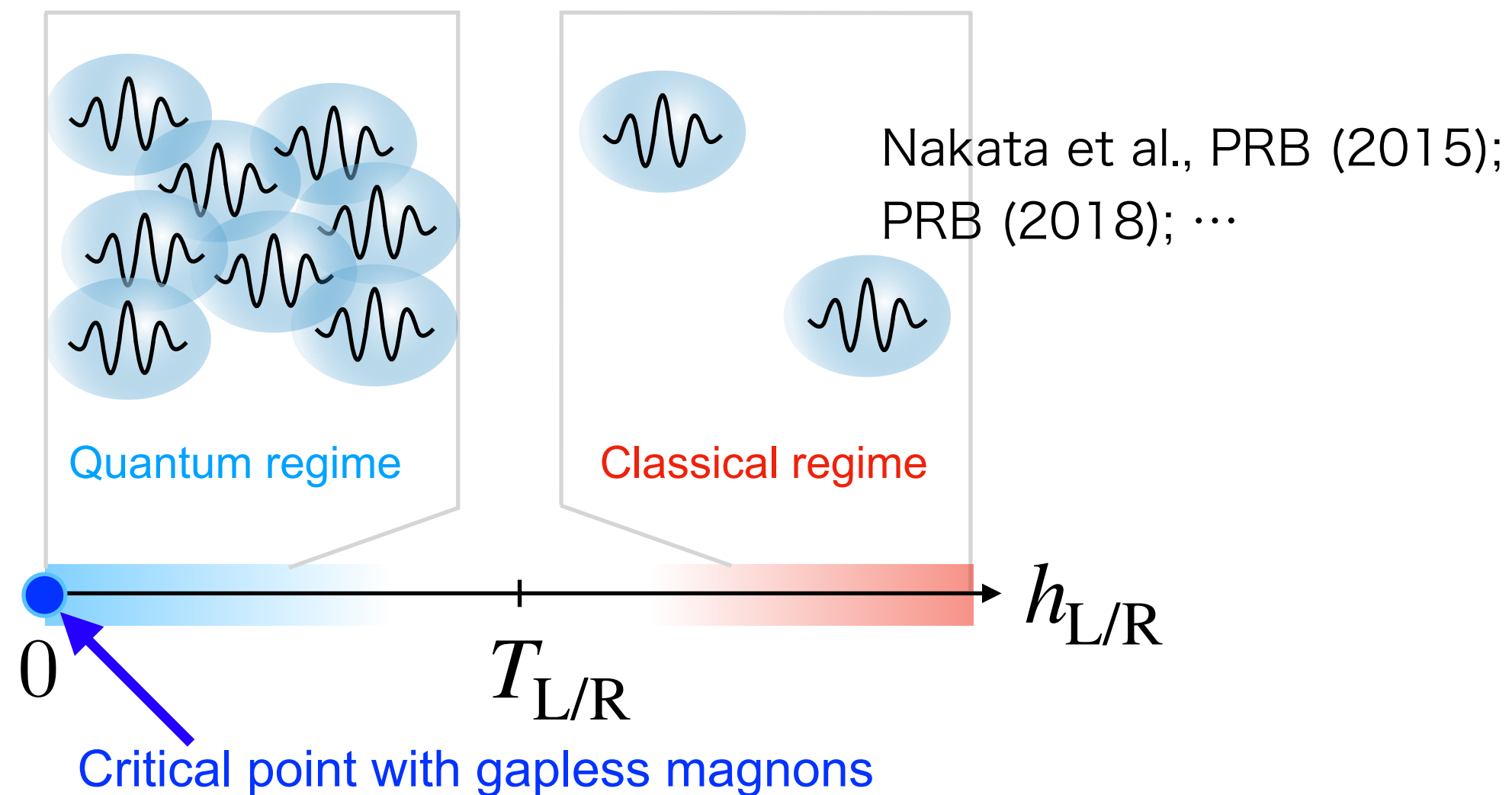
3. Interface control

Difficult to fabricate clean interface

Lattice mismatch

Change of Electron state near the surface

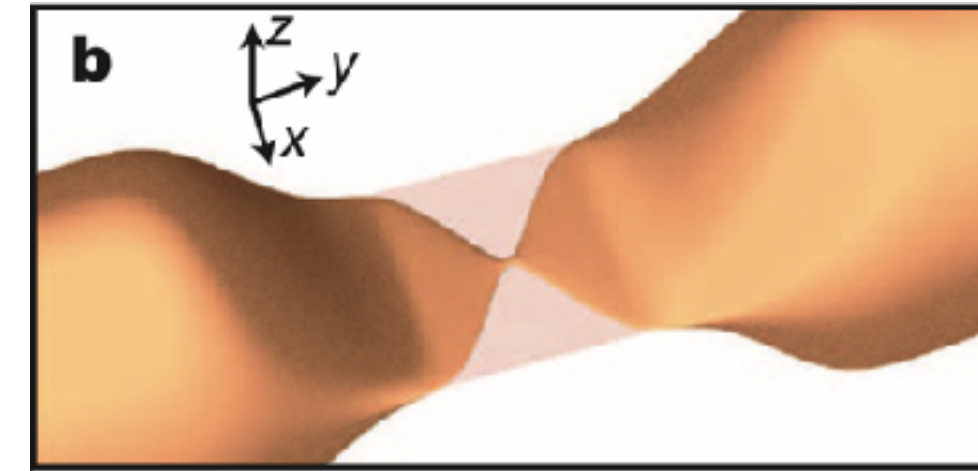
4. Magnon criticality



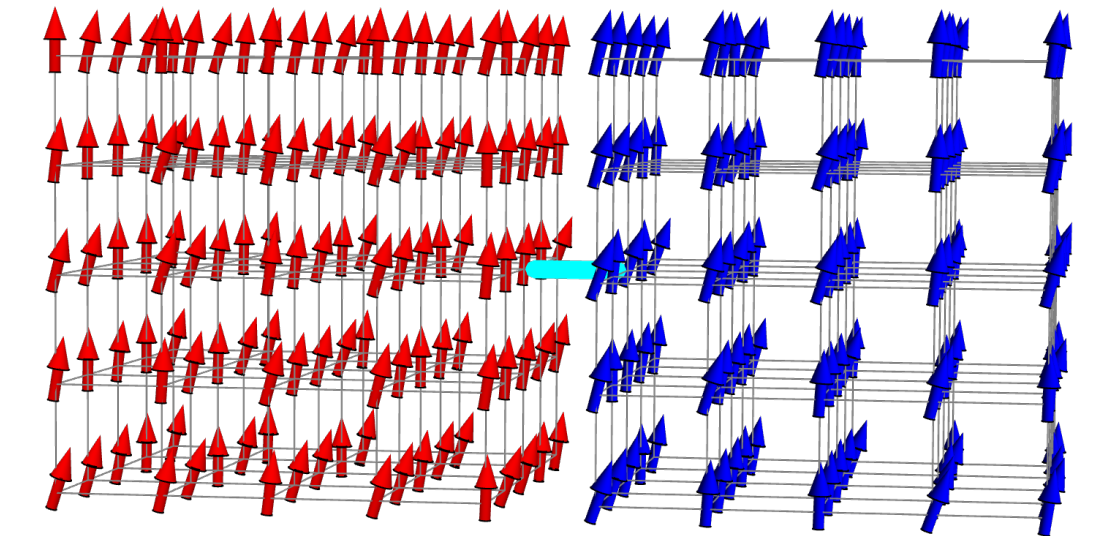
Cold atoms

3. Control of potential profile by lasers

Quantum point contact



Krinner et al., Nature (2015)



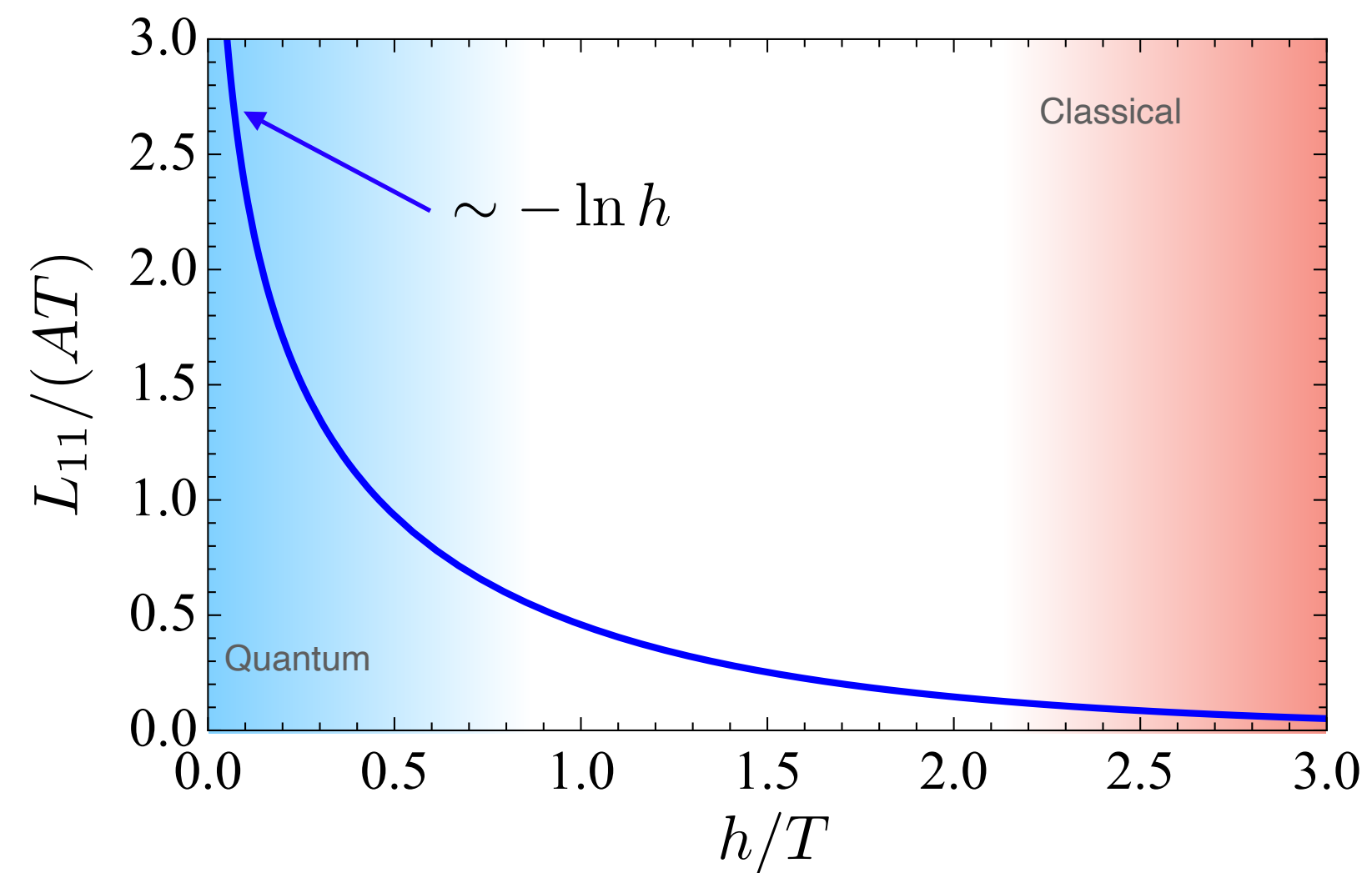
Critical behavior of Spin Conductance

Divergent behavior of spin conductance resulting from magnon criticality

$$L_{11}$$

Spin conductance

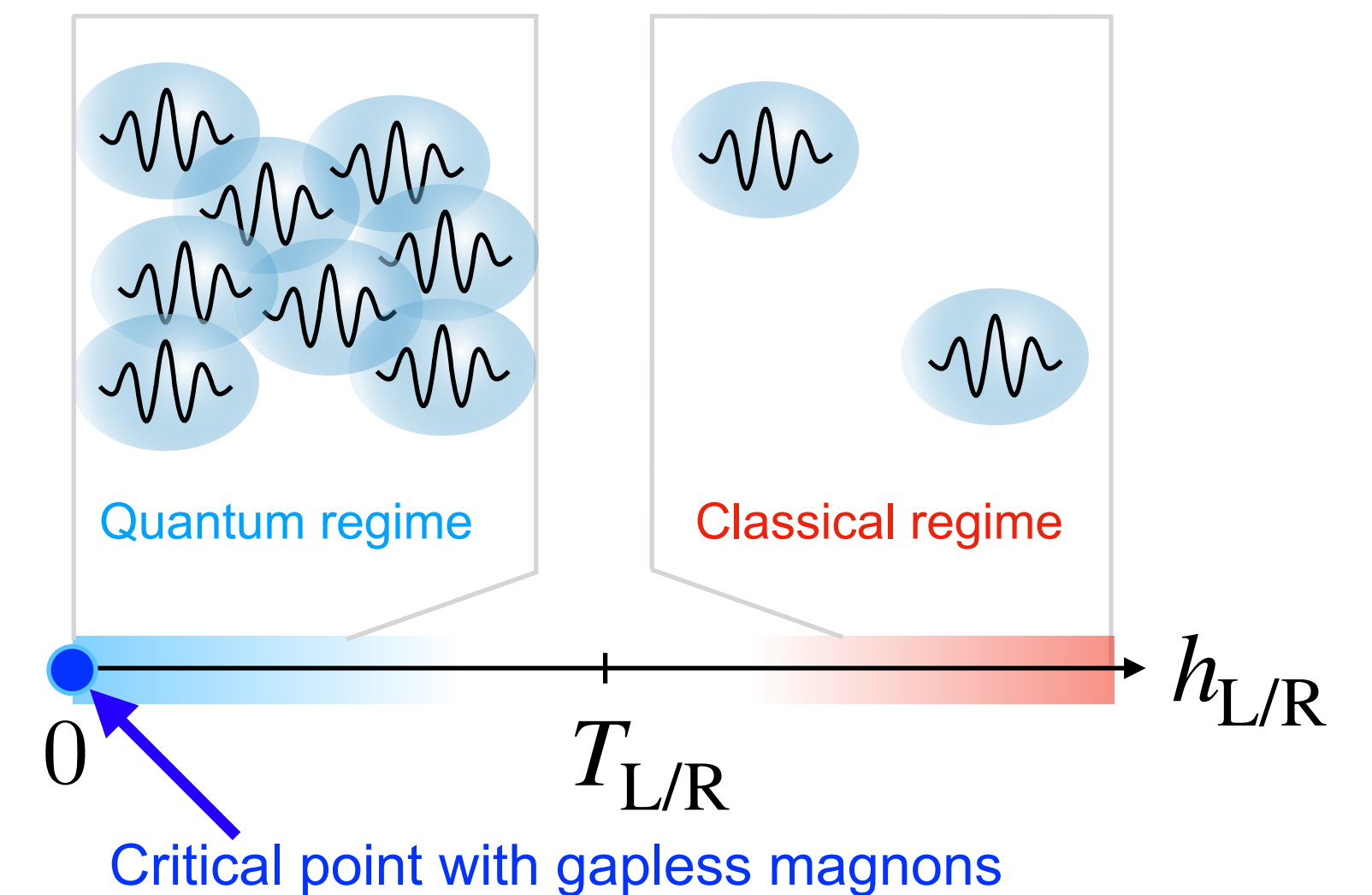
$$I_S = L_{11} \Delta h + O(\Delta h^2)$$



$$T_L = T_R$$

$$\Delta h = h_R - h_L, h = (h_L + h_R)/2$$

Criticality of magnons



$$E_{\text{gap}} = h_{L/R}$$

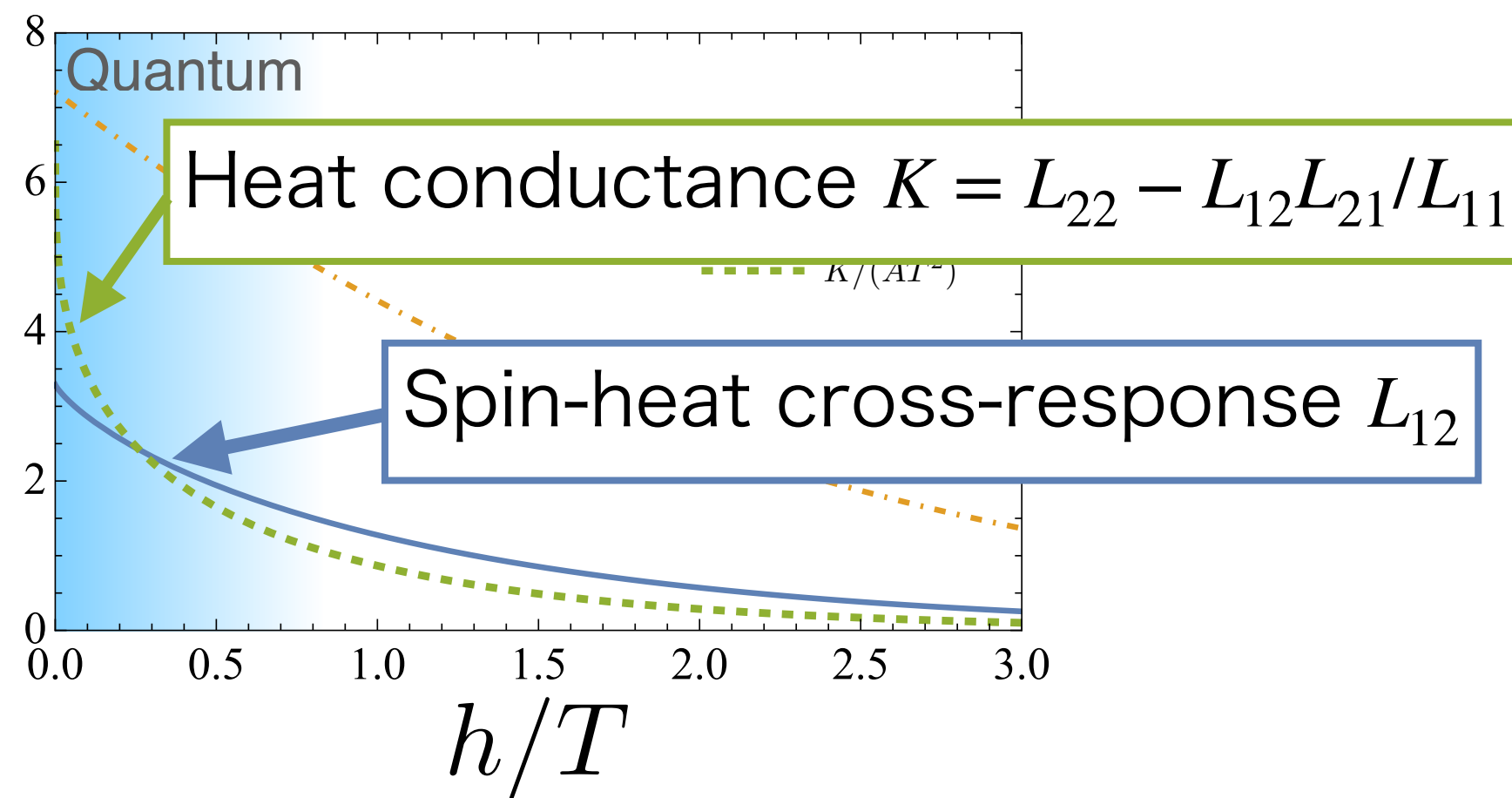
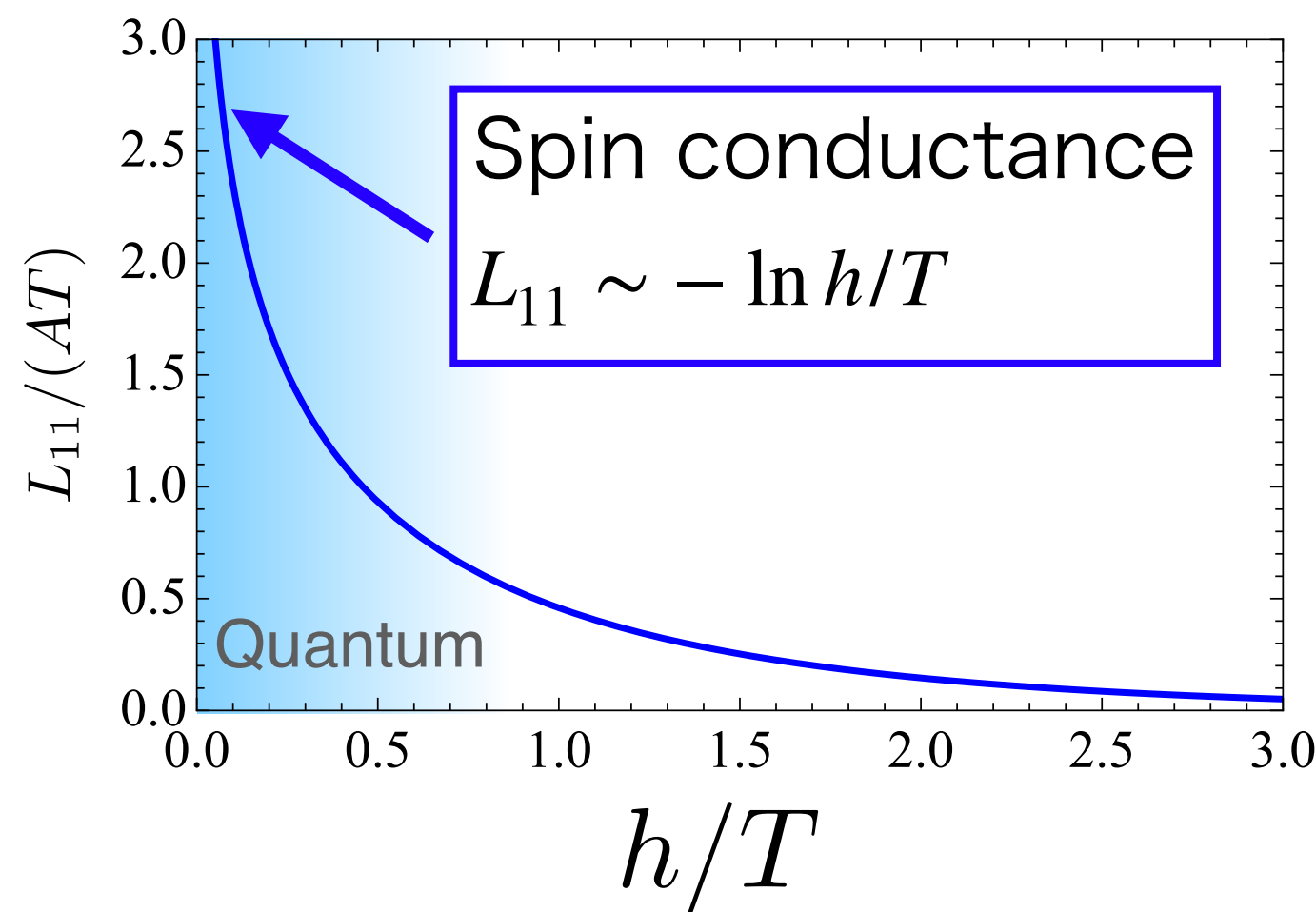
Gapless magnon as a Goldstone mode due to spontaneous magnetization

Critical behavior of transport coefficients

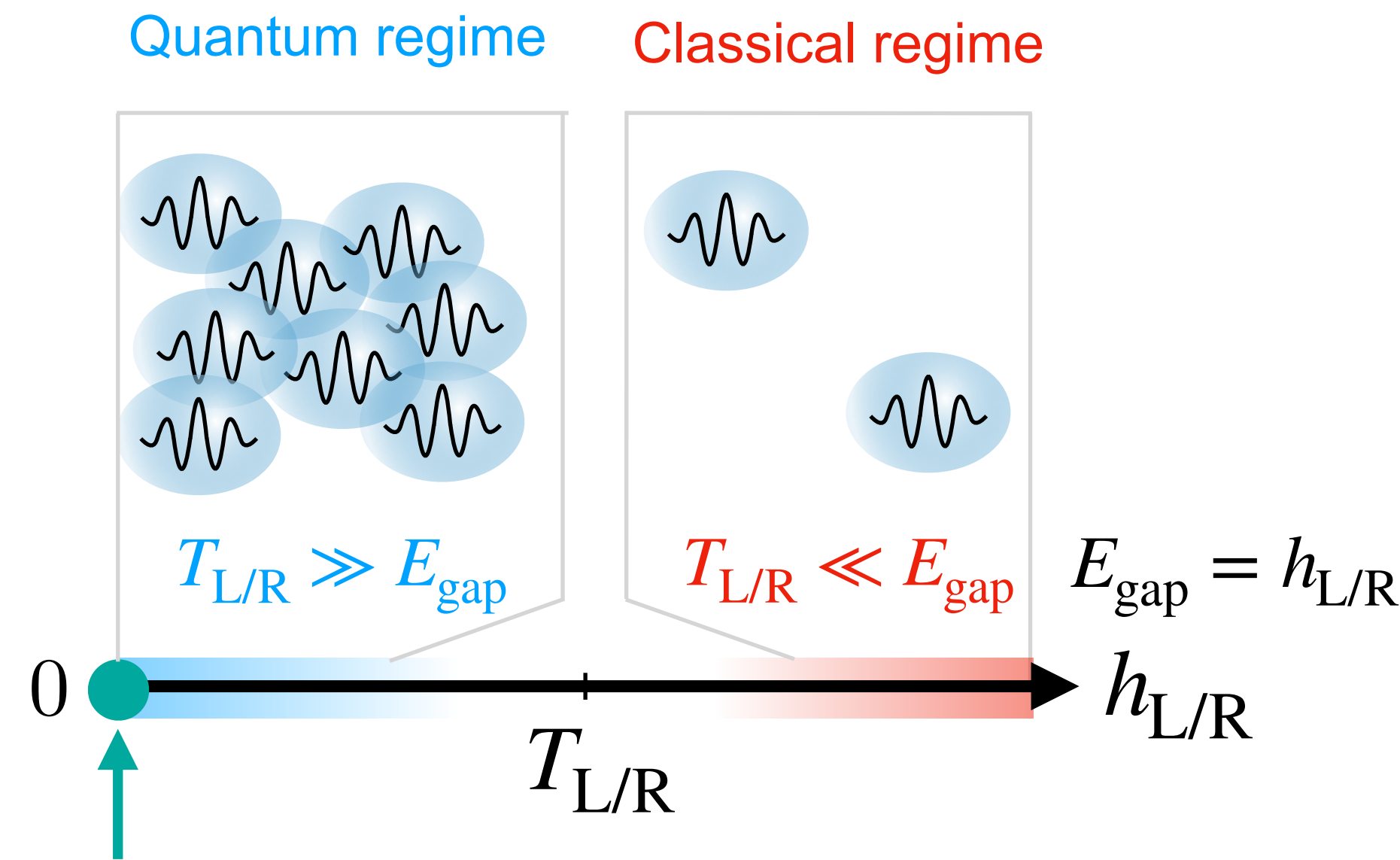
Enhancement of transport coefficients resulting from magnon criticality

Transport coefficients L_{ij}

$$\begin{pmatrix} I_S \\ I_H \end{pmatrix} = \begin{pmatrix} L_{11}L_{12} \\ L_{21}L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix} + O(\Delta T^2, \Delta h^2) \quad \begin{aligned} \Delta h &= h_R - h_L, h = (h_L + h_R)/2 \\ \Delta T &= T_L - T_R, T = (T_L + T_R)/2 \end{aligned}$$



Phase diagram of magnons at $T_{L/R} > 0$



Magnonic critical point at $E_{\text{gap}} = 0$
Gapless magnon as a Goldstone mode due to **spontaneous magnetization**

Transport coefficients are enhanced **neat the magnonic critical point**

Slowing down of magnetization relaxation

Critical behavior of magnon compressibility causes slowing down in two-terminal spin relaxation

Two-terminal relaxation in quasi-stationary case similar to Fermi-gas cases

Transport coefficients + Therm

$$\Delta M(t) = M_L(t) - M_R(t)$$

$$\Delta T(t) = T_L(t) - T_R(t)$$

Relevant to extract trans. properties

Spin conductance, ...

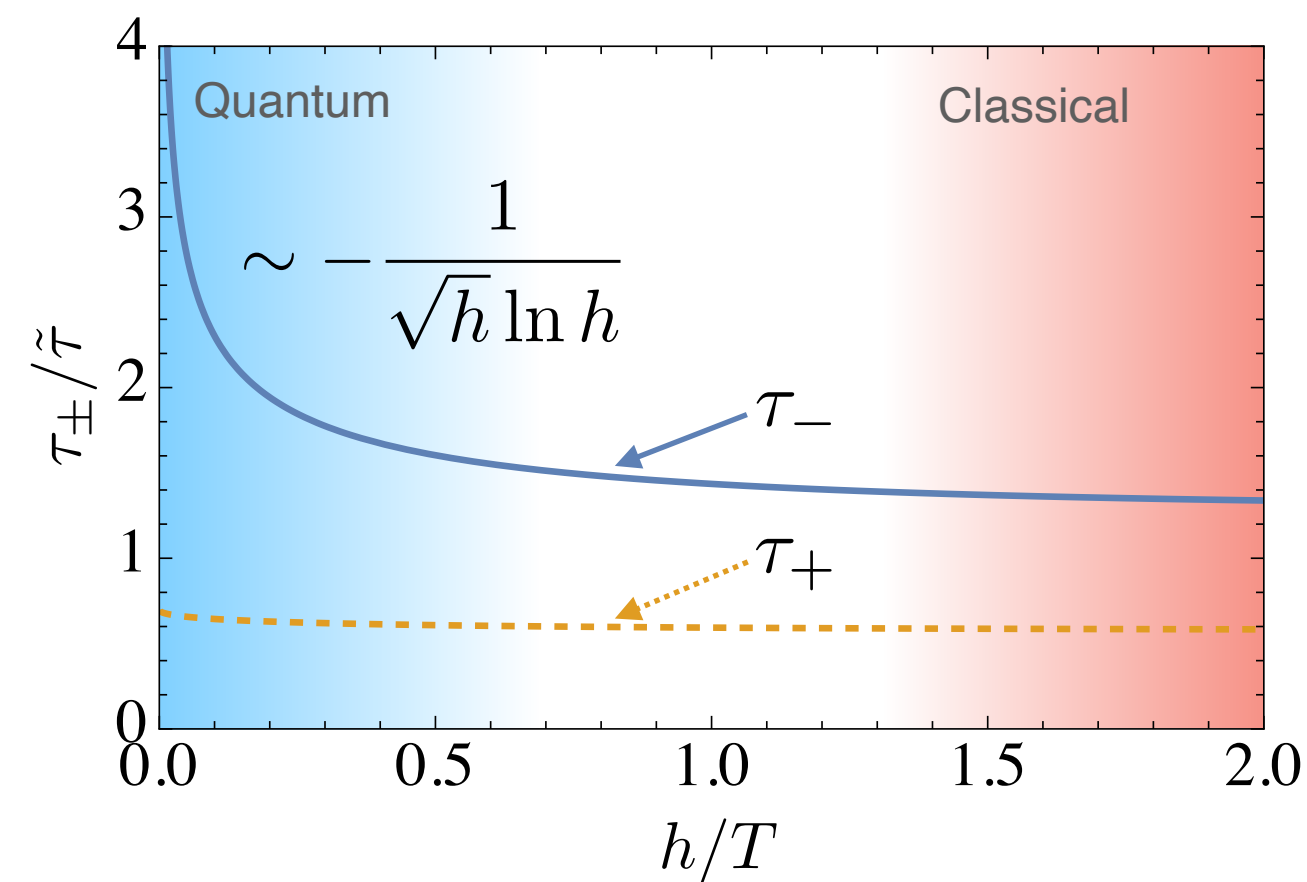
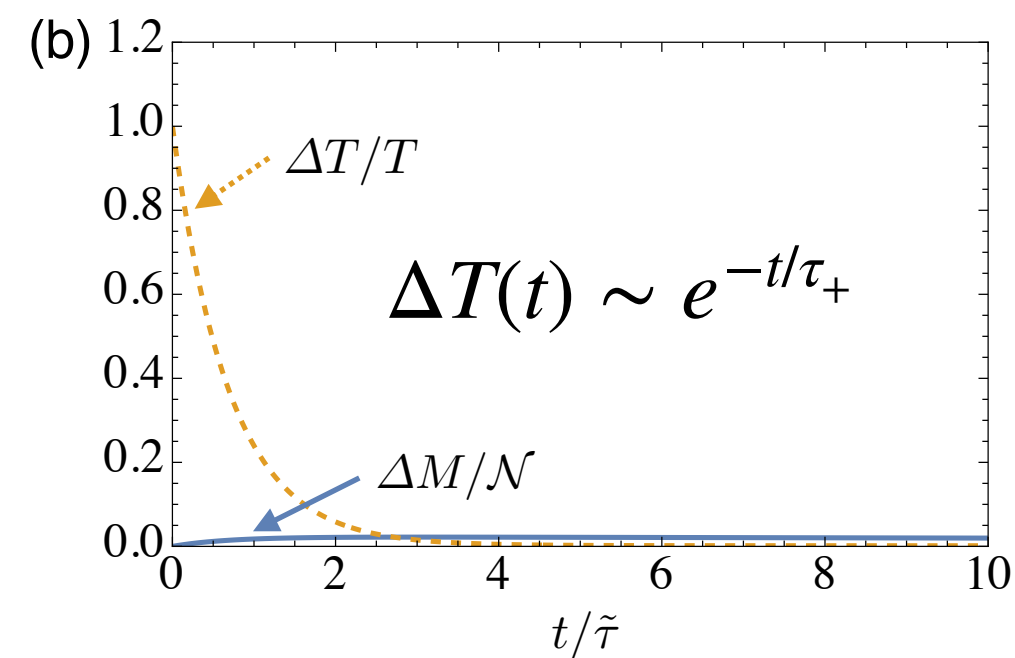
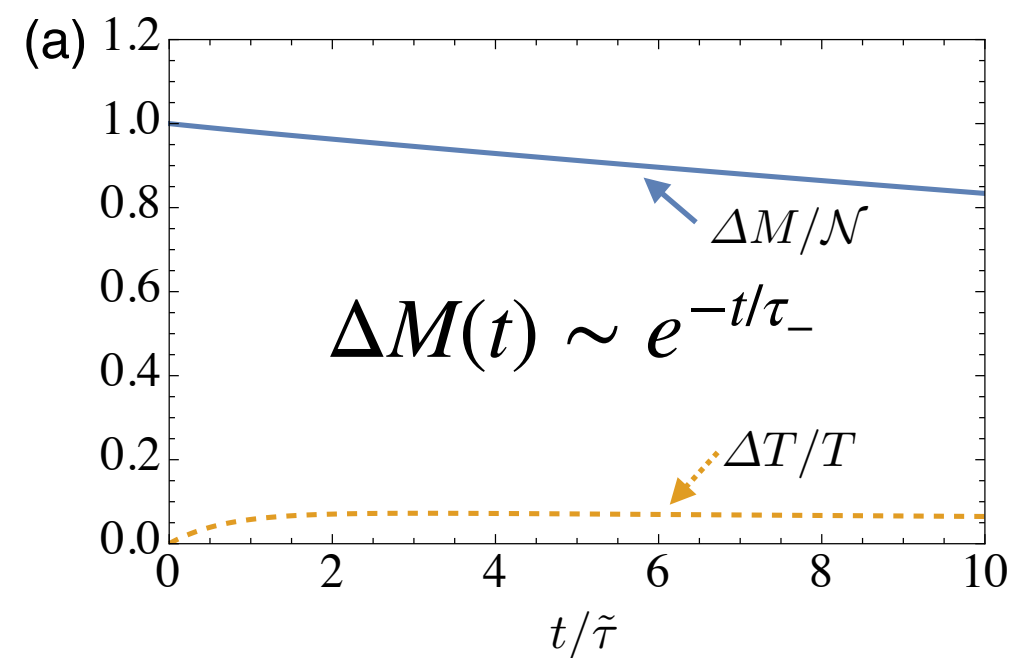
Differential sus

$$\kappa = \left(\frac{\partial M}{\partial h} \right)_T$$

Compressibility

$(\Delta M(0) > 0, \Delta T(0) = 0)$

$(\Delta M(0) = 0, \Delta T(0) > 0)$



$$\tau_- \sim \frac{\kappa}{L_{11}} \sim \frac{1/\sqrt{h}}{-\log h}$$

Decay of magnetization is very slow

$$\tau_- = 56\tilde{\tau}$$

$$\tau_+ = 0.70\tilde{\tau}$$

Relaxation time of M

Relaxation time of T

Slowing down of magnetization relaxation

Critical behavior of magnon compressibility causes slowing down in two-terminal spin relaxation

Compressibility of magnons

Differential susceptibility, ...

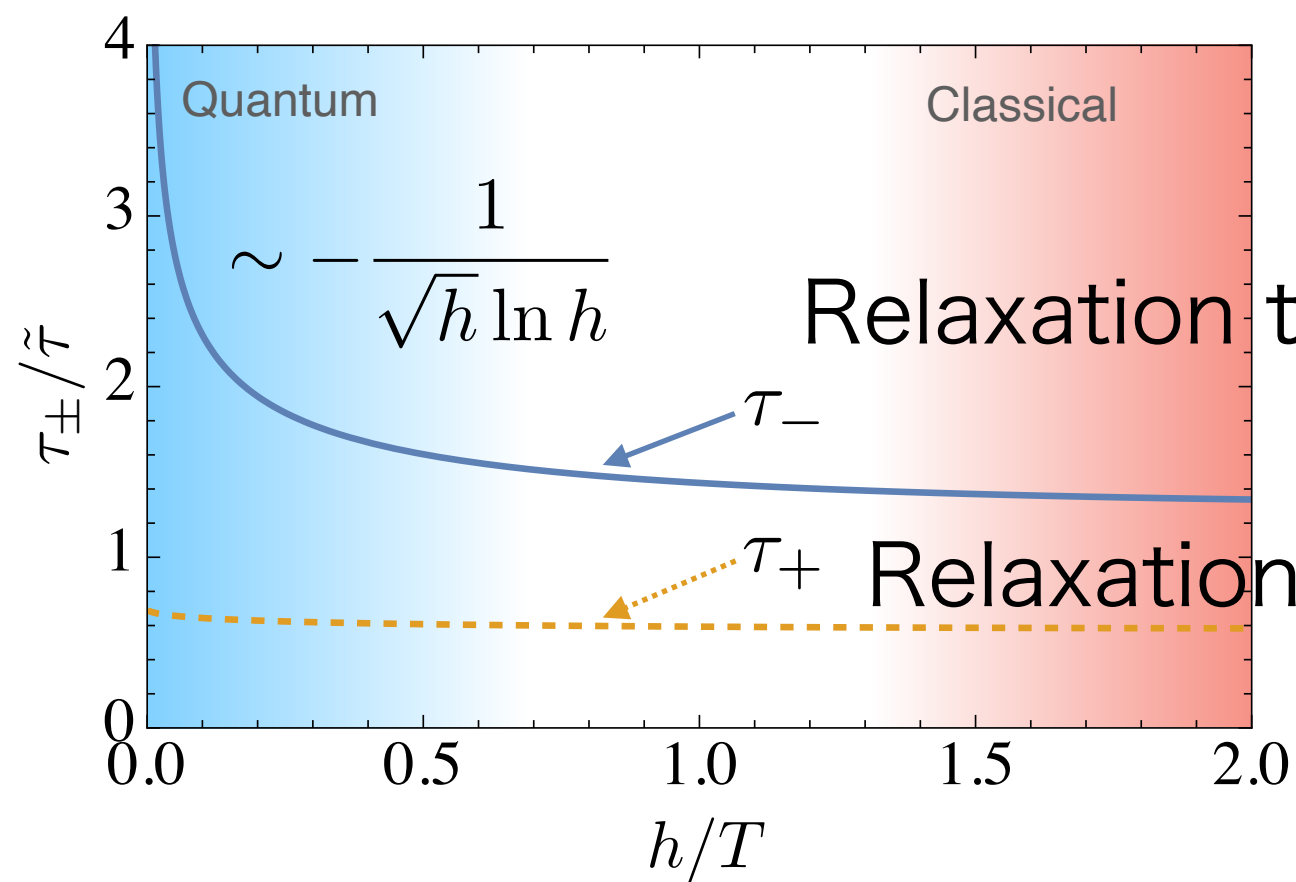
$$\kappa = \left(\frac{\partial M}{\partial h} \right)_T$$

$$\tau_- \sim \frac{\kappa}{L_{11}} \sim \frac{1/\sqrt{h}}{-\log h}$$

Spin conductance, ...

Magnon criticality \Leftrightarrow BEC transition point

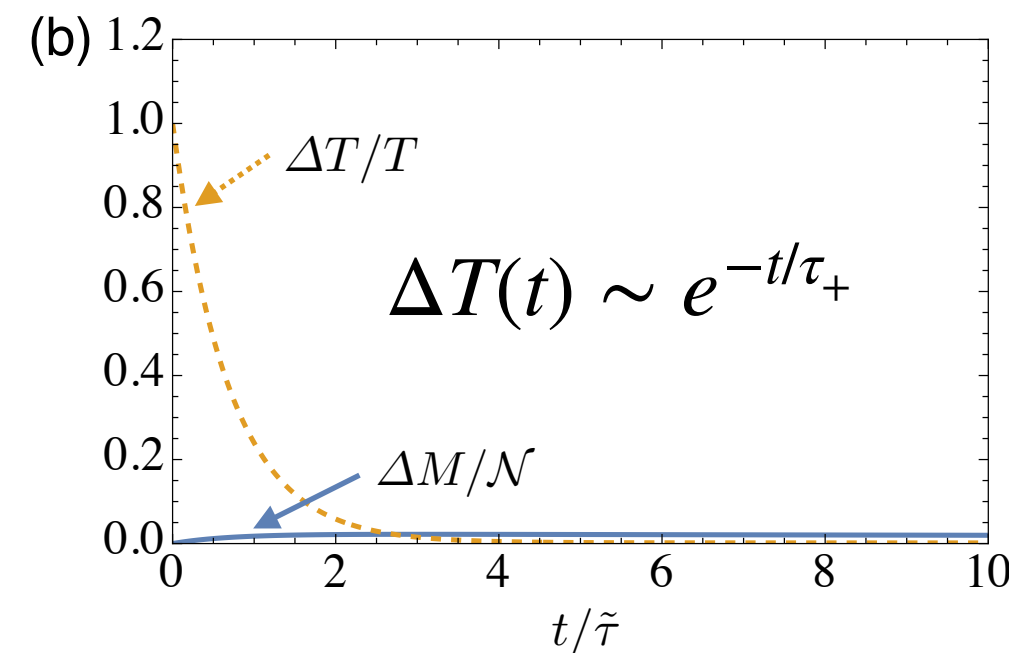
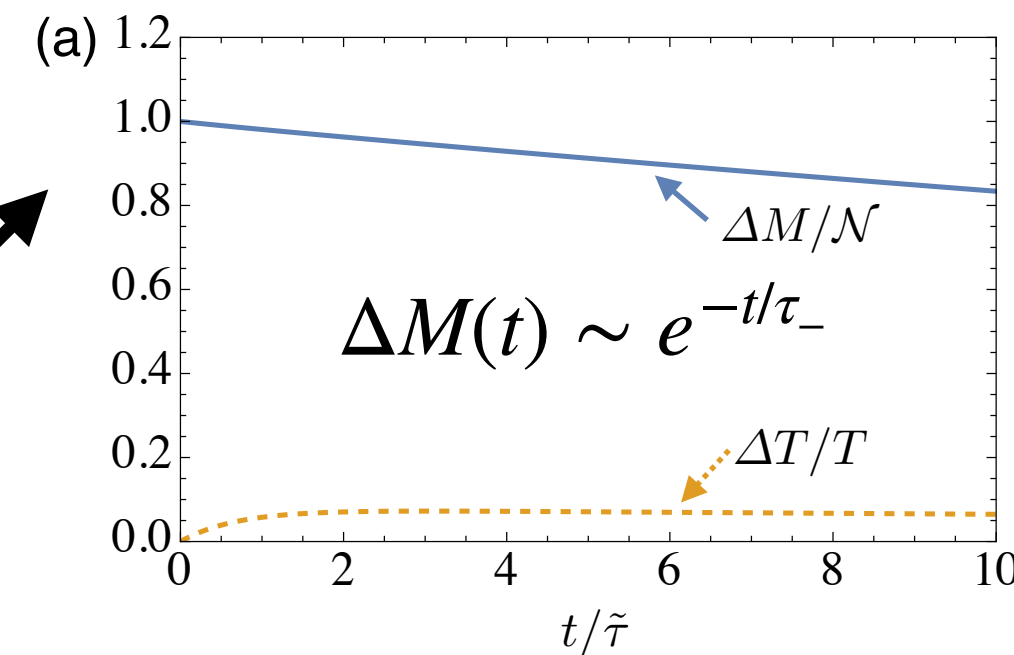
Magnon criticality \Leftrightarrow BEC transition point



Relaxation time of M

Relaxation time of T

$(\Delta M(0) > 0, \Delta T(0) = 0)$



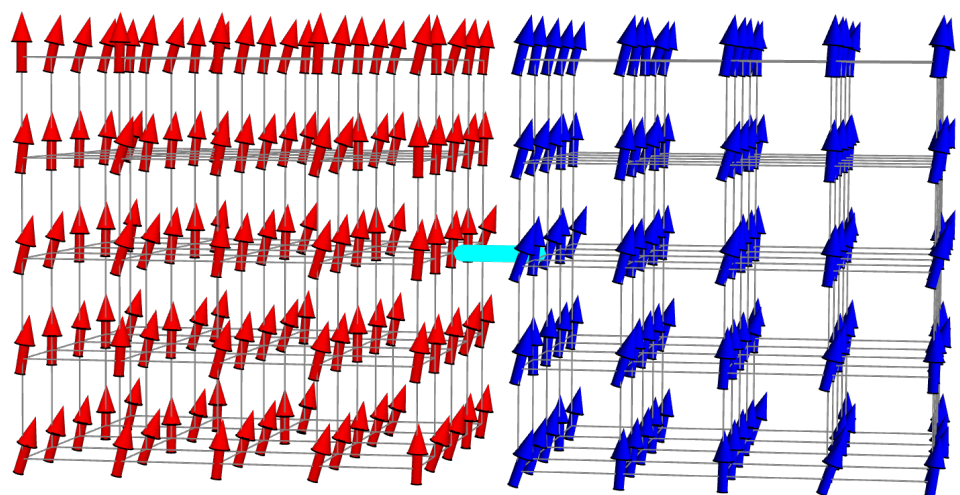
How to experimentally determine conductances?

Following the scheme proposed for Fermi-gas cases [ETH: Brantut et al., Science (2013)],

we can experimentally determine **transport properties** L_{ij} by measuring

- Relaxation dynamics of $\Delta M(t) = M_L(t) - M_R(t)$, $\Delta T(t) = T_L(t) - T_R(t)$ b/w FIs near equilibrium
- Thermodynamic quantities of one FI at equilibrium

A. Relaxation dynamics b/w FIs



$$M_L(t) \longleftrightarrow M_R(t)$$

$$T_L(t) \longleftrightarrow T_R(t)$$

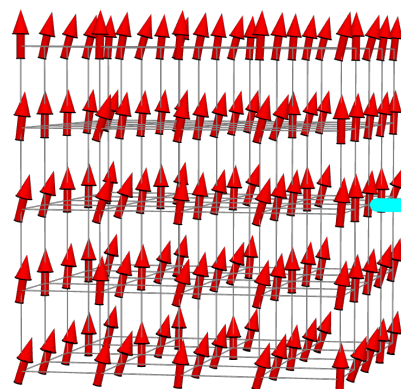
- Close the channel and prepare thermal states with $M_{L/R}(0)$, $T_{L/R}(0)$
- At $t = 0$, open the channel so that $M_{L/R}(t)$, $T_{L/R}(t)$ start time evolution
- Observe $\Delta M(t) = M_L(t) - M_R(t)$ $\Delta T(t) = T_L(t) - T_R(t)$ at time t
- Fit obtained $\Delta M(t)$ and $\Delta T(t)$ with solutions of the quasi-stationary model

$$\text{Transport relation: } \frac{d}{dt} \begin{pmatrix} -\Delta M \\ T\Delta S \end{pmatrix} = -2 \begin{pmatrix} I_S \\ I_H \end{pmatrix} = -2 \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

$$\text{Thermodynamic relation: } \begin{pmatrix} -\Delta M \\ \Delta S \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial M}{\partial h}\right)_T & -\left(\frac{\partial M}{\partial T}\right)_h \\ -\left(\frac{\partial S}{\partial h}\right)_T & \left(\frac{\partial S}{\partial T}\right)_h \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

Fitting parameters consist of L_{ij} and thermodynamic quantities

B. Measuring **thermodynamic quantities** of one FI at equilibrium



$$h = [h_L(0) + h_R(0)]/2$$

$$T = [T_L(0) + T_R(0)]/2$$

Why tunneling magnon transport in cold atoms?

1. Ultraclean systems

No impurity

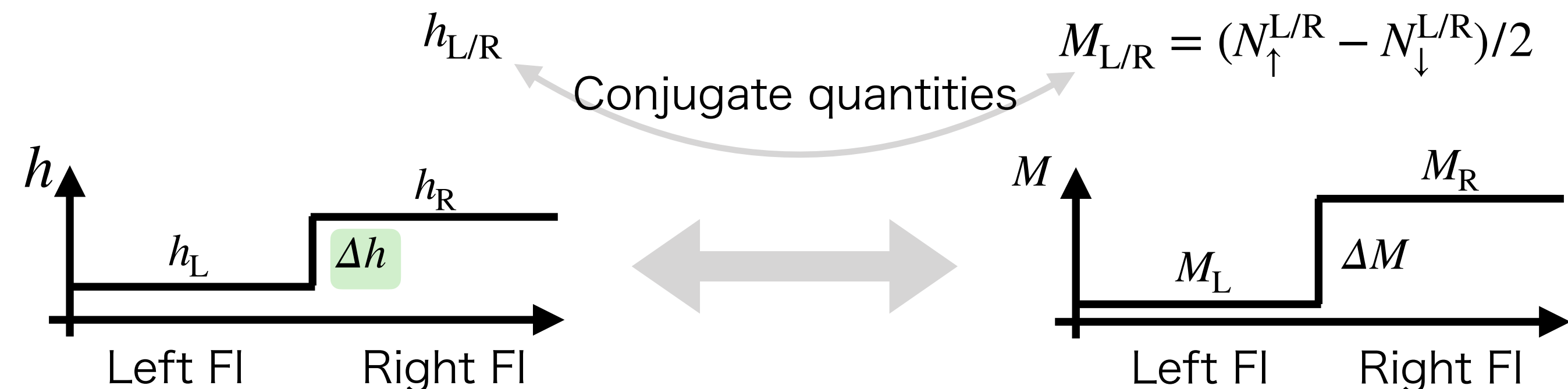
Roughness & lattice mismatch

2. Quantum controllability of effective Zeeman fields

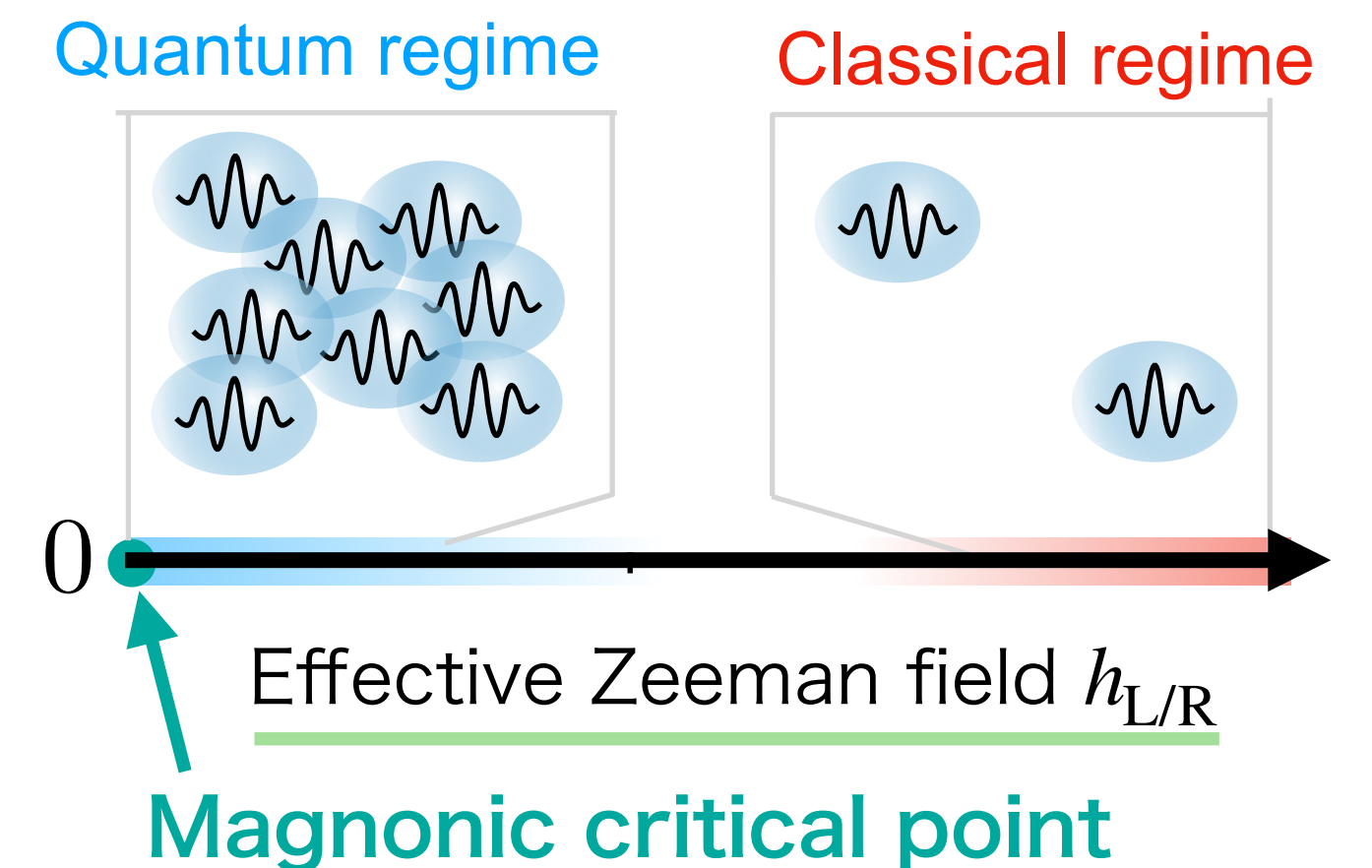
Similarly to the Fermi gases [Kriner et al., PNAS, **113** (29) 8144-8149 (2016)]

Control of spin bias Δh to generate tunneling currents

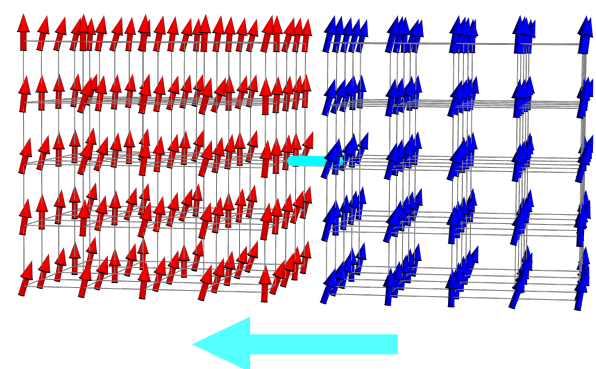
Effective Zeeman fields can be tuned by spin imbalance



Access to **quantum critical regime**



No tunneling experiment b/w FIs in solid-state systems Theoretical proposal: [Nakata et al., PRB \(2015\); PRB \(2018\)](#)



Hard to generate Δh by spatial modulation of magnetic field

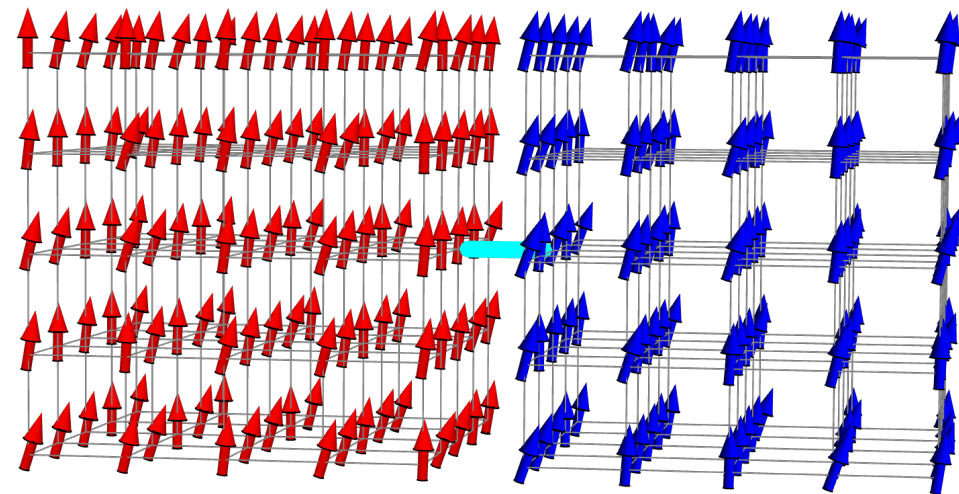
Difficult to access **magnonic quantum regime** because of magnetic domains by dipole interactions

→ Focusing on **Classical regime**

How to experimentally determine conductances?

Following the scheme proposed for Fermi-gas cases [ETH: Brantut et al., Science (2013)], we can experimentally determine **transport properties** L_{ij} by measuring

A. Near-equilibrium relaxation dynamics of $\Delta M(t) = M_L(t) - M_R(t)$, $\Delta T(t) = T_L(t) - T_R(t)$ b/w FIs from a thermal initial state



$$M_L(t) \longleftrightarrow M_R(t)$$

$$T_L(t) \longleftrightarrow T_R(t)$$

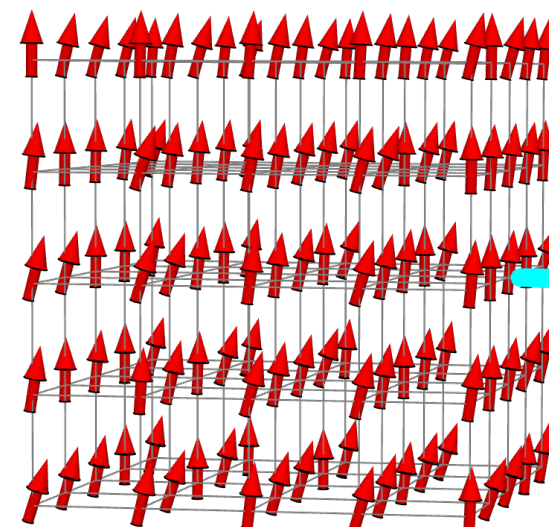
1. Observe $\Delta M(t)$ and $\Delta T(t)$ at time t
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$$\text{Thermodynamic relation: } \begin{pmatrix} -\Delta M \\ \Delta S \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial M}{\partial h}\right)_T & -\left(\frac{\partial M}{\partial T}\right)_h \\ -\left(\frac{\partial S}{\partial h}\right)_T & \left(\frac{\partial S}{\partial T}\right)_h \end{pmatrix} \begin{pmatrix} \Delta h \\ \Delta T \end{pmatrix}$$

Fitting parameters consist of L_{ij} and thermodynamic quantities

B. Thermodynamic quantities of one FI at equilibrium



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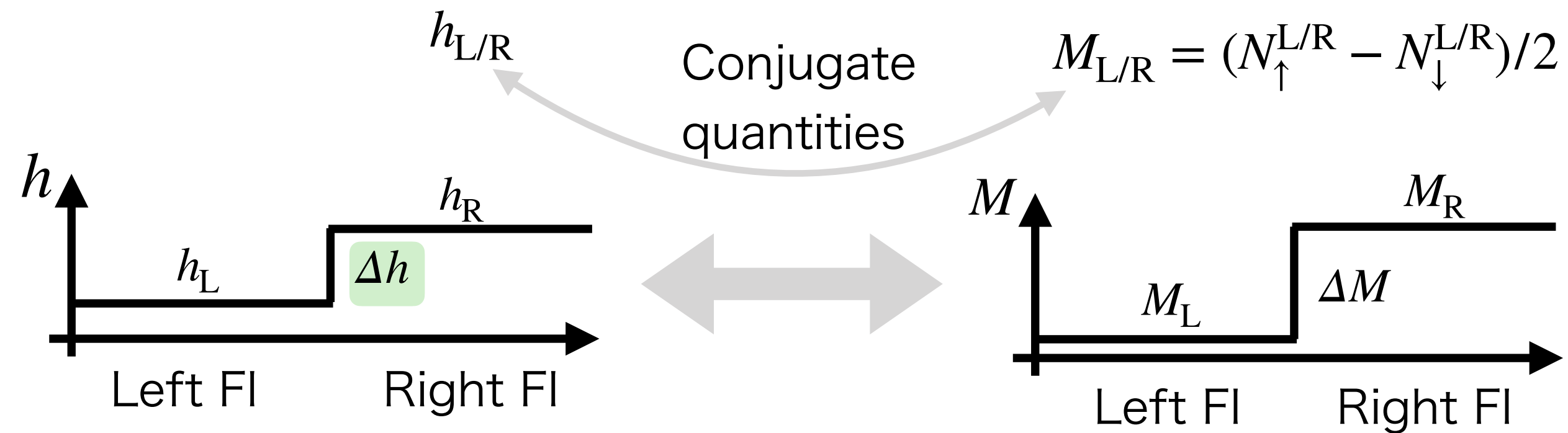
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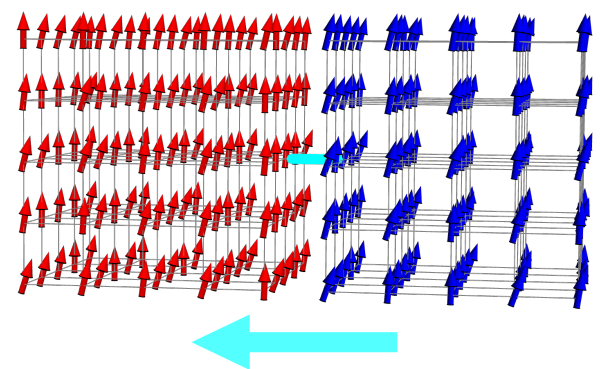
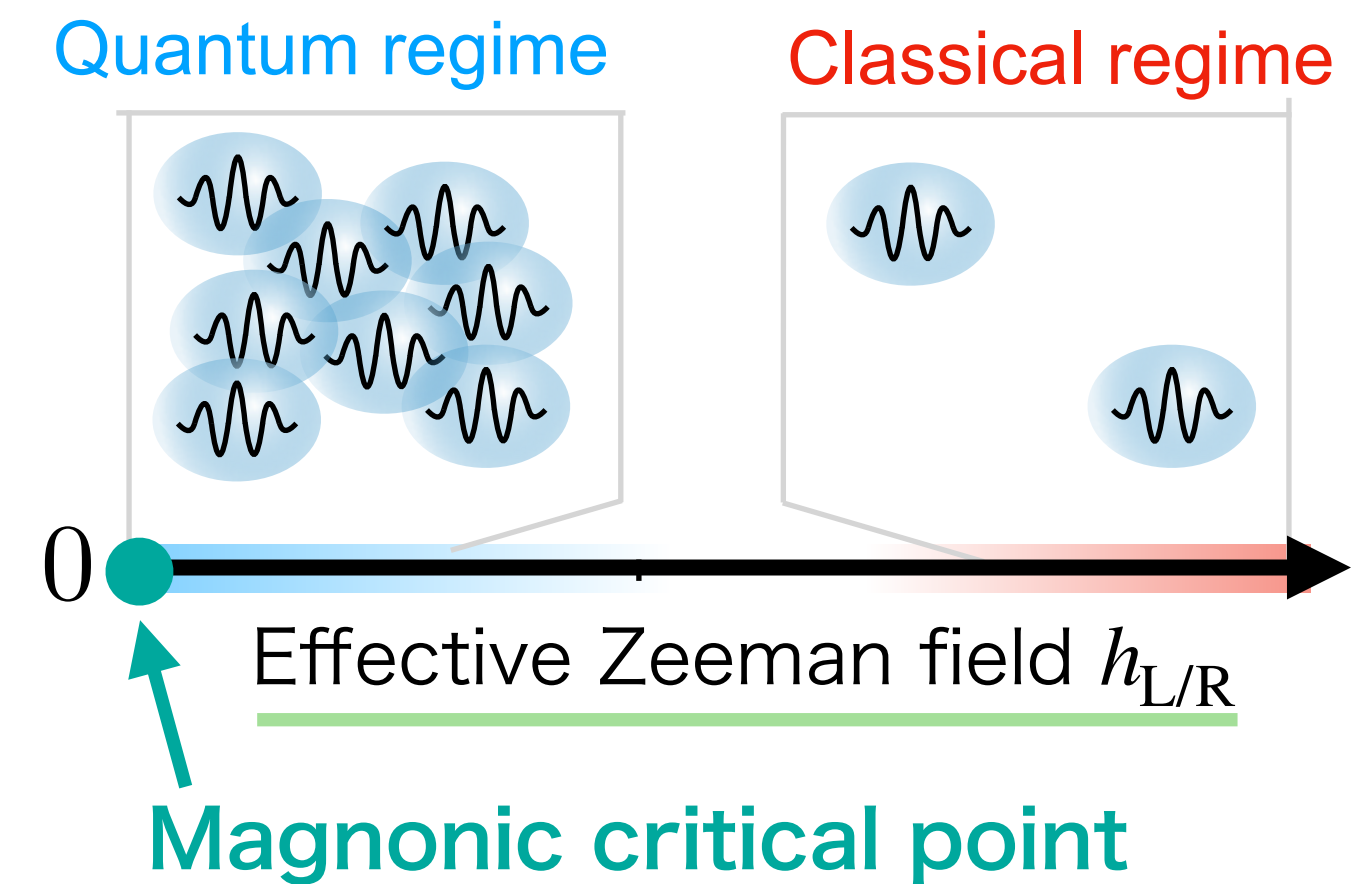
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Control of spin bias Δh to generate tunneling currents

Effective Zeeman fields can be tuned by spin imbalance



Access to quantum critical regime



No solid-state experiment b/w FIs

because inducing Δh by spatially-modulated magnetic field is difficult

Theoretical proposal for **solid FIs** focusing on classical regime [Nakata et al., PRB (2015); PRB (2018)]

Difficult to experimentally access magnonic quantum regime because of magnetic domains by dipole interactions