QUANTUM GRAVITY CONSTRAINTS ON GLOBAL SYMMETRIES

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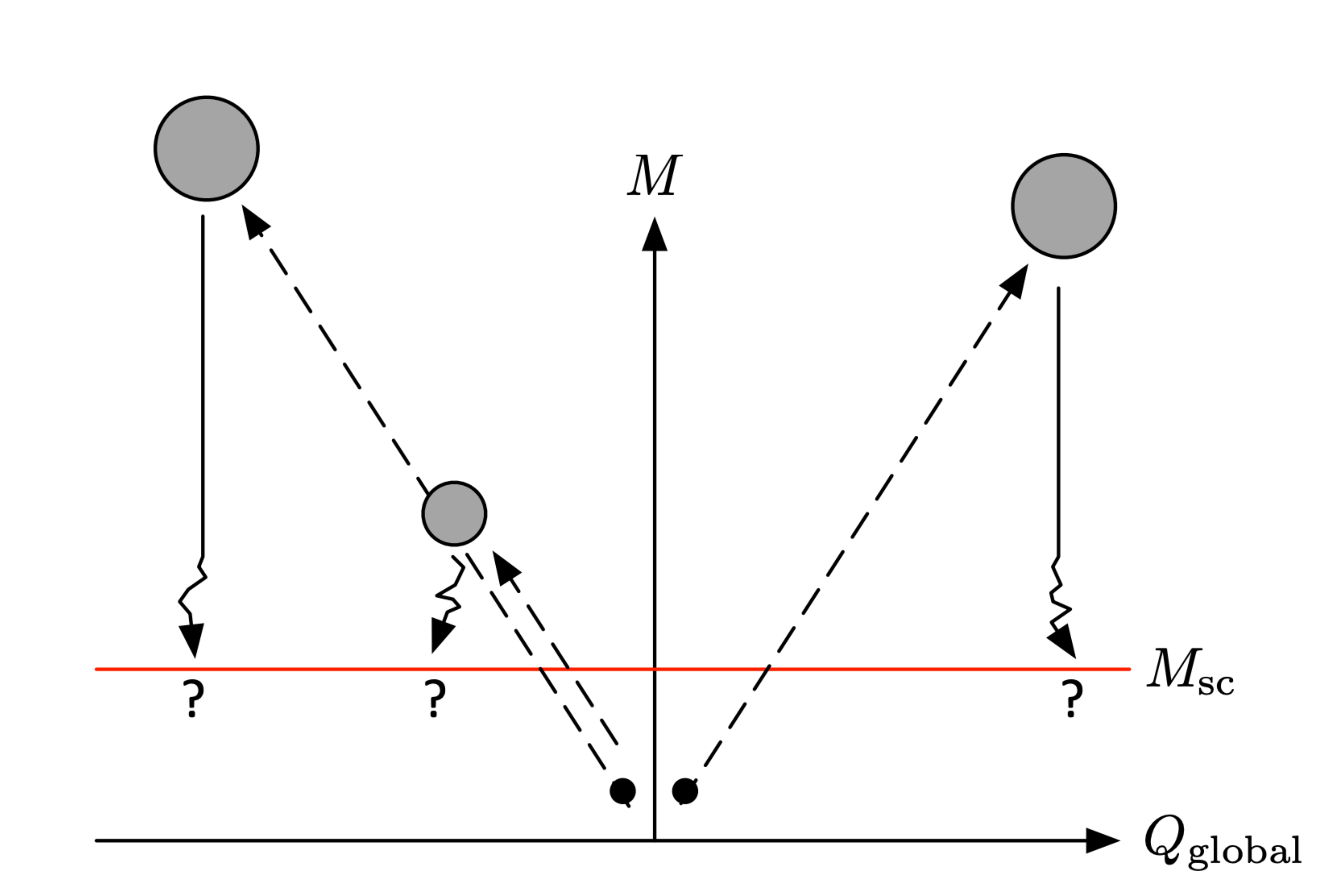
NCTS-iTHEMS workshop Aug 27th 2024

It is generally believed that all global symmetries are broken, or become gauged, in the full theory of quantum gravity

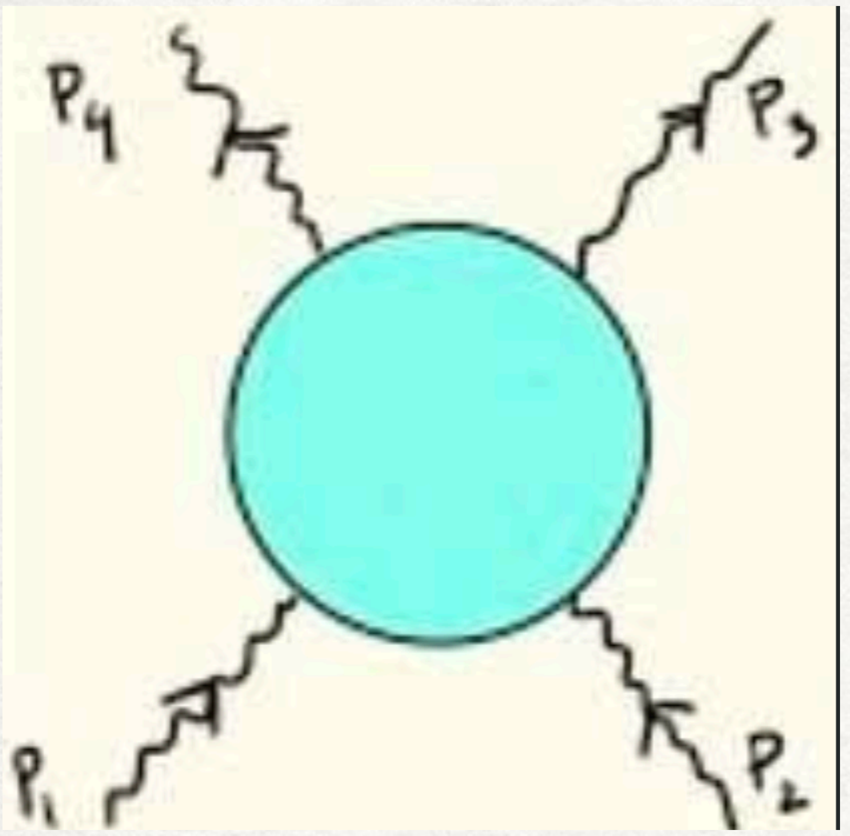


T. Banks and N. Seiberg, "Symmetries and Strings in Field Theory and Gravity," *Phys. Rev.* **D83** (2011) 084019, `arXiv:1011.5120 [hep-th]`.

The gravitational collapse of global-charged objects creates black holes of arbitrarily large global charge. After Hawking radiation, this leads to an infinite number of microstates violating he Bekenstein-Hawking entropy formula

If the symmetry is gauge, in known models of quantum gravity (string theory), gauge groups are constrained
$SO(32)$ or $E_8 \times E_8$

This constraint is directly visible in the S-matrix



$$\mathcal{A}(1^{-a},2^{-b},3^{+c},4^{+d}) = \langle 12\rangle^2\,[34]^2\left[\frac{1}{M_P^2}\left(\frac{\mathbb{P}_1^s}{s}+\frac{\mathbb{P}_1^t}{t}+\frac{\mathbb{P}_1^u}{u}\right)+\frac{g_{\rm YM}^2}{3}\left(\frac{\mathbb{P}_{\rm Adj}^s-\mathbb{P}_{\rm Adj}^t}{st}+\frac{\mathbb{P}_{\rm Adj}^t-\mathbb{P}_{\rm Adj}^u}{tu}+\frac{\mathbb{P}_{\rm Adj}^u-\mathbb{P}_{\rm Adj}^s}{su}\right)\right]$$

$$\begin{cases}\mathbb{P}_1^s & \delta^{ab}\delta^{cd}\\ \mathbb{P}_{\rm Adj}^s & f^{abe}f^{edc}\end{cases}$$

UV complete

$$\mathcal{A}^{\rm UV}(1^{-a},2^{-b},3^{+c},4^{+d}) = \Gamma^{\rm str}\mathcal{A}(1^{-a},2^{-b},3^{+c},4^{+d})$$

$$\Gamma^{\rm str} = -\frac{\Gamma(-\alpha' s)\,\Gamma(-\alpha' t)\,\Gamma(-\alpha' u)}{\Gamma(\alpha' s)\,\Gamma(\alpha' t)\,\Gamma(\alpha' u)}$$

Require the residues on factorization poles to be consistent with unitarity

$$\lim_{s\to m^2}\mathcal{A}(1^a2^b3^c4^d) \sim \frac{1}{s-m^2}\sum_J \rho_{J,\alpha}\mathbb{P}_\alpha^{abcd}\,\mathbb{G}_j(\cos\theta)$$

$$\rho_{J,\alpha} > 0$$

If the symmetry is gauge, in known models of quantum gravity (string theory), gauge groups are constrained SO(32) or E_8 x E_8

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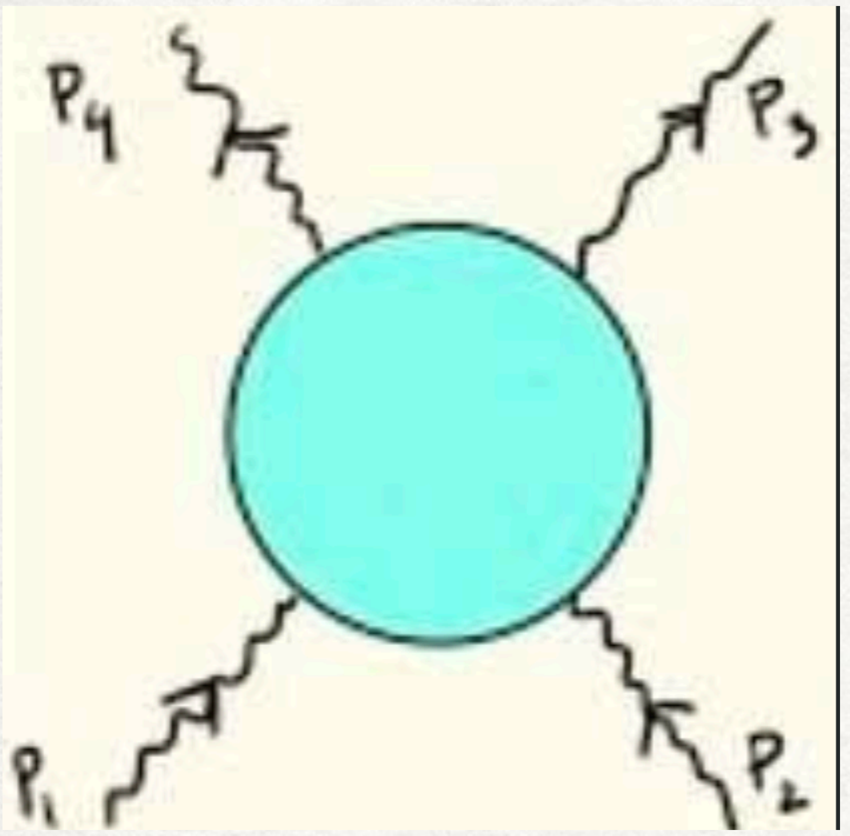
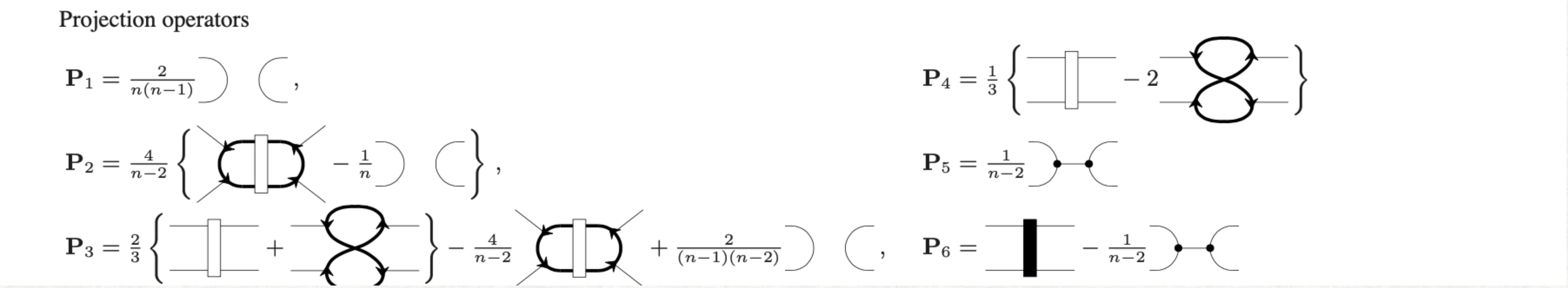


$$\lim_{s\to m^2} M(1^a 2^b 3^c 4^d) \sim \frac{1}{s-m^2}\sum_J \rho_{J,\alpha}\mathbb{P}^{abcd}_\alpha\,\mathbb{G}_j(\cos\theta)$$



Level 1

$$\left\{\frac{-1+8\,\text{gym}^2\,\text{Mp}^2+\frac{32}{(-1+n)\,n}+x^2}{8\,\text{Mp}^2},\ \frac{\text{gym}^2\,(-4+n)}{2\,(-2+n)}+\frac{4}{\text{Mp}^2\,(-1+n)\,n},\ -\frac{\text{gym}^2}{-2+n}+\frac{4}{\text{Mp}^2\,(-1+n)\,n},\ \frac{2\,\text{gym}^2}{-2+n}+\frac{4}{\text{Mp}^2\,(-1+n)\,n},\ \frac{\text{gym}^2\,x}{2},\ 0\right\}$$

→

$$\left\{\text{gym}^2\,\text{Mp}^2-\frac{1}{9}+\frac{4}{(-1+n)\,n}>0,\ \frac{4\,(-2+n)}{(-1+n)\,n}>\text{gym}^2\,\text{Mp}^2\right\}$$

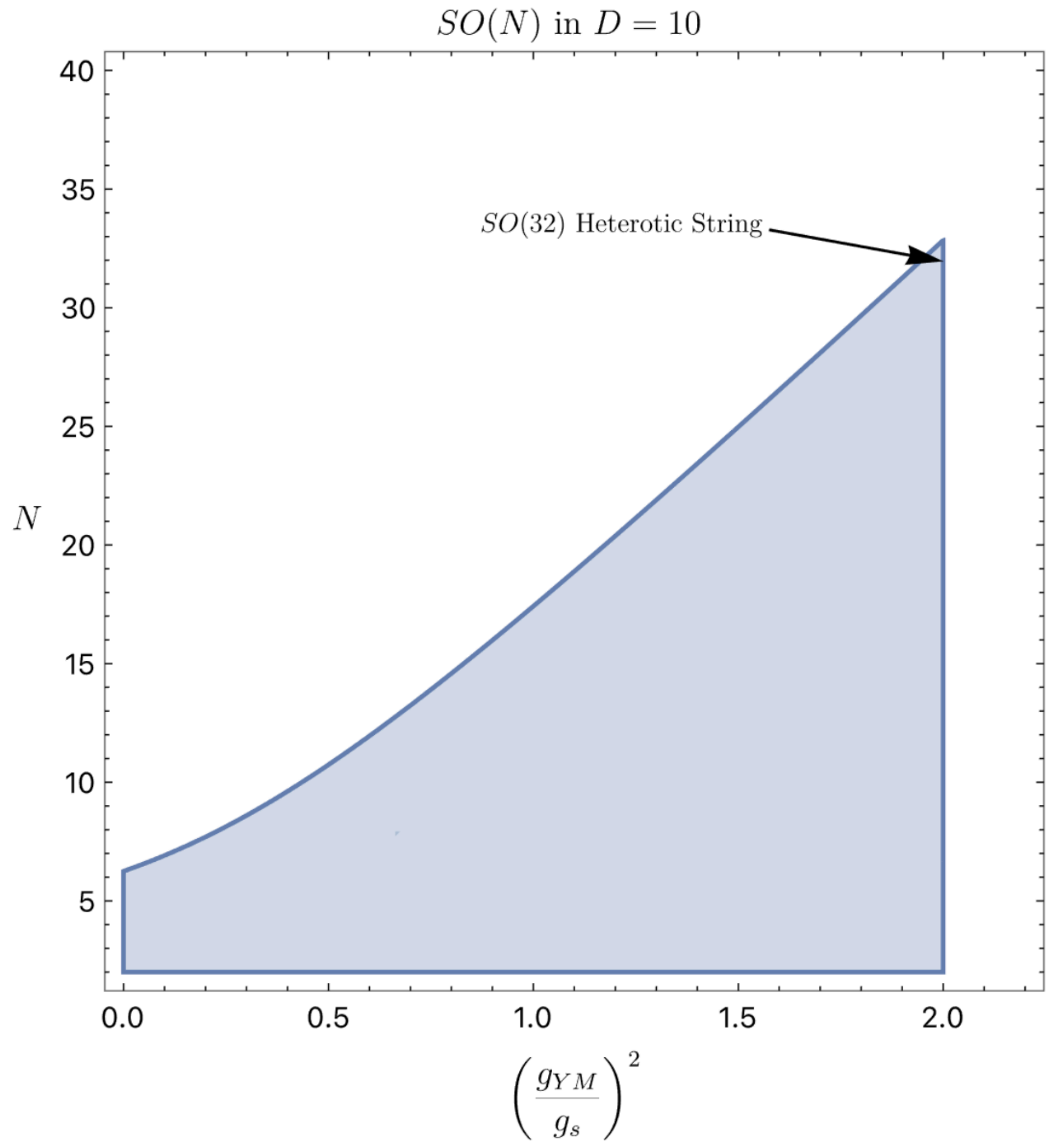
No solutions for n> 35

Require the residues on factorization poles to be consistent with unitarity

$$\lim_{s\to m^2} \mathcal{A}(1^a 2^b 3^c 4^d) \sim \frac{1}{s-m^2} \sum_J \rho_{J,\alpha} \mathbb{P}_\alpha^{abcd}\, \mathbb{G}_j(\cos\theta)$$

$$\rho_{J,\alpha} > 0$$

Brad Bachu, Aaron Hillman, 2212.03871



Can we see this tension in a more general context ?

The Gravitation EFT

Why now ?

- Experience in bootstrapping perturbative amplitudes with high multiplicity and loop level
- AdS/CFT allows us to put properties of S-matrix in quantum gravity on firm footing
- Numeric methods developed in the CFT bootstrap -> semi-definite programming (ML ?)

. arXiv:2102.08951 [pdf, other] hep-th doi 10.1007/JHEP07(2021)110

Sharp Boundaries for the Swampland

Authors: Simon Caron-Huot, Dalimil Mazac, Leonardo Rastelli, David Simmons-Duffin

arXiv:2205.01495 [pdf, other] hep-th gr-qc

Graviton partial waves and causality in higher dimensions

Authors: Simon Caron-Huot, Yue-Zhou Li, Julio Parra-Martinez, David Simmons-Duffin

. arXiv:2103.12728 [pdf, other] hep-th gr-qc hep-ph doi 10.1088/1751-8121/ac0e51

Gravitational Effective Field Theory Islands, Low-Spin Dominance, and the Four-Graviton Amplitude

Authors: Zvi Bern, Dimitrios Kosmopoulos, Alexander Zhiboedov

. arXiv:2102.02847 [pdf, other] hep-th doi 10.1103/PhysRevLett.127.081601

Where is String Theory?

Authors: Andrea Guerrieri, Joao Penedones, Pedro Vieira

arXiv:2201.07177 [pdf, other] hep-th

(Non)-projective bounds on gravitational EFT

Authors: Li-Yuan Chiang, Yu-tin Huang, Wei Li, Laurentiu Rodina, He-Chen Weng

. arXiv:2104.09682 [pdf, other] hep-th gr-qc hep-ph doi 10.1103/PhysRevLett.127.091602

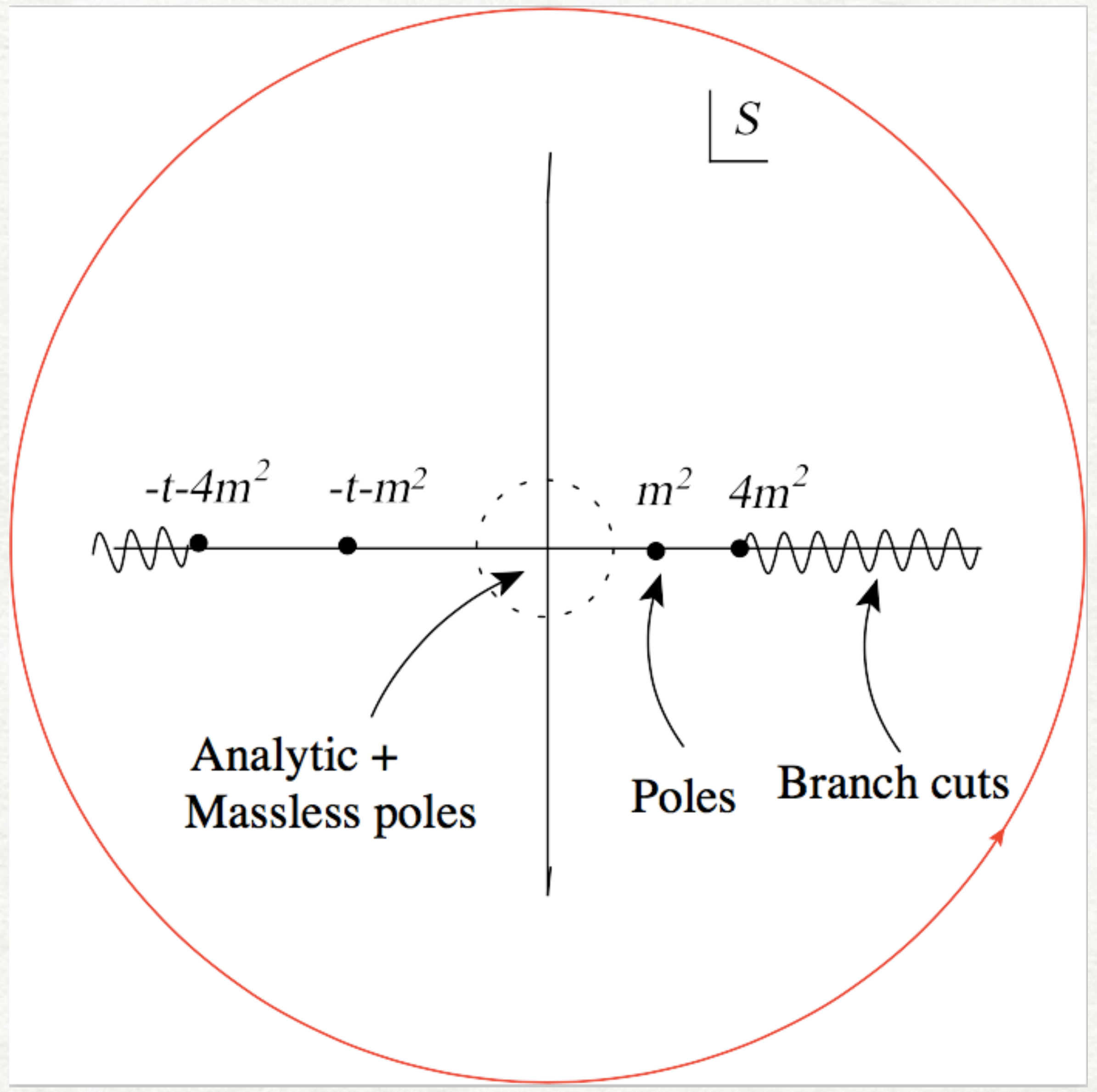
Is the Standard Model in the Swampland? Consistency Requirements from Gravitational Scattering

Authors: Katsuki Aoki, Tran Quang Loc, Toshifumi Noumi, Junsei Tokuda

Dispersion with gravity

Causality implies twice subtraction

Haring, Zhiboedov 2202.08280



$$\oint_\infty \frac{ds'}{2\pi i(s'-s)} \frac{M^{abcd}(s',t)}{s'(s'+t)} = 0\,,$$

Relates the low energy parameters with unitarity of UV

$$(\text{Res}_{s'=0} + \text{Res}_{s'=-t} + \text{Res}_{s'=s}) \frac{M^{abcd}(s',t)}{(s'-s)s'(s'+t)} =$$

$$\int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \left(\frac{\text{Im}M^{abcd}(s',t)}{(s'-s)} + \frac{\text{Im}M^{abcd}(-s'-t,t)}{(-s'-t-s)} \right)$$

$$\left\langle \frac{suP_J(1+\frac{2t}{s'})}{s'+t} \left(\frac{P_R^s}{(s'-s)} + \frac{P_R^u}{(s'+t+s)} \right) \right\rangle$$

We expand in t on both sides to get dispersive representation for the Wilson coefficients, but not with gravity

A proof of species bound

Simon Caron-Huot, Yue-Zhou Li arXiv:2408.06440

A longstanding observation is that in the presence of a large number N of matter fields below the scale Λ, the “quantum gravity cutoff” Λ should be parametrically lower than the Planck mass

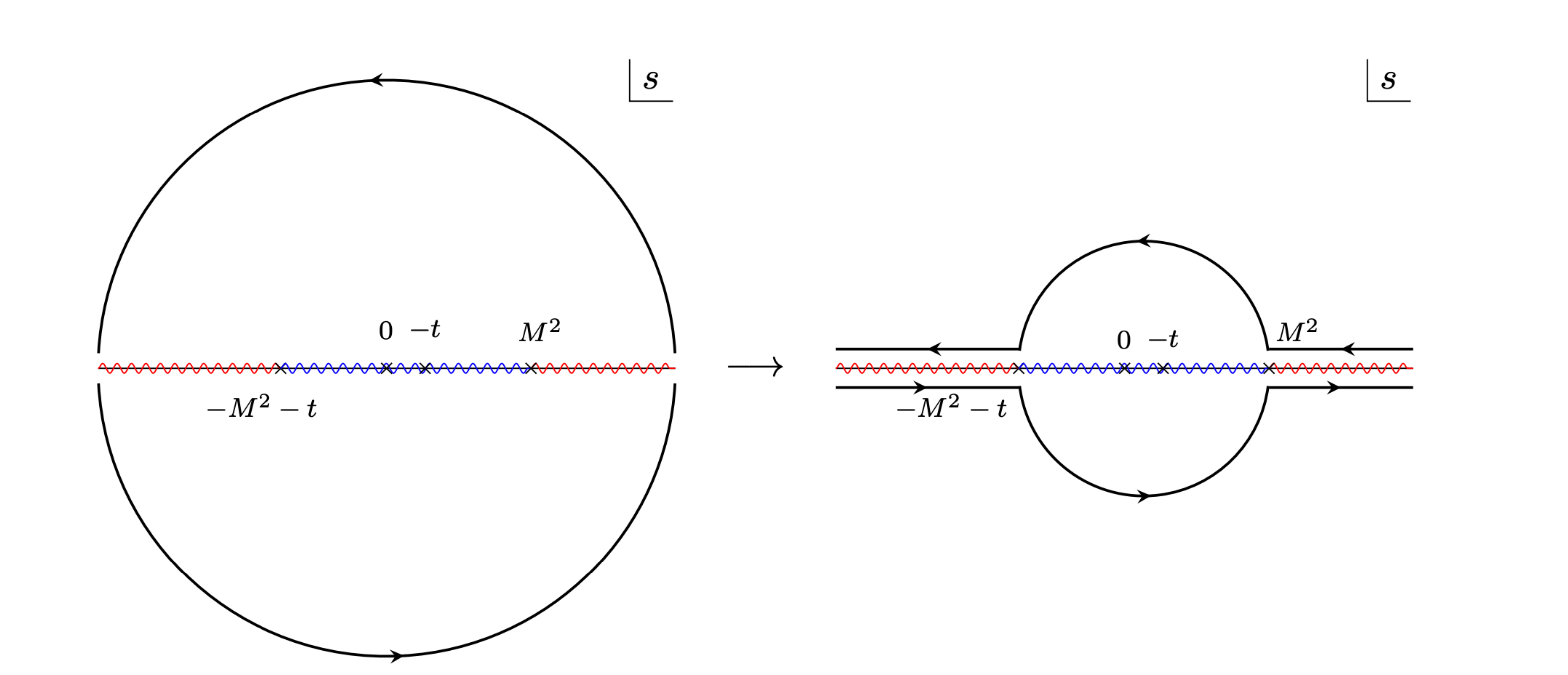
$$\Lambda^{d-2} N(\Lambda) \lesssim \mathcal{O}(1) M_{\rm pl}^{d-2}$$

Consider the four-graviton amplitude $\mathcal{M}(1^+2^-3^-4^+) = \langle 23\rangle^4 [14]^4 f(s,u)$

Which satisfies Kramers-Kronig-type sum rules

$$B_2(p) \equiv \oint_{\mathcal{C}_+\cup\mathcal{C}_-} \frac{ds}{2\pi i}(s-t) f(s, u=-p^2) = 0$$

$\longrightarrow$ $$\frac{8\pi G}{p_\perp^2} = \int \frac{ds}{\pi}(2s-p^2)\,\mathrm{Im}\, f(s,-p_\perp^2)$$



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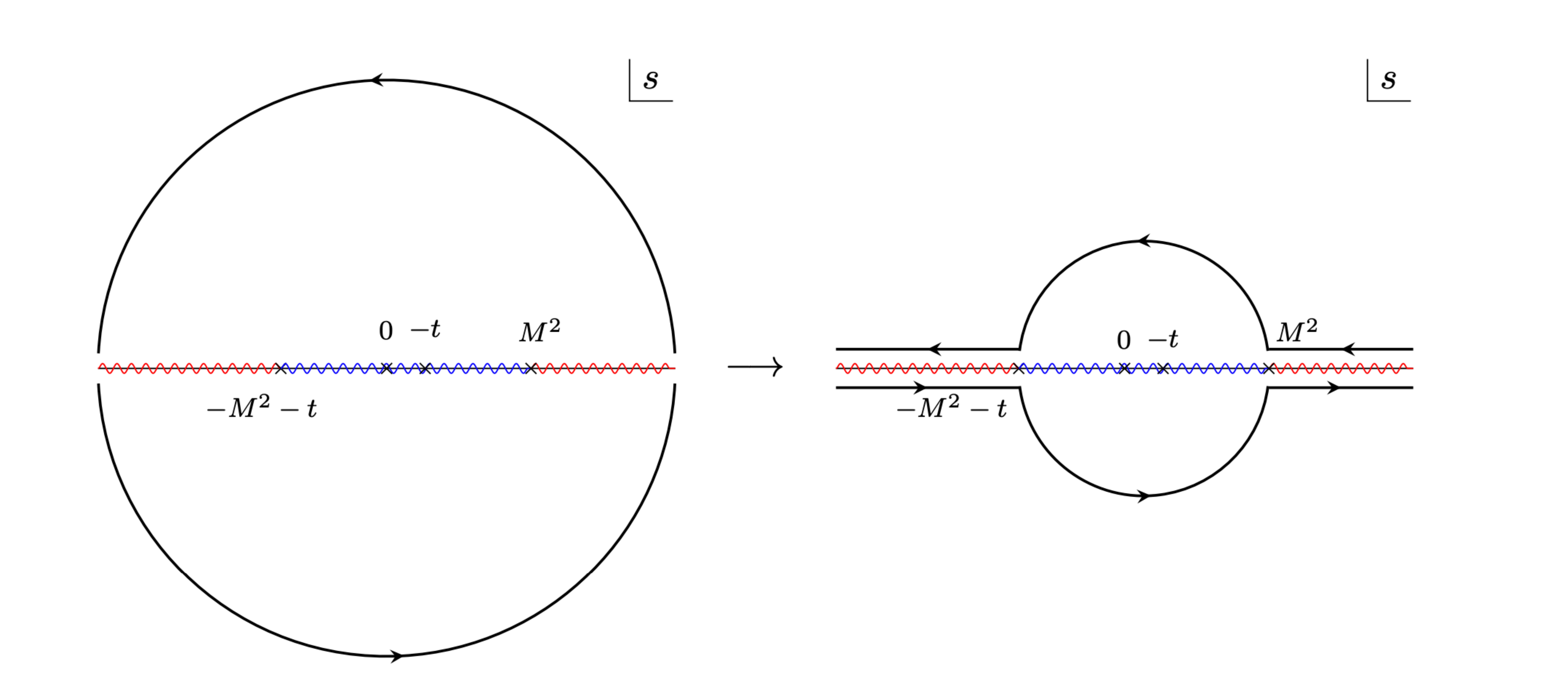
$$-\sum_{k=2,3}\int_0^M pdp\,\psi_k(p)B_k(p)\Big|_{\text{low}} = \sum_{k=2,3}\int_0^M pdp\,\psi_k(p)B_k(p)\Big|_{\text{high}}$$

$$-B_2(p)\Big|_{\text{low}} = \sum_{\pm}\int_{M^2}^{p^2-M^2}\frac{ds}{2\pi i}(p^2-2s)f(s,-p^2) = \frac{8\pi G}{p^2} + \text{loops}\,,$$



$$B_2(p)\Big|_{\text{high}} = 16\int_{M^2}^{\infty}\frac{ds}{s^4}(2s-p^2)\left[\sum_{J\geq 0,\,\text{even}}|\bar{c}_J^{++}(s)|^2 P_J\left(1-\tfrac{2p^2}{s}\right) + \sum_{J\geq 4}|\bar{c}_J^{+-}(s)|^2\tilde{d}^J_{4,4}\left(1-\tfrac{2p^2}{s}\right)\right]$$

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Simon Caron-Huot, Yue-Zhou Li arXiv:2408.06440

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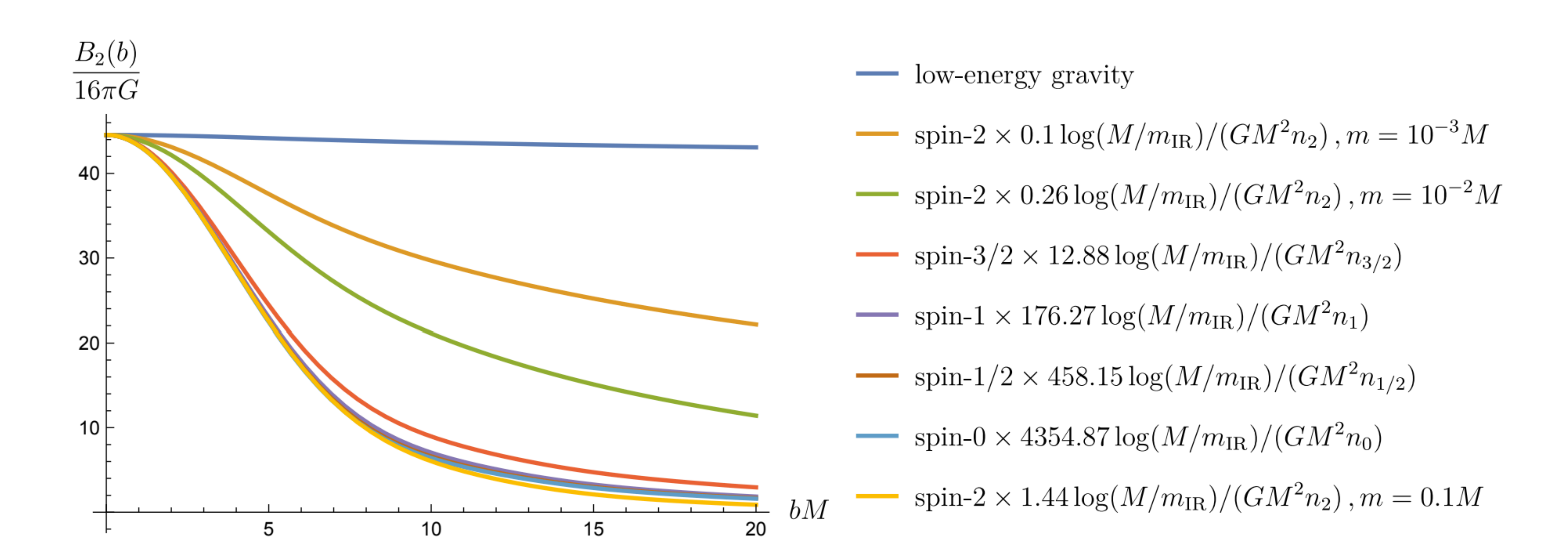
$$\Lambda^{d-2} N(\Lambda) \lesssim \mathcal{O}(1) M_{\rm pl}^{d-2}$$

$$B_2(p) \equiv \oint_{\mathcal{C}_+ \cup \mathcal{C}_-} \frac{ds}{2\pi i}(s-t) f(s, u=-p^2)\,,$$

smeared bounds →

$$B_2(b) = \int_0^M dp\,(1-p)^2 p J_0(bp) B_2(p)$$



Exp

$$n_0 + 5.6 n_{1/2} + 9.2 n_1 + 157 n_{3/2} + 491.7 n_2 \log\frac{M}{m_2} < \left(1429.6 \log\frac{M}{m_{\rm IR}} - 1735.6\right)\frac{1}{GM^2}$$

Plan:

Consider a general colored EFT (Adjoint or Fundamental), derive optimal bounds on the Wilson coefficients

$$M^{abcd}(s,t) = g^2\left(P^s_{adj}\frac{t-u}{s} + P^t_{adj}\frac{u-s}{t} + P^u_{adj}\frac{s-t}{u}\right) + 8\pi G\left(P^s_I\frac{tu}{s} + P^t_I\frac{us}{t} + P^u_I\frac{st}{u}\right) + B^{abcd}(s,t),$$

$$B^{abcd}(s,t) = \sum_{\sigma\in S_3}\mathrm{Tr}(T^aT^{\sigma(b)}T^{\sigma(c)}T^{\sigma(d)})B_1(1,\sigma(2),\sigma(3),\sigma(4)) + B_2(s,t)\mathrm{Tr}(T^aT^b)\mathrm{Tr}(T^cT^d) + B_2(t,u)\mathrm{Tr}(T^aT^c)\mathrm{Tr}(T^bT^d) + B_2(u,s)\mathrm{Tr}(T^aT^d)\mathrm{Tr}(T^cT^b).$$

$$B_1(1234) = \sum_{k,q\leq k,q\in\text{even}} g_{kq}t^{k-q}(s-u)^q,$$

$$B_2(s,t) = \sum_{k,q\leq k,q\in\text{even}} G_{kq}s^{k-q}(t-u)^q,$$

Dispersion with gravity

Improved dispersion relation

$$\bar{A}^{abcd}(s,t). \equiv \frac{B^{abcd}(s,t)}{s(s+t)} - \frac{B^{abcd}(0,t)}{st} + \frac{B^{abcd}(-t,t)}{(s+t)t} - \frac{8\pi G}{t}P_I^t =$$
$$16\pi \sum_{JR}(2J+1)P_J(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \mathrm{Im}(a_{JR}(s')) \left(\frac{P_R^s}{(s'-s)} + \frac{P_R^u}{(s'+t+s)} \right)$$

$$\frac{1}{t}\Big[\tilde{A}^{(1,0)abcd}(0,t) - \tilde{A}^{(0,1)acbd}(t,0) + \tilde{A}^{(0,1)acbd}(t,0)|_{\mathcal{O}(2)} -$$
$$\frac{1}{t}\Big(- \tilde{A}^{acbd}(t,0) + \tilde{A}^{acbd}(t,0)|_{\mathcal{O}(3)} + \tilde{A}^{acdb}(t,0) - \tilde{A}^{acdb}(t,0)|_{\mathcal{O}(3)}\Big)\Big] =$$
$$c^{abcd}(t) + 8\pi G\frac{P_I^t}{t},$$

$$c^{abcd}(t) = \begin{pmatrix} -4(tg_{32}+g_{22}) \\ 2t(g_{30}-g_{32})-g_{20}-g_{22} \\ t(g_{30}-3g_{32})-g_{20}-g_{22} \\ 2t(g_{30}-g_{32})-g_{20}-g_{22} \\ t(g_{30}-3g_{32})-g_{20}-g_{22} \\ -4(tg_{32}+g_{22}) \\ t(G_{30}-3G_{32})-G_{20}-G_{22} \\ -4(tG_{32}+G_{22}) \\ 2t(G_{30}-G_{32})-G_{20}-G_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{Tr}(T^aT^bT^cT^d) \\ \mathrm{Tr}(T^aT^bT^dT^c) \\ \mathrm{Tr}(T^aT^cT^bT^d) \\ \mathrm{Tr}(T^aT^cT^dT^b) \\ \mathrm{Tr}(T^aT^dT^bT^c) \\ \mathrm{Tr}(T^aT^dT^cT^b) \\ \mathrm{Tr}(T^aT^b)\mathrm{Tr}(T^cT^d) \\ \mathrm{Tr}(T^aT^c)\mathrm{Tr}(T^bT^d) \\ \mathrm{Tr}(T^aT^d)\mathrm{Tr}(T^bT^c) \end{pmatrix}$$

We can reduce to three couplings in a dispersion relation

Dispersion with gravity

The low energy couplings are in the trace basis, but the UV is in the projector basis

It will be convenient to put in t-channel projector basis where the graviton pole is isolated

$$\begin{pmatrix} \mathrm{Tr}[a,b,c,d] \\ \mathrm{Tr}[a,b,d,c] \\ \mathrm{Tr}[a,d,b,c] \\ \mathrm{DTr}[a,b;c,d] \\ \mathrm{DTr}[a,c;b,d] \\ \mathrm{DTr}[a,d;b,c] \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -1 & 0 & 0 \\ \frac{N-1}{2} & \frac{N-2}{4} & 0 & 0 & \frac{N-2}{4} & 0 \\ \frac{N-1}{2} & \frac{N-2}{4} & 0 & 0 & \frac{2-N}{4} & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ \frac{N(N-1)}{2} & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} P_1^t \\ P_2^t \\ P_3^t \\ P_4^t \\ P_5^t \\ P_6^t \end{pmatrix}$$

$$\vec{P}^s = M_{st}\vec{P}^t$$

$$\begin{pmatrix} \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(1-N)N} & \frac{2}{(1-N)N} \\ \frac{N+2}{N} & \frac{N^2-8}{2(N-2)N} & \frac{N-4}{(N-2)N} & \frac{2(N+2)}{(2-N)N} & \frac{(4-N)(N+2)}{2(N-2)N} & \frac{4}{(N-2)N} \\ \frac{N^3-7N-6}{6(N-1)} & \frac{(N-4)(N-3)(N+1)}{6(N-2)(N-1)} & \frac{N^2-6N+11}{3(N-2)(N-1)} & \frac{(N+1)(N+2)}{3(N-2)(N-1)} & \frac{N^3-7N-6}{6(N-2)(N-1)} & \frac{(N-4)(N+1)}{3(N-2)(N-1)} \\ \frac{(N-3)(N-2)}{12} & \frac{3-N}{6} & \frac{1}{6} & \frac{1}{6} & \frac{3-N}{6} & \frac{1}{6} \\ 1 & \frac{N-4}{2(N-2)} & \frac{1}{2-N} & \frac{2}{N-2} & -\frac{1}{2} & 0 \\ \frac{(N-3)(N+2)}{4} & \frac{N-3}{2-N} & \frac{N-4}{2(2-N)} & \frac{N+2}{2(2-N)} & 0 & -\frac{1}{2} \end{pmatrix}$$

while M_{ut} given as:

$$\begin{pmatrix} \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} & \frac{2}{(N-1)N} \\ \frac{N+2}{N} & \frac{N^2-8}{2(N-2)N} & \frac{N-4}{(N-2)N} & \frac{2(N+2)}{(2-N)N} & \frac{(N-4)(N+2)}{2(N-2)N} & \frac{4}{(2-N)N} \\ \frac{(N-3)(N+1)(N+2)}{6(N-1)} & \frac{(N-4)(N-3)(N+1)}{6(N-2)(N-1)} & \frac{N^2-6N+11}{3(N-2)(N-1)} & \frac{(N+1)(N+2)}{3(N-2)(N-1)} & \frac{(N-3)(N+1)(N+2)}{6(2-N)(N-1)} & \frac{(N-4)(N+1)}{3(2-N)(N-1)} \\ \frac{(N-3)(N-2)}{12} & \frac{3-N}{6} & \frac{1}{6} & \frac{1}{6} & \frac{N-3}{6} & -\frac{1}{6} \\ 1 & \frac{N-4}{2(N-2)} & \frac{1}{2-N} & \frac{2}{N-2} & \frac{1}{2} & 0 \\ \frac{(N-3)(N+2)}{4} & \frac{N-3}{2-N} & \frac{N-4}{2(2-N)} & \frac{N+2}{2(2-N)} & 0 & \frac{1}{2} \end{pmatrix}$$

We utilize semidefinite programming (SDPB)

D. Simmons-Duffin, *A Semidefinite Program Solver for the Conformal Bootstrap*, *JHEP* **06** (2015) 174, [1502.02033].

W. Landry and D. Simmons-Duffin, *Scaling the semidefinite program solver SDPB*, 1909.09745.

$$\sum_a \begin{pmatrix} p_a^{++} & p_a^{--} & p_a^{+-} \end{pmatrix} \begin{pmatrix} [\mathbf{B_1}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ [\mathbf{B_2}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ [\mathbf{N_1}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ [\mathbf{N_2}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ \vdots \end{pmatrix} \begin{pmatrix} p_a^{*++} \\ p_a^{*--} \\ p_a^{*+-} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ 0 \\ \vdots \end{pmatrix}$$

Search for all possible 2+n dimensional vectors $\vec{v}$ such that

$$(0, -1, 0, \cdots) \cdot \vec{v} = 1, \ \& \quad \vec{v}^T \cdot \vec{\vec{F}}_{x,\ell} \succeq 0 \quad \forall x \geq 0, \ \ell = 0, 1, ..., \ell_{max}$$

$$\vec{F}_{m_a,\ell_a} = \begin{pmatrix} \frac{B_{k_1,q_1}(\ell_a)}{m_a^{2(k_1+1)}} \\ \frac{B_{k_2,q_2}(\ell_a)}{m_a^{2(k_2+1)}} \\ \frac{N_k(\ell_a)}{m_a^{2(k+1)}} \\ \vdots \end{pmatrix}$$

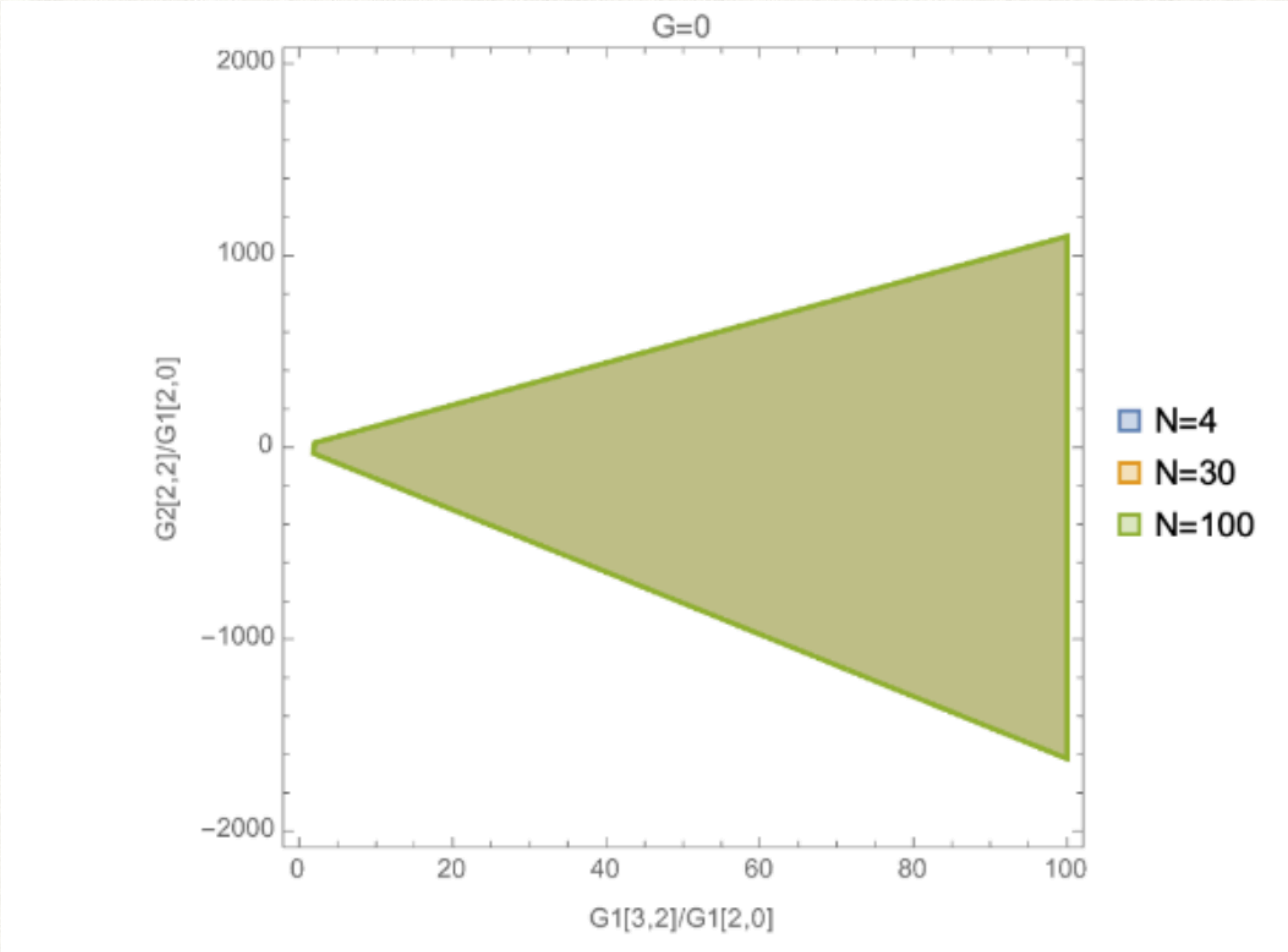
For each $\vec{v}$ we have

$$\vec{v}^T \begin{pmatrix} b_{k_1,q_1} \\ b_{k_2,q_2} \\ 0 \\ \vdots \end{pmatrix} = v_1 b_{k_1,q_1} - b_{k_2,q_2} \geq 0 \, ,$$

Minimize v_1 gives the upper bound on the ratio

Explicit bounds

It is difficult to find bounds due to both signs appearing in the dispersion relation from the projectors
But when we do find bounds



Without gravity they are independent on the rank of SO(N), only dependence comes from spacetime D

First consider the QFT (EFT) limit G=0

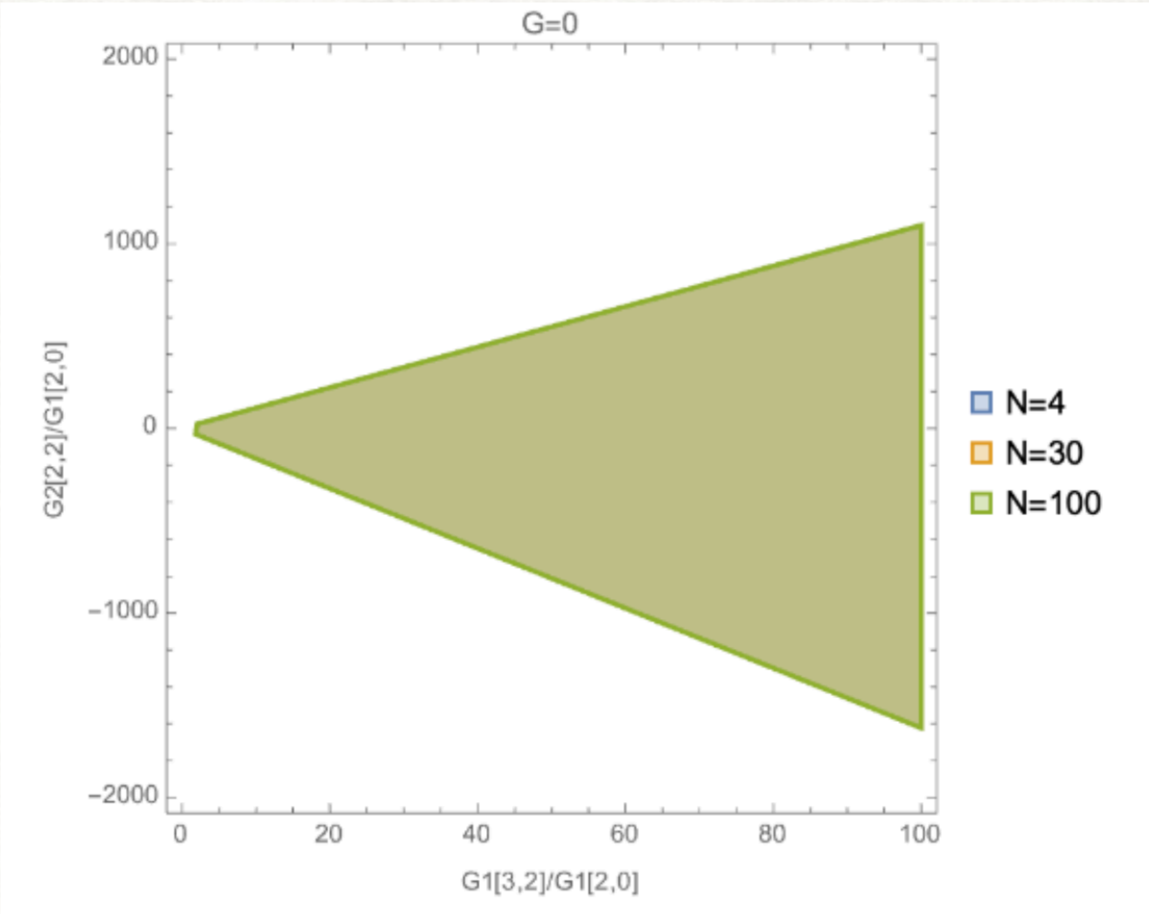
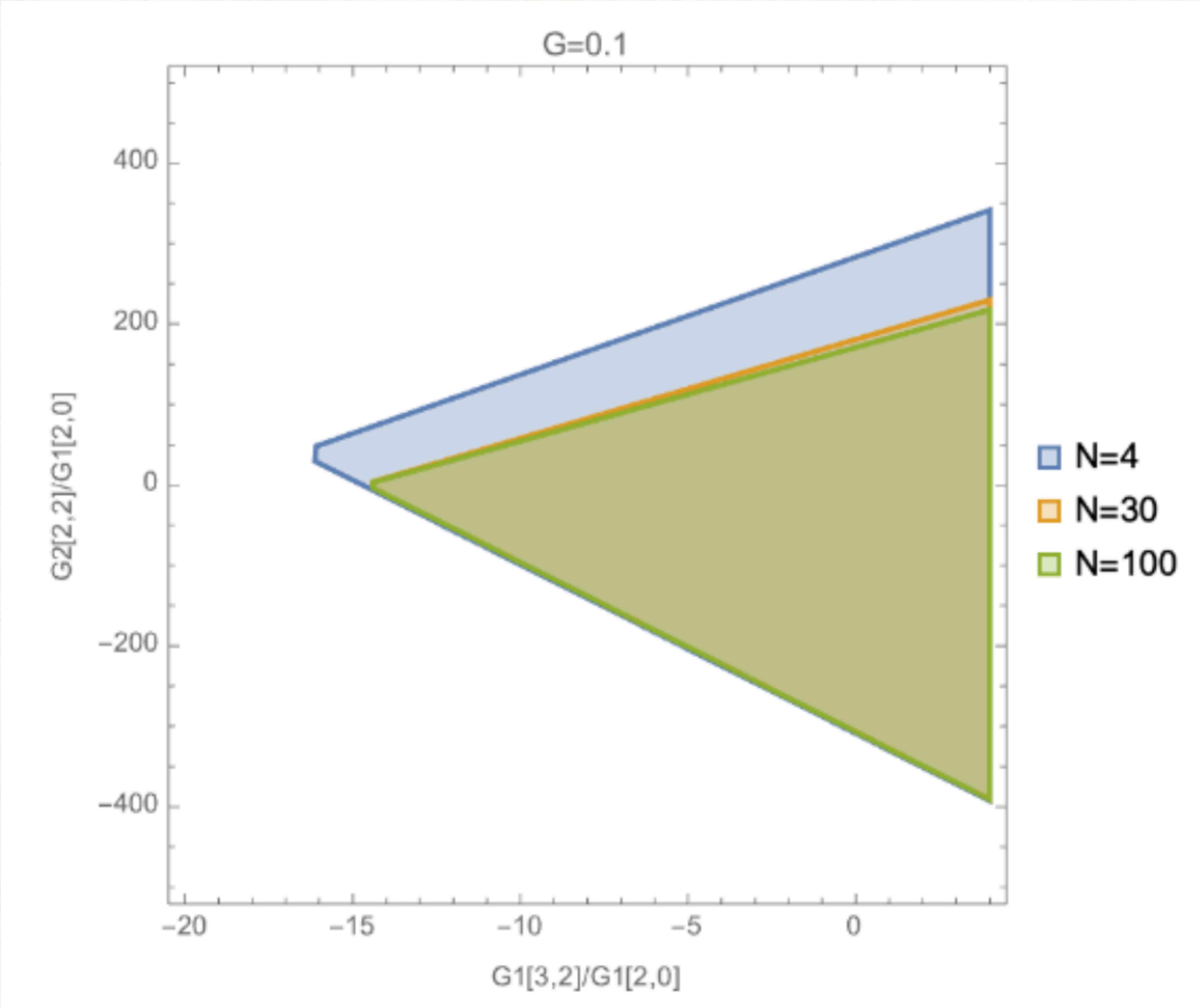


Figure 2. Adjoint representation, $G = 0$

We obtain two sided bounds that are Independent of the rank

Let's turn on gravity



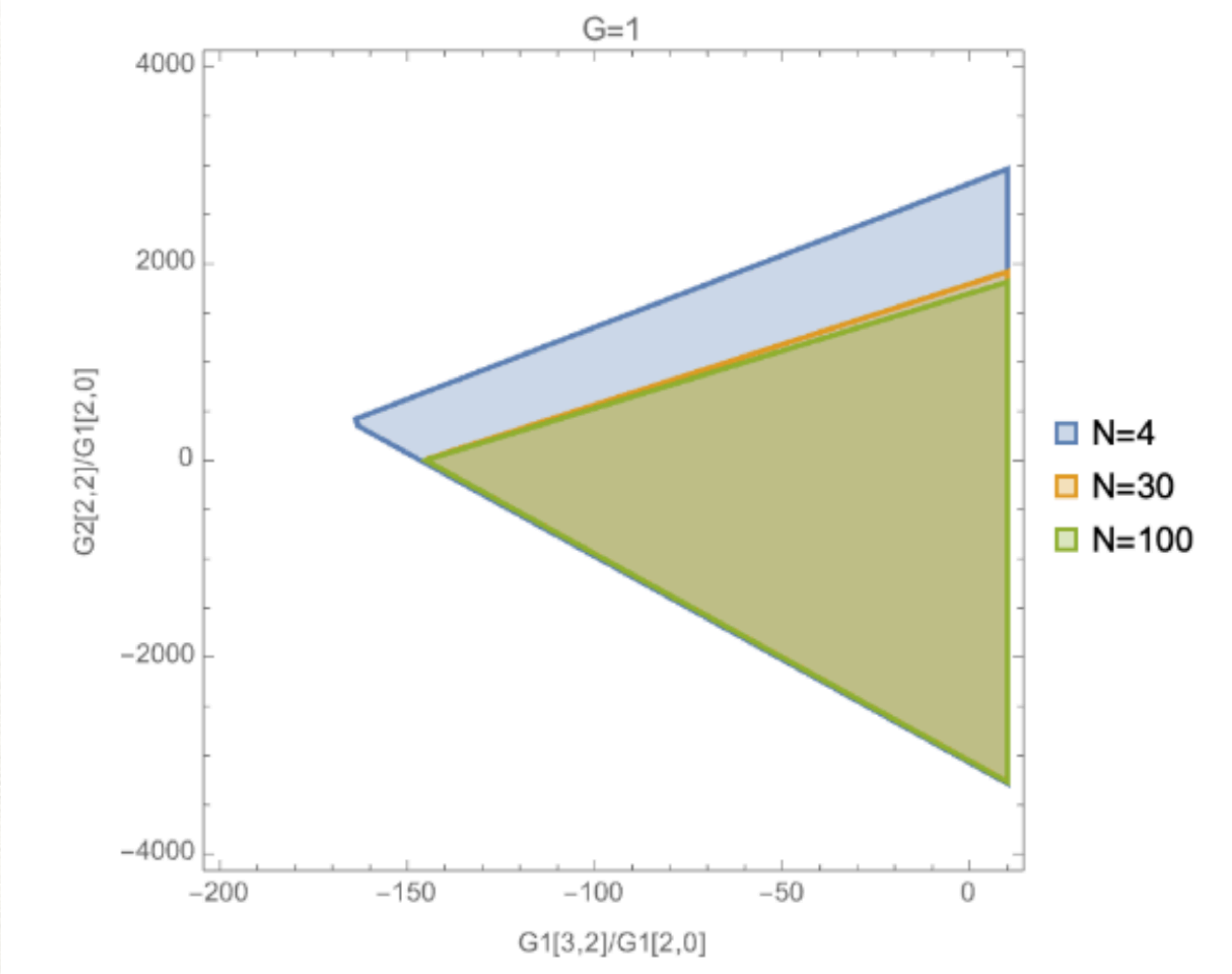


Figure 4. Adjoint representation, $G = 1$

We see N dependence!

The dependence Converges for large N, an emergent Universality

We can also consider supersymmetry. Consider maximal SUSY (16 super charges)

$$M^{abcd} = \delta^8(Q) f(s,t) \qquad \delta^8(Q) \sim s^2$$

This leads to zero subtraction dispersion relations for the couplings since

$$\lim_{s\to\infty} f(s,t) < s^0$$

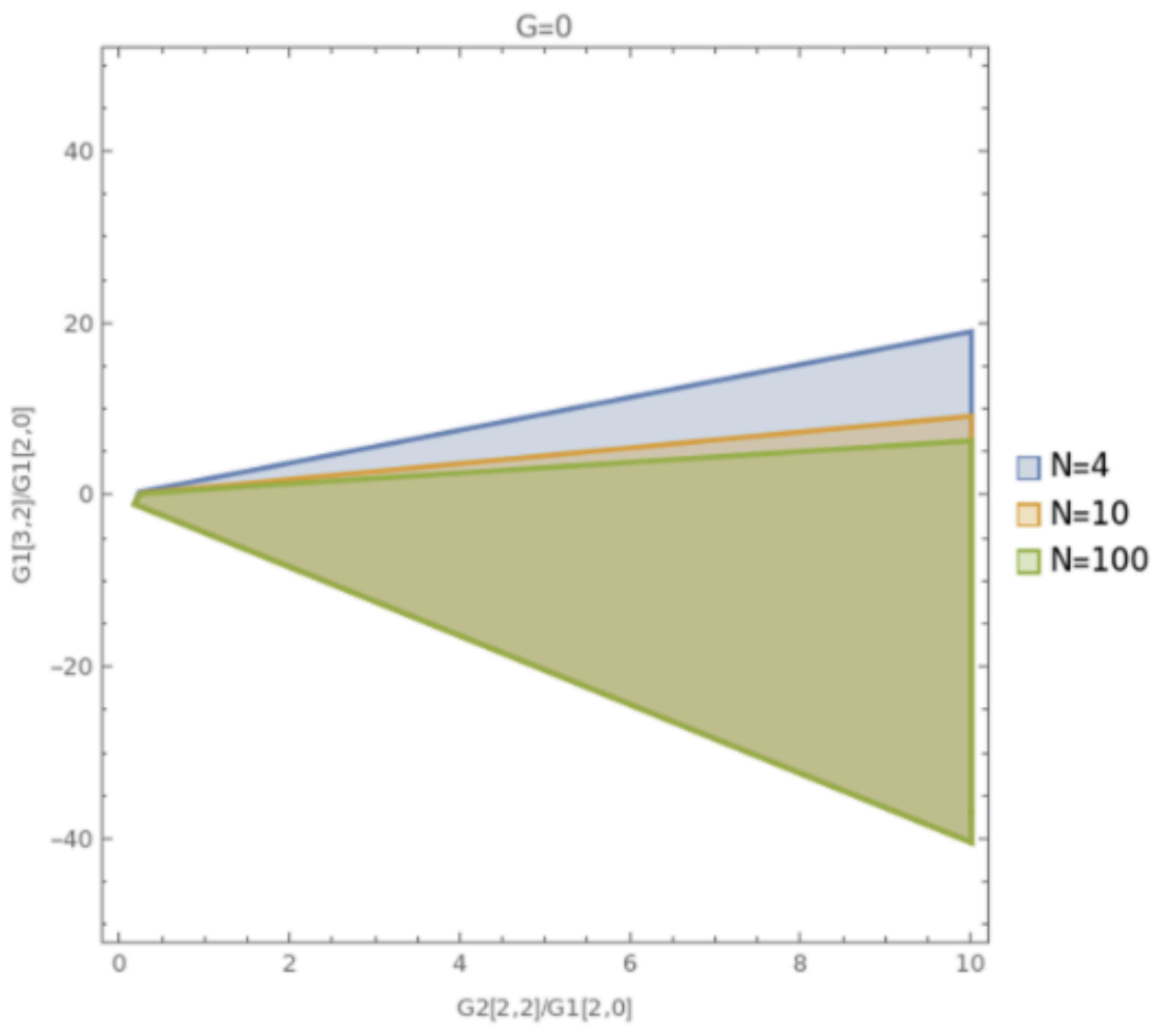


Figure 5. Adjoint representation, maximal SUSY, $G = 0$

We see N dependence even without Gravity !

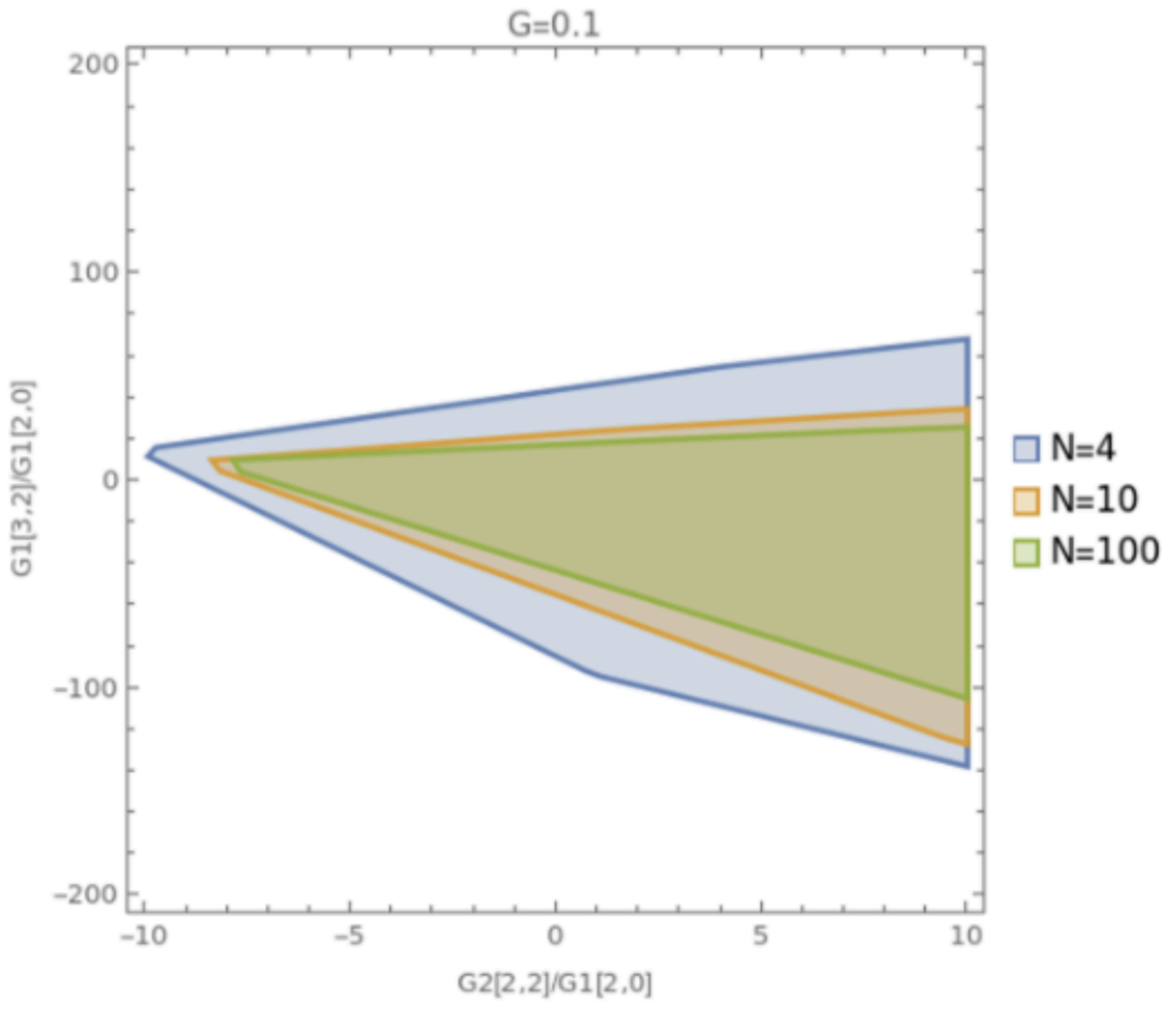
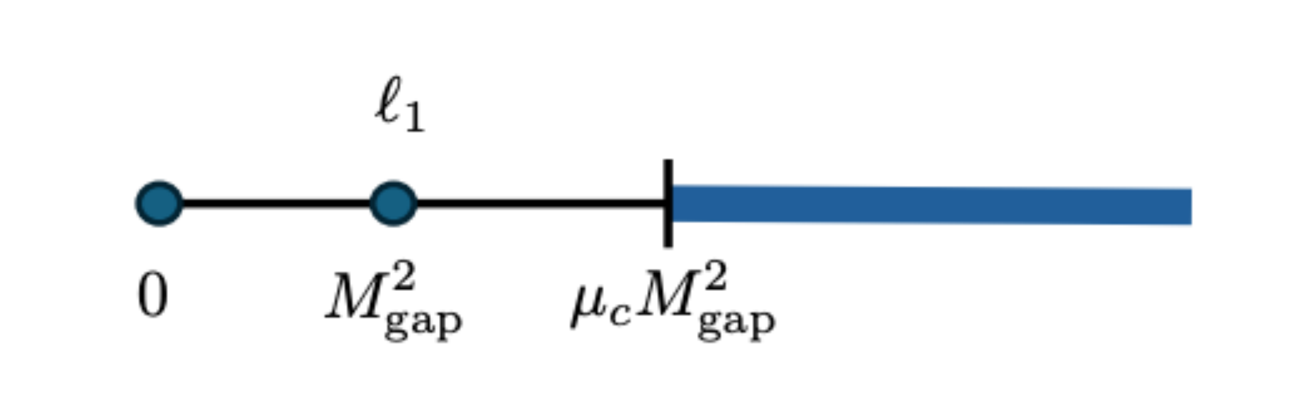
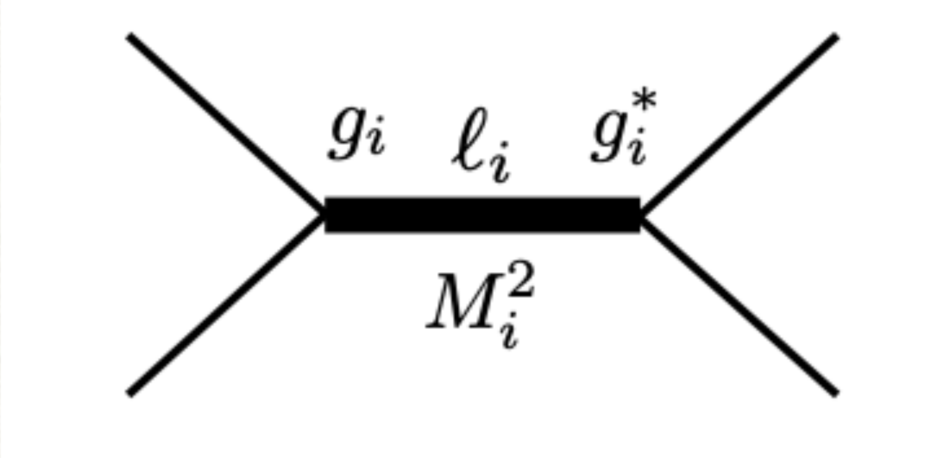


Figure 6. Adjoint representation, maximal SUSY, $G = 0.1$

Additional N dependence when gravity is turned on

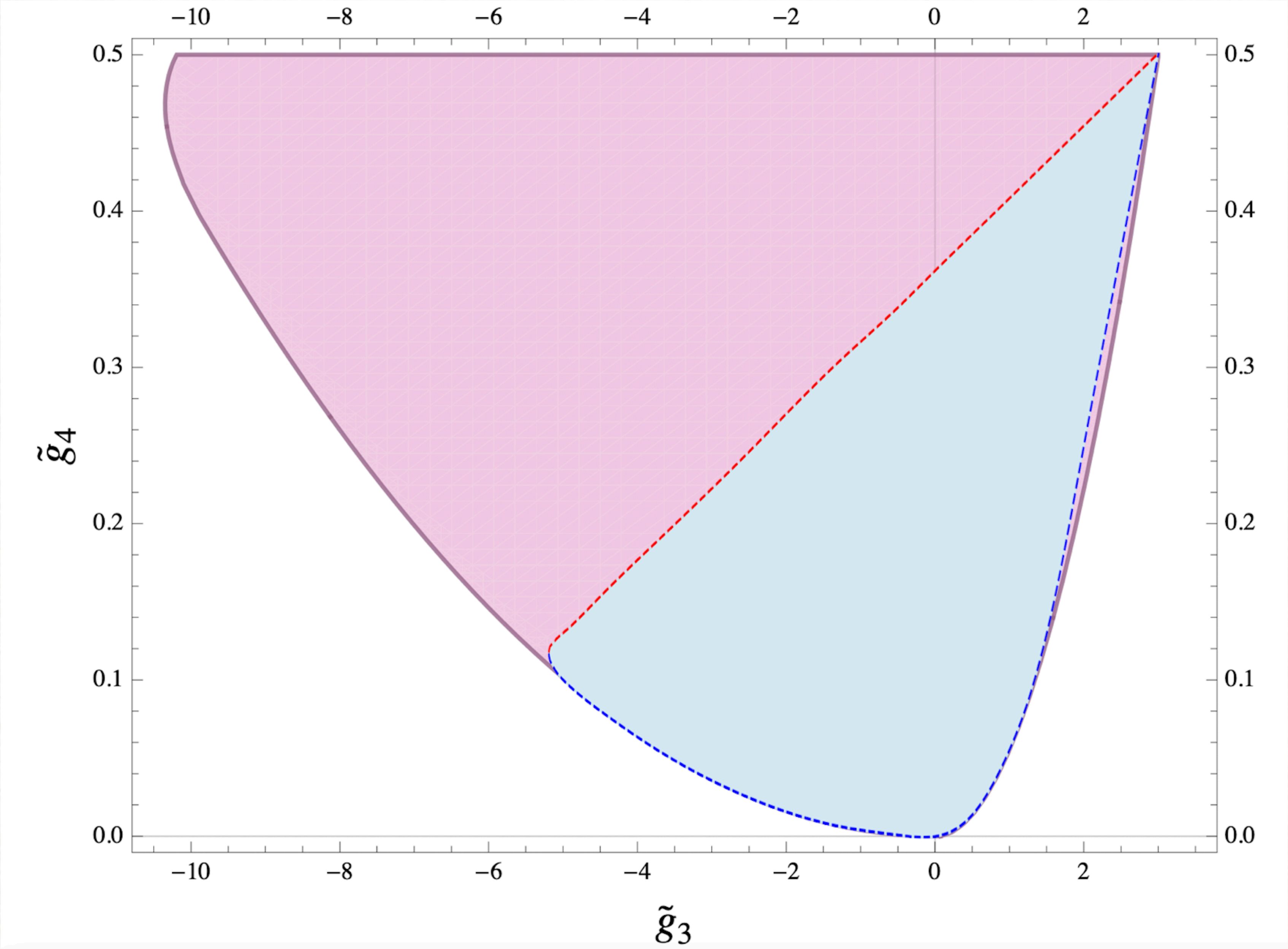
Let us assume that at the gap, there is an isolated state



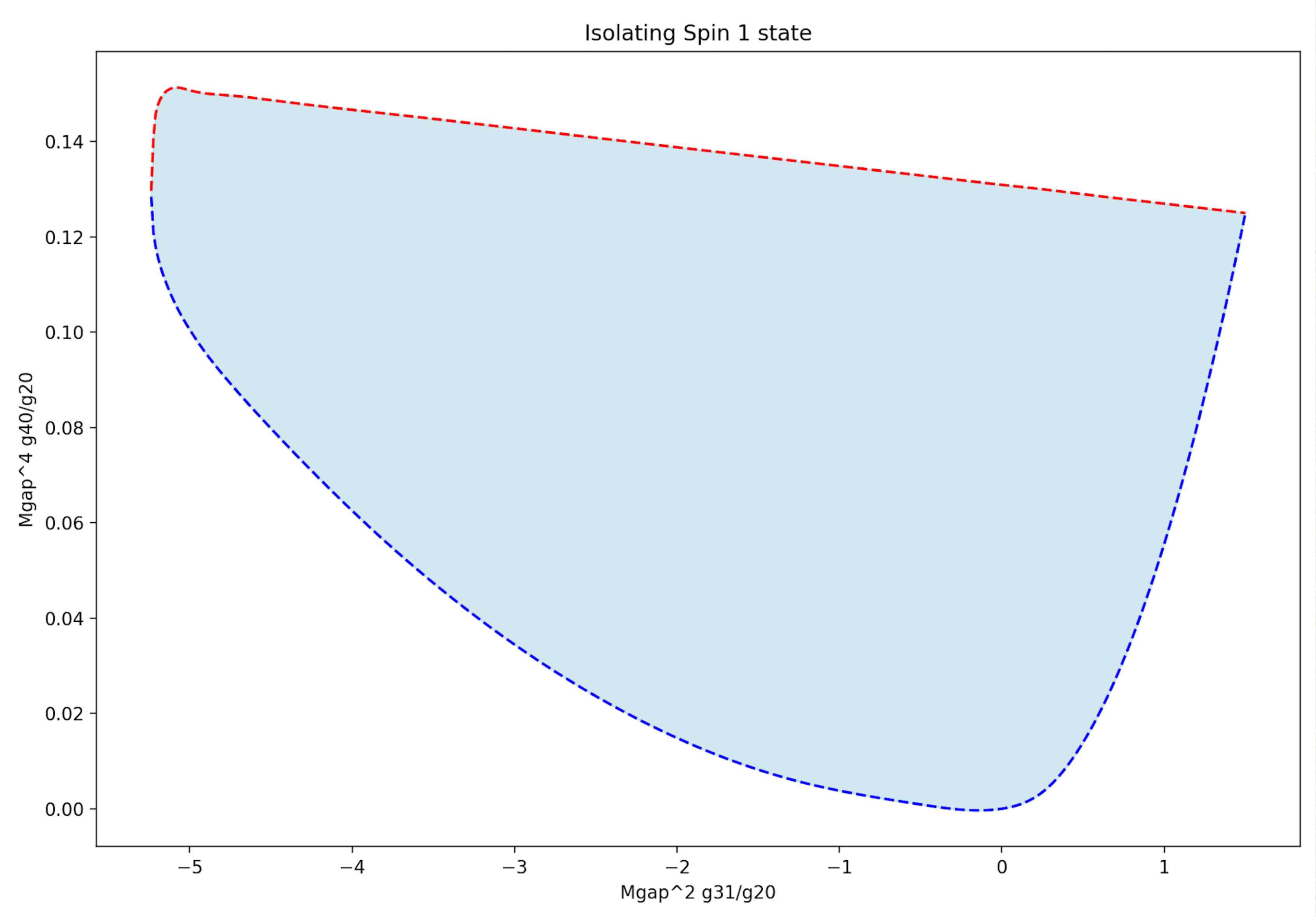


Assuming just a single scalar state, vs higher spin

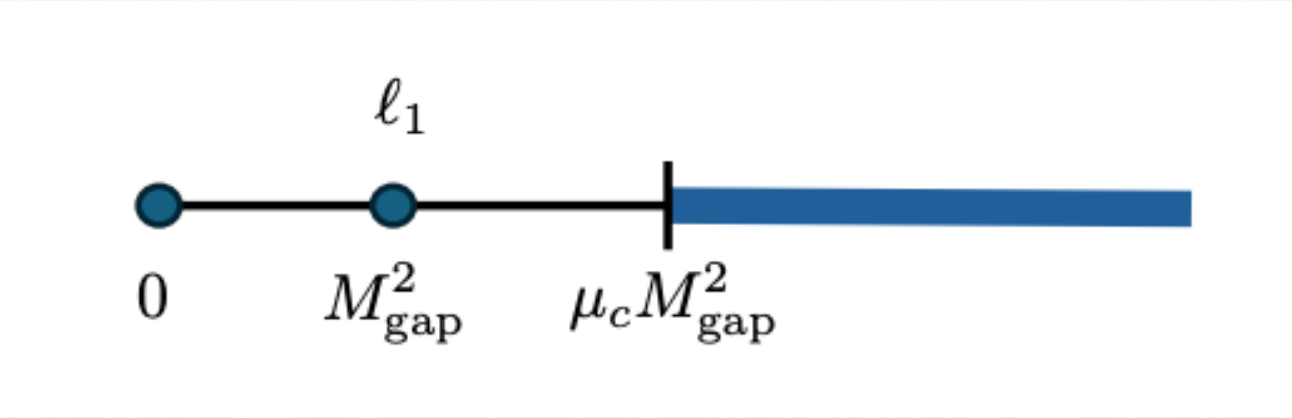
S=O

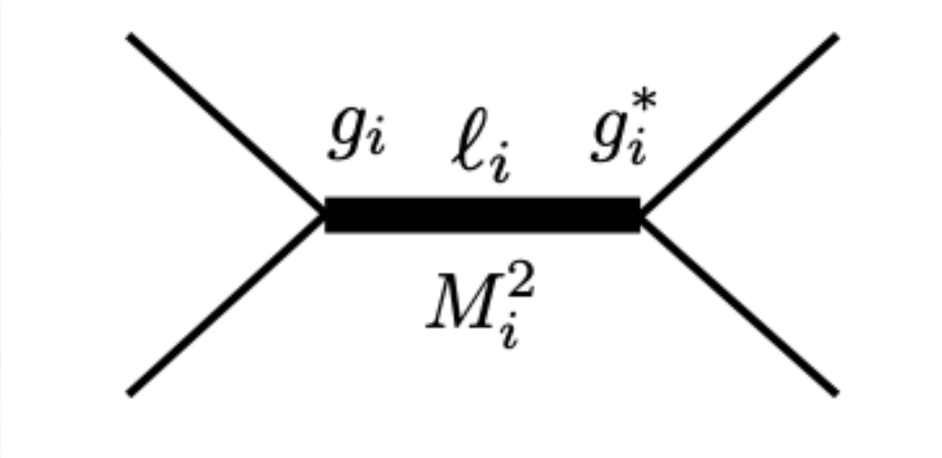


S=1

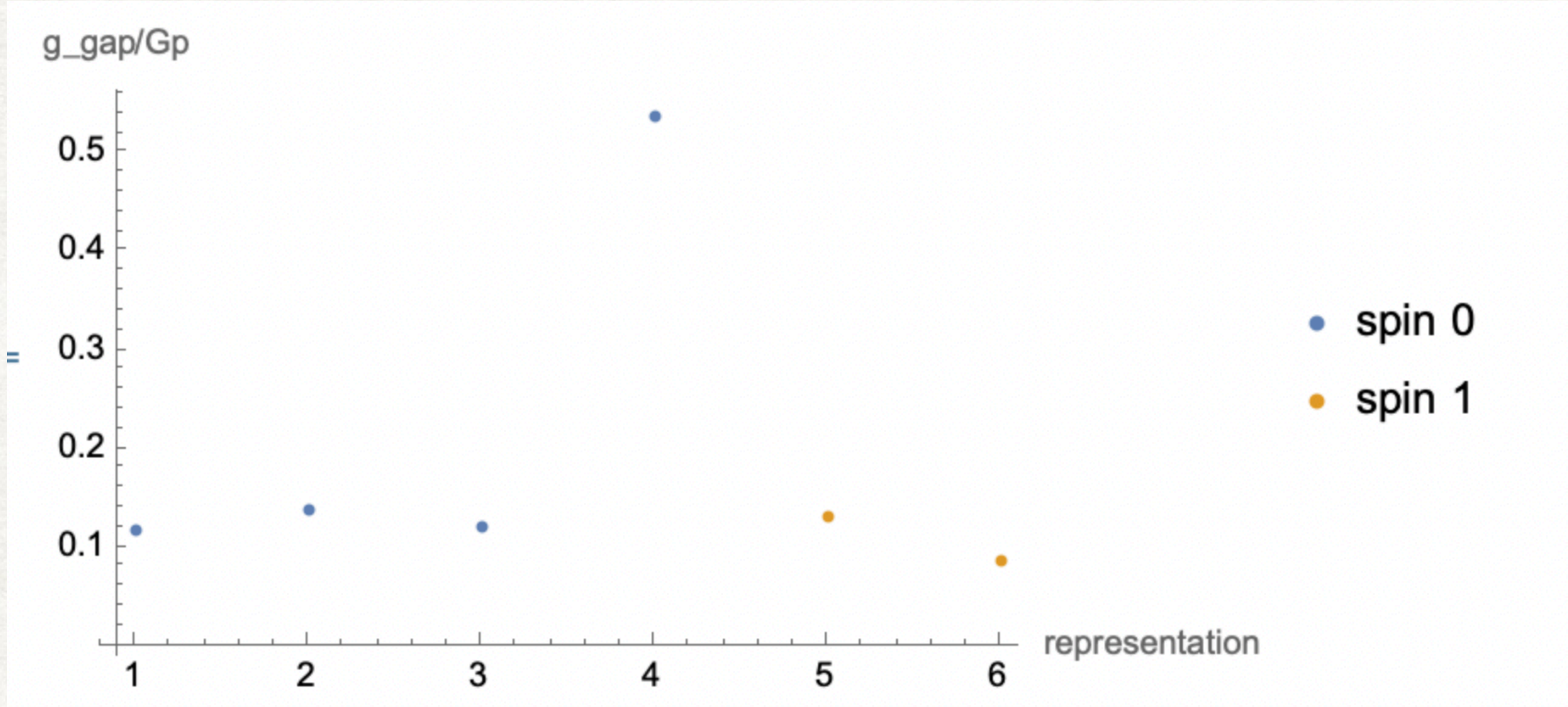


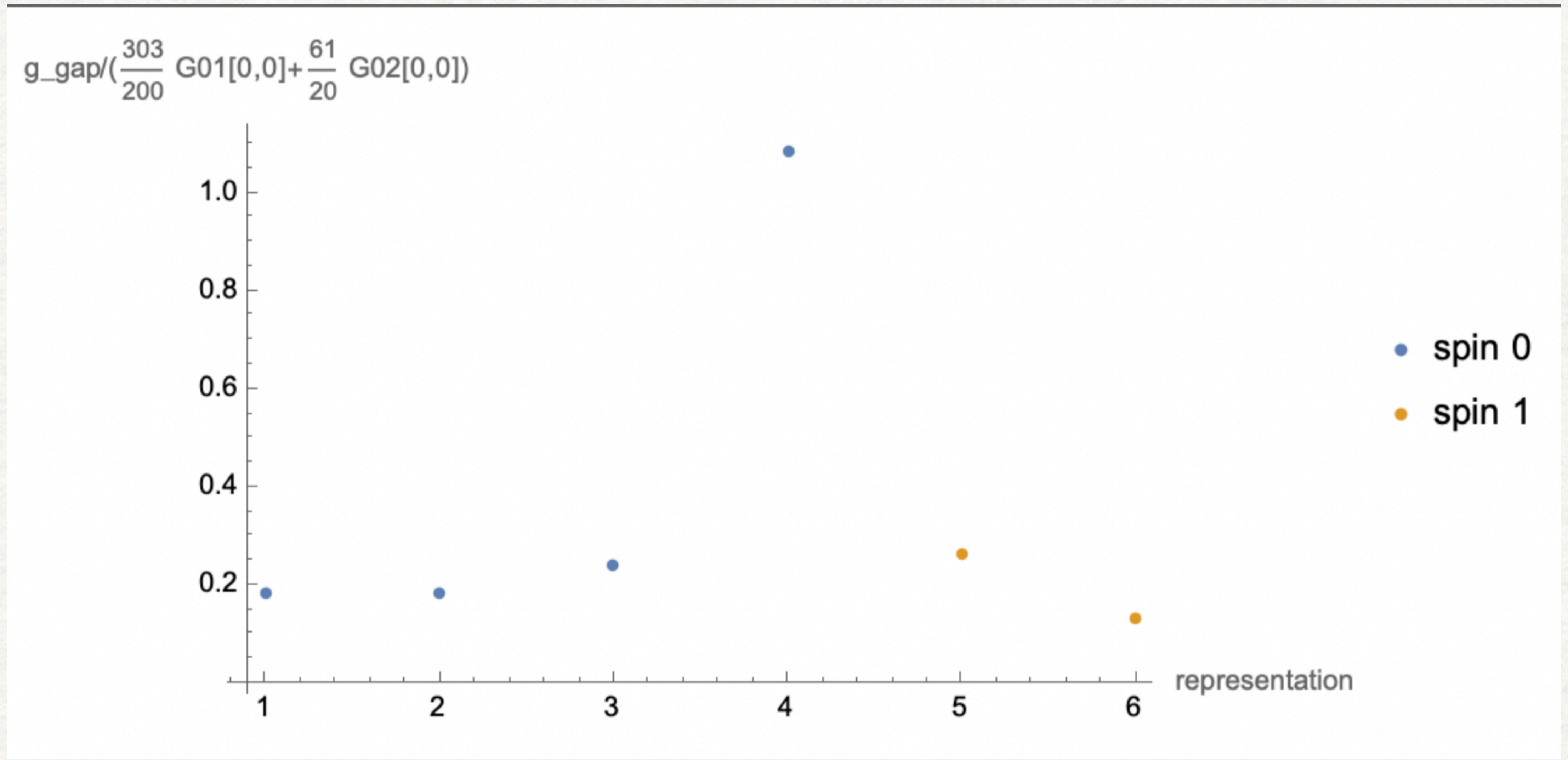
Let us assume that at the gap, there is an isolated state





We can probe the maximal value of the coupling vs EFT Wilson coefficients





Dominated by the symmetric representation !

Comments:

- We observe that the introduction of graviton poles always induce new rank-dependence on the Wilson coefficients
- Assuming maximal SUSY, we also see rank-dependence. (Coloumb branch Wilson coefficients are far from the boundary)
- For maximal SUSY the bounds are stable with respect to dimensions.

Current/Next stage:

- (Global vs Gauge): We are doing twice subtraction, which does not capture the massless gauge pole. There is no-distinction between local and global symmetries.
- Consider one subtraction for certain smeared amplitudes (Haring, Zhiboedov 2202.08280)
- Consider four-dimensional helicity states

The Gravitation EFT

Essentially a ``bottom up" approach where the EFT operators serve as IR parameterization of UV completions

$$\mathcal{L} = \int dx^D \sqrt{-g}\,(M_{\rm pl}^{D-2} R + \alpha_1 R^2 + \alpha_2 R^3 + \alpha_4 R^4 \cdots)$$

Arising from perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat\alpha_1 \frac{R^2}{M^2} + \hat\alpha_2 \frac{R^3}{M^4} + \hat\alpha_4 \frac{R^4}{M^6} \cdots\right)$$

$$M \ll M_{\rm pl}$$

From non-perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat\alpha_1 \frac{R^2}{M_{\rm pl}^2} + \hat\alpha_2 \frac{R^3}{M_{\rm pl}^4} + \hat\alpha_4 \frac{R^4}{M_{\rm pl}^6} \cdots\right)$$

String theory provides solutions to both scenarios

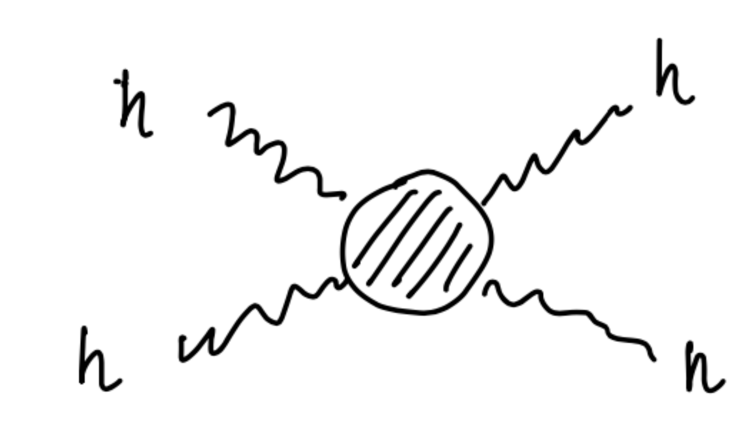
$$M_{\rm s} \propto (g_s)^{\frac{1}{4}} M_{\rm p}$$

Can we confine the space for allowed $\hat\alpha_i$ → Can we carve out the landscape of perturbative strings ?

The Gravitation S-matrix

The EFT operators are encoded in the four-graviton S-matrix which is subject to it's own consistency

$$M(s,t)$$



$$s = (p_1+p_2)^2 = E_c^2$$

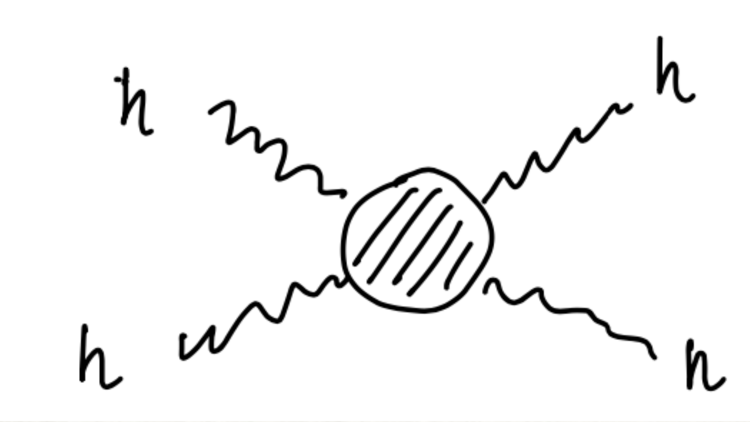
$$t = (p_1-p_4)^2 = -\frac{E_c^2}{2}(1-\cos\theta)$$

It is well defined (infrared finite) for D>4 but divergent in D=4

→ work with regulated observables, their axiomatic properties are less understood

→ restrict ourselves to perturbative (tree)-limit

The Gravitation S-matrix

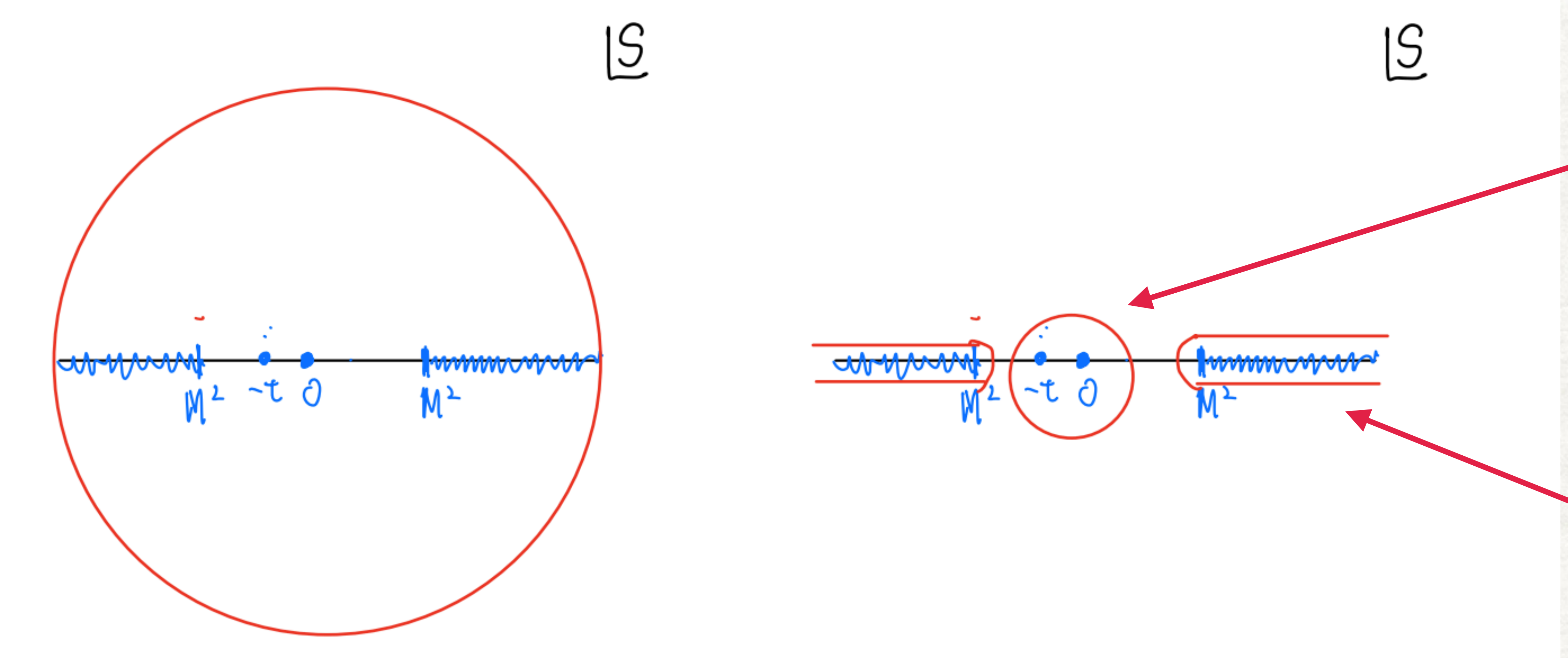


EFT information is embedded in the low-energy limit of $M(s,t)$

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2}\left(R + \hat{\alpha}_1 \frac{R^2}{M^2} + \hat{\alpha}_2 \frac{R^3}{M^4} + \hat{\alpha}_4 \frac{R^4}{M^6} \cdots \right)$$

For perturbative completion we can keep M_{pl} large, loops are suppressed

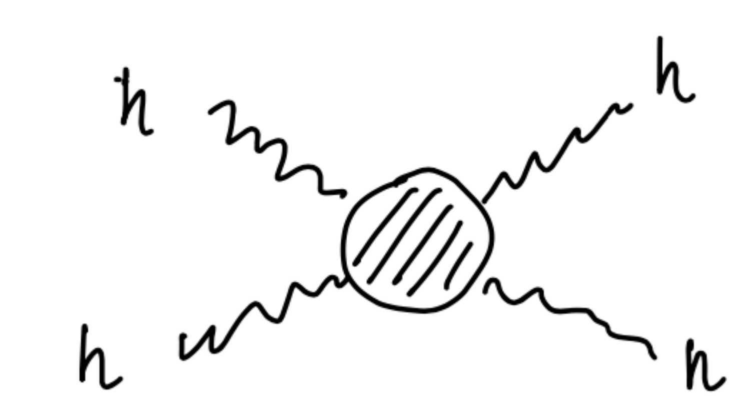
$$M^{\rm IR}(s,t) = R^4 \left(\frac{1}{stu} + \{\text{massless poles from } R^2, R^3\} + \sum_{k,q} b_{k,q} s^{k-q} t^q \right)$$



$$b_{k,q} = \frac{1}{2\pi i} \frac{\partial^q}{\partial t^q} \oint \frac{ds}{s^{k-q+1}} M(s,t)$$

$$\frac{1}{\pi} \frac{\partial^q}{\partial t^q} \int_{m^2}^{\infty} \frac{ds}{s^{k-q+1}} \mathrm{Im}[M(s,t)]$$

The Gravitation S-matrix

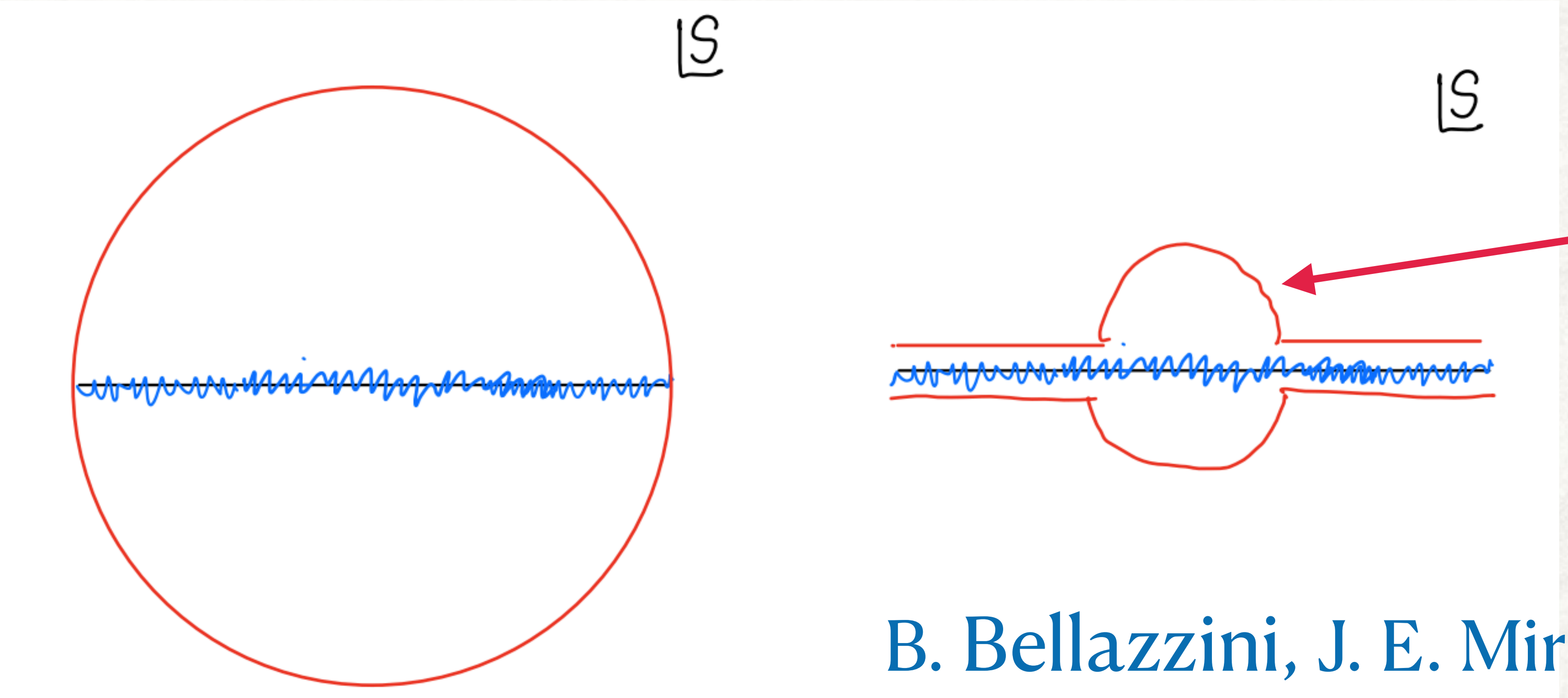


EFT information is embedded in the low-energy limit of $M(s,t)$

From non-perturbative completion

$$\int dx^D \sqrt{-g} M_{\text{pl}}^{D-2} \left(R + \hat{\alpha}_1 \frac{R^2}{M_{\text{pl}}^2} + \hat{\alpha}_2 \frac{R^3}{M_{\text{pl}}^4} + \hat{\alpha}_4 \frac{R^4}{M_{\text{pl}}^6} \cdots \right)$$

$$M^{\text{IR}}(s,t) = \{\text{massless poles from } R,\ R^2,\ R^3\} + \text{polynomials} + \text{massless branch cuts}$$



calculable from EFT

$$b_{k,q}^{\mathcal{C}} = \frac{1}{2\pi i} \frac{\partial^q}{\partial t^q} \int_{\mathcal{C}} \frac{ds}{s^{k-q+1}} M(s,t)$$

$$\frac{1}{\pi} \frac{\partial^q}{\partial t^q} \int_{m^2}^{\infty} \frac{ds}{s^{k-q+1}} \text{Im}[M(s,t)]$$

B. Bellazzini, J. E. Miró, R. Rattazzi, M. Riembau, F. Riva 2011.00037

Dispersion relations for S-matrix

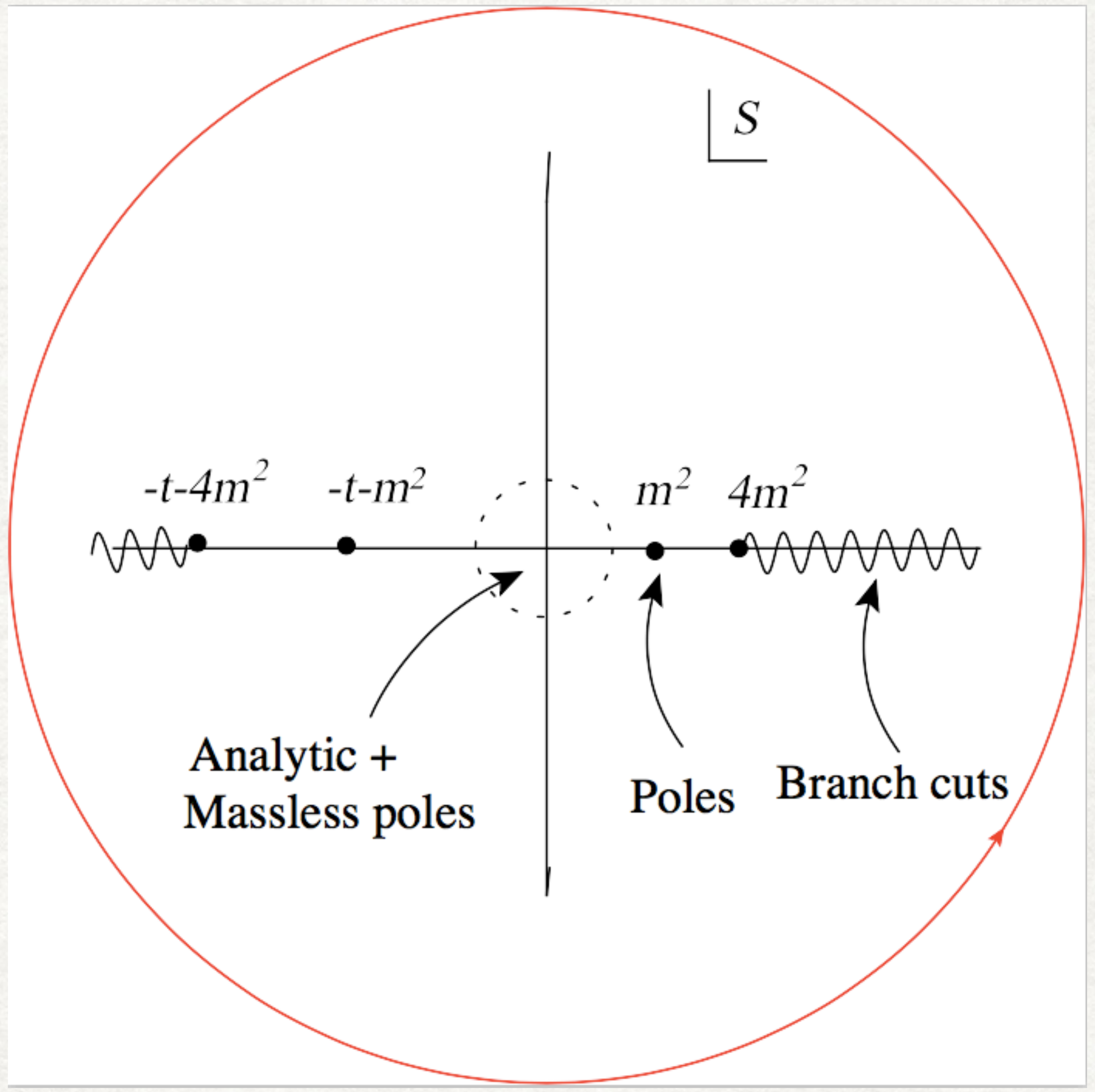
Arising from perturbative completion

$$\int dx^D \sqrt{-g} M_{\rm pl}^{D-2} \left(R + \hat{\alpha}_1 \frac{R^2}{M^2} + \hat{\alpha}_2 \frac{R^3}{M^4} + \hat{\alpha}_4 \frac{R^4}{M^6} \cdots \right)$$

The coefficients can be derived from a contour integral of $M(s,t)$

$$b_{n+q,q} = \frac{\partial^q}{\partial t^q} \oint \frac{ds}{s^{n+1}} M(s,t)$$

$M^{\rm sub}$



- Analyticity: M(s,t) is analytic away from the real s-axes for fixed t

$$M(s,t^*) = M^{\rm sub} + \int_{M^2}^{\infty} \frac{{\rm Im}[M(s,t^*)]}{s-m^2} + \int_{M^2}^{\infty} \frac{{\rm Im}[M(s,t^*)]}{u-m^2}$$

$$M(s,t^*) = M^{\rm sub} + \int_{M^2}^{\infty} \frac{{\rm Im}[\rho_j(s)] P_j^s \left(1+\frac{2t^*}{m^2}\right)}{s-m^2} + \int_{M^2}^{\infty} \frac{{\rm Im}[\rho_j(s)] P_j^u \left(1+\frac{2t^*}{m^2}\right)}{u-m^2}$$

- Unitarity: $0 \leq {\rm Im}[\rho_j(s)]$

positivity
optical theorem

The High Energy Behavior

At large s the amplitude satisfies twice subtraction (at fixed t<0)

Haring, Zhiboedov 2202.08280

$$\lim_{|s|\to\infty} \frac{M(s,t)}{|s|^2} = 0$$

At fixed large impact parameter b scattering is well described by GR

Since fixed b at large energy corresponds to large spin $b \equiv \frac{2J}{\sqrt{s}}$

$$M(s,t) = \frac{1}{2}\sum_{J=0}^{\infty} n_J^d f_J(s) P_J^d(1+2t/s) = -\frac{8\pi G_N s^2}{t}\left(1 + O(G_N s|t|^{\frac{d-4}{2}})\right) \rightarrow f_J(s) \simeq \frac{\Gamma\left(\frac{d-4}{2}\right)}{(4\pi)^{\frac{d-4}{2}}}\frac{G_N s}{J^{d-4}}, \quad J \to \infty$$

The Graviton Pole

- Since the amplitude is bounded by s^2 at $s \to \infty$

for $n \geq 2$
$$b_{n+q,q} = \frac{1}{\pi}\frac{\partial^q}{\partial t^q}\int_{m^2}^{\infty}\frac{ds}{s^{k-q+1}}\,\mathrm{Im}[M(s,t)]$$

- For $n = 2$ the low energy graviton pole contributes

$$M^{\mathrm{IR}}(s,t) = R^4\left(\frac{1}{stu} + \{\text{massless poles from } R^2, R^3\} + \sum_{k,q} b_{k,q} s^{k-q} t^q\right)\Bigg|_{t\to 0} = \frac{s^2}{t} + \cdots$$

The fact that the subtraction term is absent means that the the imaginary part must reproduce the t-pole, i.e. it Reggeizes

$$-\frac{8\pi G_N s^2}{t}\left(1 + O(G_N s|t|^{\frac{d-4}{2}})\right) = \int_{M^2}^{\infty}\frac{\mathrm{Im}[\rho_j(s)]P_j^s\left(1+\frac{2t^*}{m^2}\right)}{s-m^2} + \int_{M^2}^{\infty}\frac{\mathrm{Im}[\rho_j(s)]P_j^u\left(1+\frac{2t^*}{m^2}\right)}{u-m^2}$$

the sum will not converge as t->o

The Graviton Pole

- Consider smeared amplitude

Caron-Huot, Mazac, Rastelli, Simmons-Duffin 2102.08951

$$\int_0^\infty dp\, f(p) \frac{8\pi G}{p^2}$$

but the EFT coefficients can no longer be identified via forward limit expansion

$$\begin{aligned}
\mathcal{C}_{2,u}\big|_{\mathrm{EFT}} &= \frac{8\pi G}{-u} + 2g_2 - g_3 u + 8g_4 u^2 - 2g_5 u^3 + 24 g_6 u^4 - 4 g_7 u^5 \dots, \\
\mathcal{C}_{4,u}\big|_{\mathrm{EFT}} &= 4g_4 - 2g_5 u + (24 g_6 + g_6')u^2 - 8 g_7 u^3 + \dots, \\
\mathcal{C}_{6,u}\big|_{\mathrm{EFT}} &= 8g_6 - 4g_7 u + \dots .
\end{aligned}$$

$$\longrightarrow \quad \mathcal{C}_{2,u}^{\mathrm{improved}}\Big|_{\mathrm{EFT}} = \frac{8\pi G}{-u} + 2g_2 - g_3 u$$

- Regge subtractions

J. Tokuda, K. Aoki, S. Hirano, 2007.15009

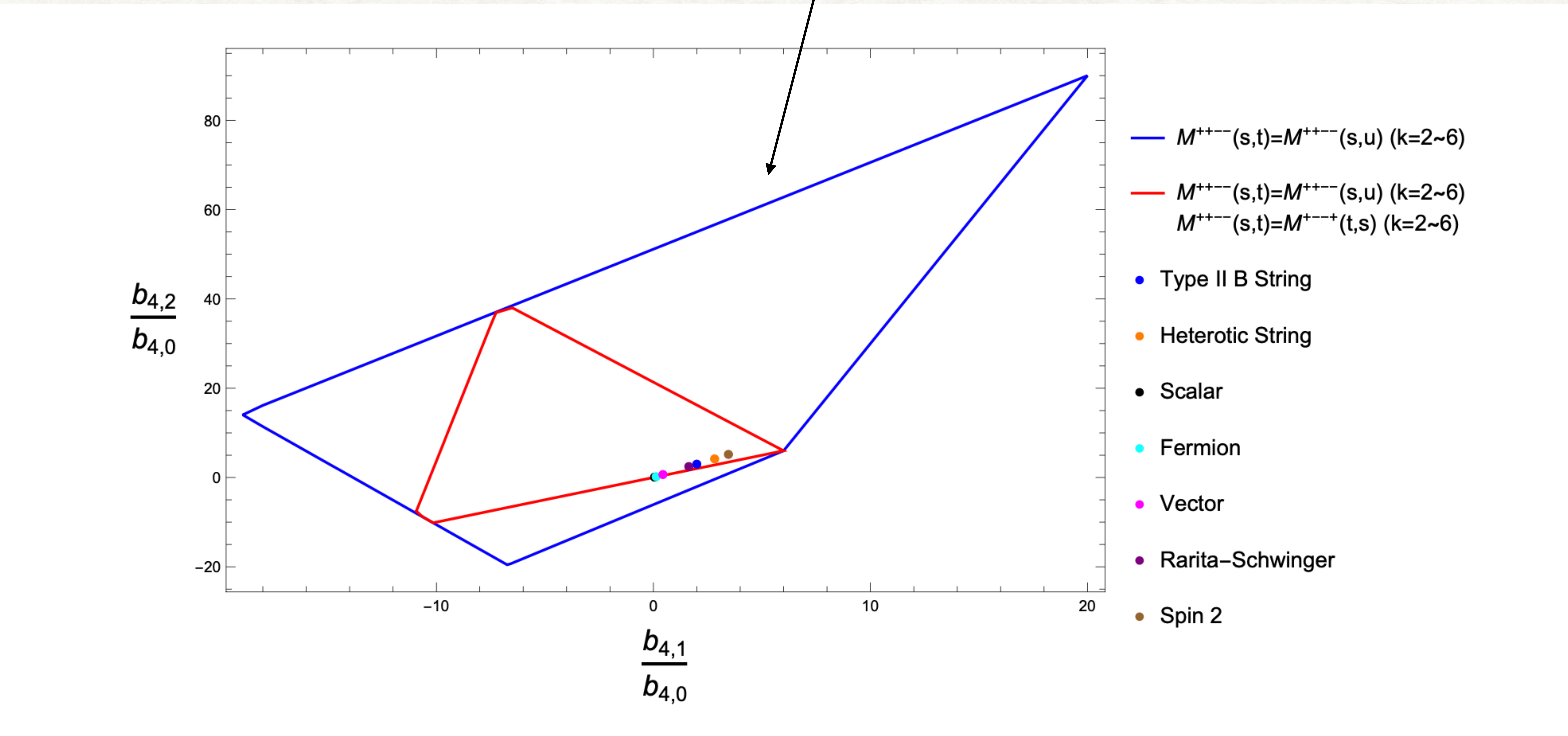
K. Aoki, T-Q Loc, T. Noumi, J. Tokuda, 2104.09682

$$c_2(t) = \frac{4}{\pi}\int_{M_s^2}^\infty \mathrm{d}s\, \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{(s+(t/2))^3} + \frac{2}{M_{\mathrm{pl}}^2 t} \quad \longrightarrow \quad c_2(0) > F_0 > -\mathcal{O}(M_{\mathrm{pl}}^{-2} M^{-2})$$

For fixed derivative couplings, with sdpb

$D^8 R^4$

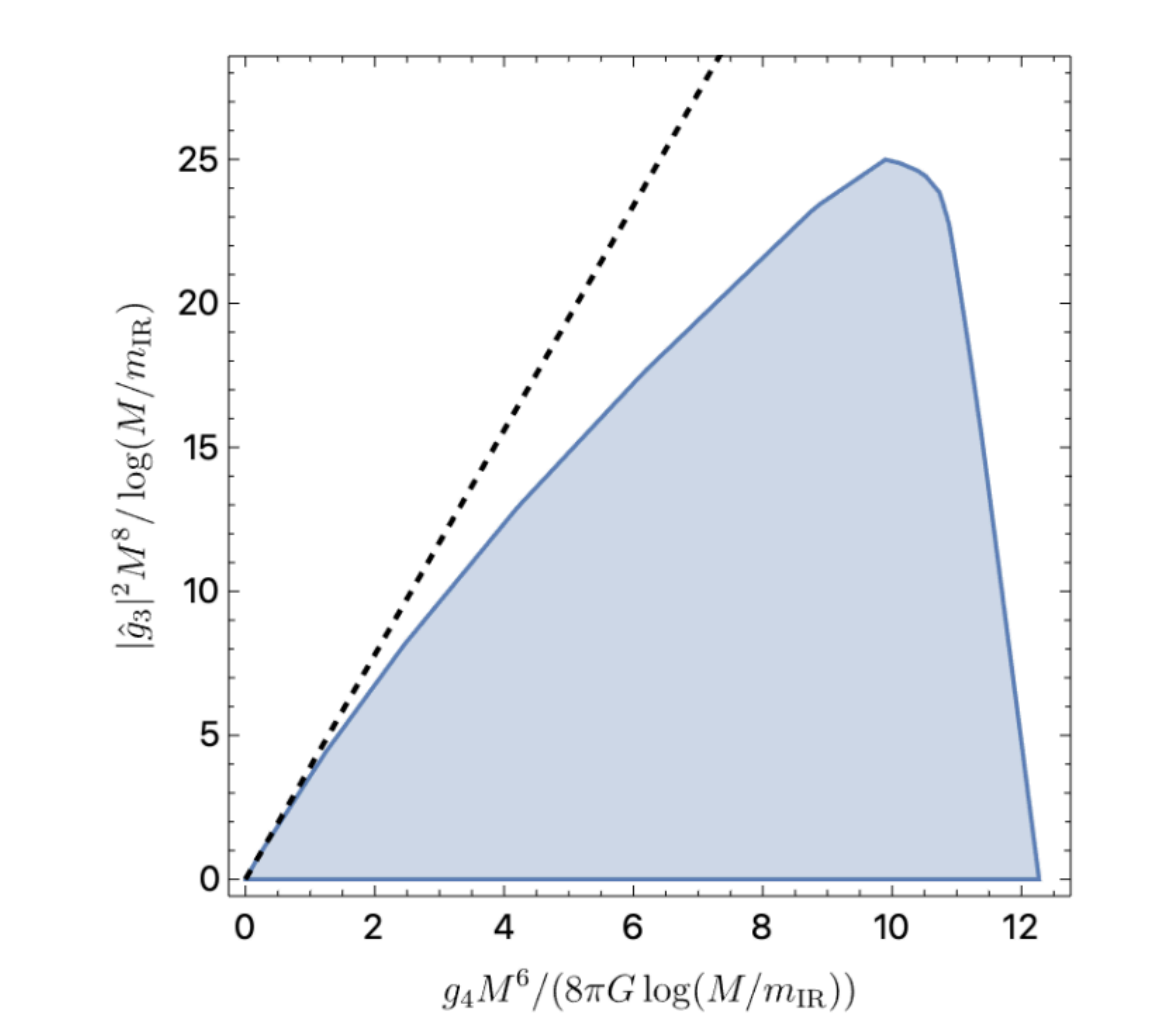
Z. Bern D. Kosmopoulos, A Zhiboedov 2103.12729



Bounds with respect to G_N

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R - \frac{1}{3!}\left(\alpha_3 R^{(3)} + \tilde{\alpha}_3\tilde{R}^{(3)}\right) + \frac{1}{4}\left(\alpha_4 (R^{(2)})^2 + \alpha_4'(\tilde{R}^{(2)})^2 + 2\tilde{\alpha}_4 R^{(2)}\tilde{R}^{(2)}\right) + \ldots\Big]$$

$$\widehat{g}_3 = \alpha_3 + i\tilde{\alpha}_3, \qquad g_4 = 8\pi G(\alpha_4 + \alpha_4'), \qquad \widehat{g}_4 = 8\pi G(\alpha_4 - \alpha_4' + i\tilde{\alpha}_4)$$



Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2201.06602

Let us now combine color with gravity

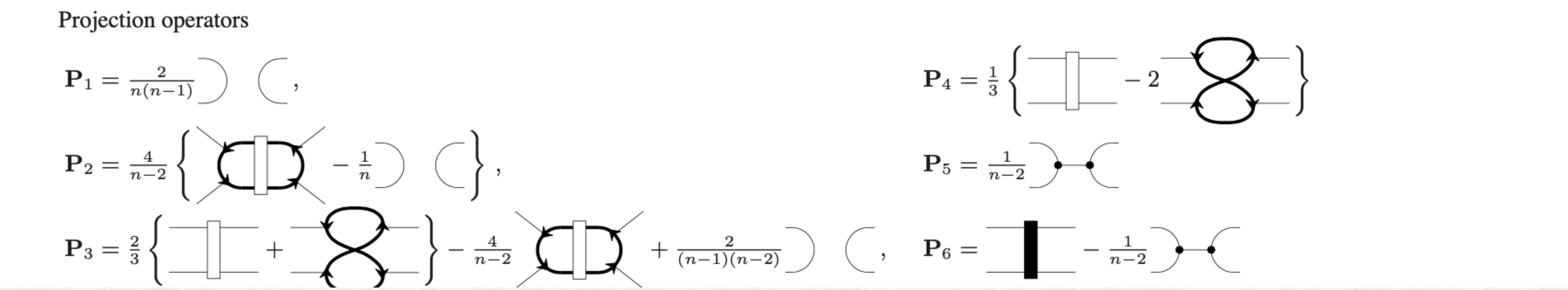
Let us begin with general low energy amplitude of 4 adjoint scalars

$$M^{abcd}(s,t) = g^2\left(P^s_{adj}\frac{t-u}{s} + P^t_{adj}\frac{u-s}{t} + P^u_{adj}\frac{s-t}{u}\right) + 8\pi G\left(P^s_I\frac{tu}{s} + P^t_I\frac{us}{t} + P^u_I\frac{st}{u}\right) + B^{abcd}(s,t),$$

$$B^{abcd}(s,t) = \sum_{\sigma\in S_3}\mathrm{Tr}(T^aT^{\sigma(b)}T^{\sigma(c)}T^{\sigma(d)})B_1(1,\sigma(2),\sigma(3),\sigma(4)) + B_2(s,t)\mathrm{Tr}(T^aT^b)\mathrm{Tr}(T^cT^d) + B_2(t,u)\mathrm{Tr}(T^aT^c)\mathrm{Tr}(T^bT^d) + B_2(u,s)\mathrm{Tr}(T^aT^d)\mathrm{Tr}(T^cT^b).$$

One now requires positivity both on the Gegenbauer polynomials and the color projectors

$$\lim_{s\to m^2} M(1^a2^b3^c4^d) \sim \frac{1}{s-m^2}\sum_J \rho_{J,\alpha}\mathbb{P}^{abcd}_\alpha\,\mathbb{G}_j(\cos\theta)$$



The twice subtraction dispersion relation reads

$$\oint_{\infty} \frac{ds'}{2\pi i(s'-s)} \frac{M^{abcd}(s',t)}{s'(s'+t)} = 0\,,$$

Combined with crossing yields

$$\left(\mathrm{Res}_{s'=0} + \mathrm{Res}_{s'=-t} + \mathrm{Res}_{s'=s}\right) \frac{M^{abcd}(s',t)}{(s'-s)s'(s'+t)} =$$
$$\int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \left(\frac{\mathrm{Im}M^{abcd}(s',t)}{(s'-s)} + \frac{\mathrm{Im}M^{abcd}(-s'-t,t)}{(-s'-t-s)}\right).$$

Unitarity then implies

$$\frac{B^{abcd}(s,t)}{s(s+t)} - \frac{B^{abcd}(0,t)}{st} + \frac{B^{abcd}(-t,t)}{(s+t)t} - \frac{8\pi G}{t} P_I^t =$$
$$16\pi \sum_{JR} (2J+1) P_J(cos(\theta)) \int_{M^2}^{\infty} \frac{ds'}{\pi s'(s'+t)} \mathrm{Im}(a_{JR}(s')) \left(\frac{P_R^s}{(s'-s)} + \frac{P_R^u}{(s'+t+s)}\right)$$

At this stage, the LHS is in the trace basis, while the RHS involves projectors of the s and u-channel

The color projectors can be linearly map to the trace basis

$$
\begin{aligned}
P_1^s &= \frac{2}{N(N-1)}\mathrm{DTr}[a,b;c,d] \\
P_2^s &= \frac{4}{(N-2)}\left(\mathrm{Tr}[a,b,(c,d)] - \frac{1}{N}\mathrm{DTr}[a,b;c,d]\right) \\
P_3^s &= \frac{2}{3}\left(\mathrm{DTr}[a,(c;d),b]+\mathrm{Tr}[a,d,b,c]\right) - \frac{4}{N-2}\mathrm{Tr}[a,b,(c,d)]+\frac{2}{(N-1)(N-2)}\mathrm{DTr}[a,b;c,d] \\
P_4^s &= \frac{1}{3}\left(\mathrm{DTr}[a,(c;d),b]-2\mathrm{Tr}[a,d,b,c]\right) \\
P_5^s &= \frac{2}{(N-2)}\mathrm{Tr}[[a,b],[c,d]] \\
P_6^s &= \mathrm{DTr}[a,[c;d],b] - \frac{2}{(N-2)}\mathrm{Tr}[[a,b],[c,d]] \qquad (31)
\end{aligned}
$$

Equating the RHS and the LHS, with judicious linear combinations we obtain

$$
\frac{1}{t}\Big[\tilde{A}^{(1,0)abcd}(0,t) - \tilde{A}^{(0,1)acbd}(t,0) + \tilde{A}^{(0,1)acbd}(t,0)|_{\mathcal{O}(2)} - \frac{1}{t}\Big(-\tilde{A}^{acbd}(t,0) + \tilde{A}^{acbd}(t,0)|_{\mathcal{O}(3)} + \tilde{A}^{acdb}(t,0) - \tilde{A}^{acdb}(t,0)|_{\mathcal{O}(3)}\Big)\Big] = c^{abcd}(t) + 8\pi G\frac{P_I^t}{t},
$$

$$
\tilde{A}^{abcd}(s,t) = \left\langle \frac{suP_J(1+\frac{2t}{s'})}{s'+t}\left(\frac{P_R^s}{(s'-s)} + \frac{P_R^u}{(s'+t+s)}\right)\right\rangle
$$

$$
c^{abcd}(t) = \begin{pmatrix} -4(tg_{32}+g_{22}) \\ 2t(g_{30}-g_{32})-g_{20}-g_{22} \\ t(g_{30}-3g_{32})-g_{20}-g_{22} \\ 2t(g_{30}-g_{32})-g_{20}-g_{22} \\ t(g_{30}-3g_{32})-g_{20}-g_{22} \\ -4(tg_{32}+g_{22}) \\ t(G_{30}-3G_{32})-G_{20}-G_{22} \\ -4(tG_{32}+G_{22}) \\ 2t(G_{30}-G_{32})-G_{20}-G_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{Tr}(T^aT^bT^cT^d) \\ \mathrm{Tr}(T^aT^bT^dT^c) \\ \mathrm{Tr}(T^aT^cT^bT^d) \\ \mathrm{Tr}(T^aT^cT^dT^b) \\ \mathrm{Tr}(T^aT^dT^bT^c) \\ \mathrm{Tr}(T^aT^dT^cT^b) \\ \mathrm{Tr}(T^aT^b)\mathrm{Tr}(T^cT^d) \\ \mathrm{Tr}(T^aT^c)\mathrm{Tr}(T^bT^d) \\ \mathrm{Tr}(T^aT^d)\mathrm{Tr}(T^bT^c) \end{pmatrix}
$$

To avoid infrared divergences for smeared amplitudes, we first consider D=5

We utilize semidefinite programming (SDPB)

D. Simmons-Duffin, *A Semidefinite Program Solver for the Conformal Bootstrap*, *JHEP* **06** (2015) 174, [1502.02033].

W. Landry and D. Simmons-Duffin, *Scaling the semidefinite program solver SDPB*, 1909.09745.

$$\sum_a \left(p_a^{++}\; p_a^{--}\; p_a^{+-}\right) \begin{pmatrix} [\mathbf{B_1}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ [\mathbf{B_2}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ [\mathbf{N_1}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ [\mathbf{N_2}(\ell_a, \mathbf{m_a})]_{3\times 3} \\ \vdots \end{pmatrix} \begin{pmatrix} p_a^{*++} \\ p_a^{*--} \\ p_a^{*+-} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ 0 \\ \vdots \end{pmatrix}$$

Search for all possible 2+n dimensional vectors $\vec{v}$ such that

$$(0, -1, 0, \cdots) \cdot \vec{v} = 1, \;\; \& \quad \vec{v}^T \cdot \vec{\vec{F}}_{x,\ell} \succeq 0 \quad \forall x \geq 0,\; \ell = 0, 1, ..., \ell_{max}$$

$$\vec{F}_{m_a,\ell_a} = \begin{pmatrix} \frac{B_{k_1,q_1}(\ell_a)}{m_a^{2(k_1+1)}} \\ \frac{B_{k_2,q_2}(\ell_a)}{m_a^{2(k_2+1)}} \\ \frac{N_k(\ell_a)}{m_a^{2(k+1)}} \\ \vdots \end{pmatrix}$$

For each $\vec{v}$ we have

$$\vec{v}^T \begin{pmatrix} b_{k_1,q_1} \\ b_{k_2,q_2} \\ 0 \\ \vdots \end{pmatrix} = v_1 b_{k_1,q_1} - b_{k_2,q_2} \geq 0\,,$$

Minimize v_1 gives the upper bound on the ratio

First consider the QFT (EFT) limit G=0

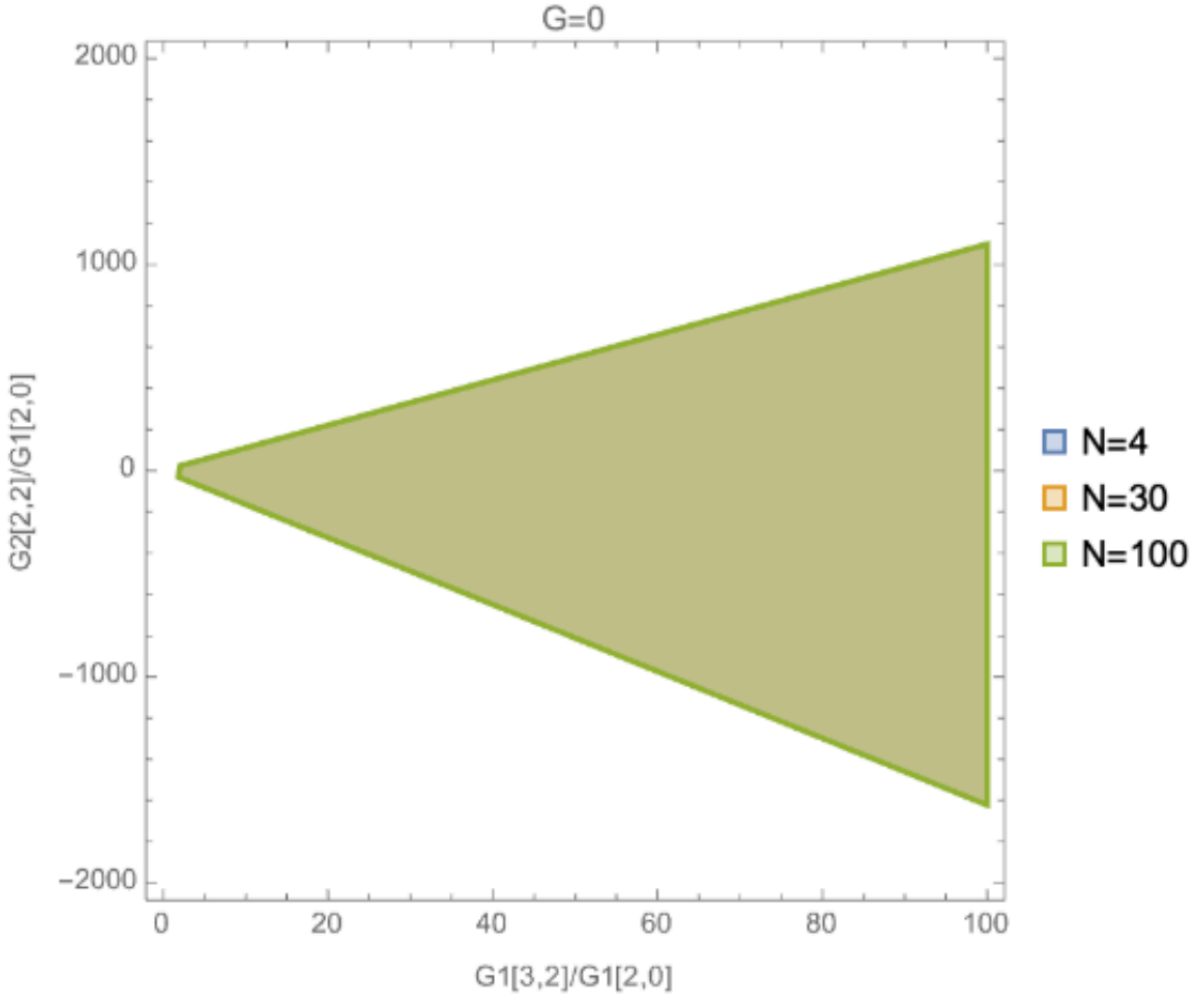


Figure 2. Adjoint representation, $G = 0$

We obtain two sided bounds that are
Independent of the rank

First consider the QFT (EFT) limit G=0

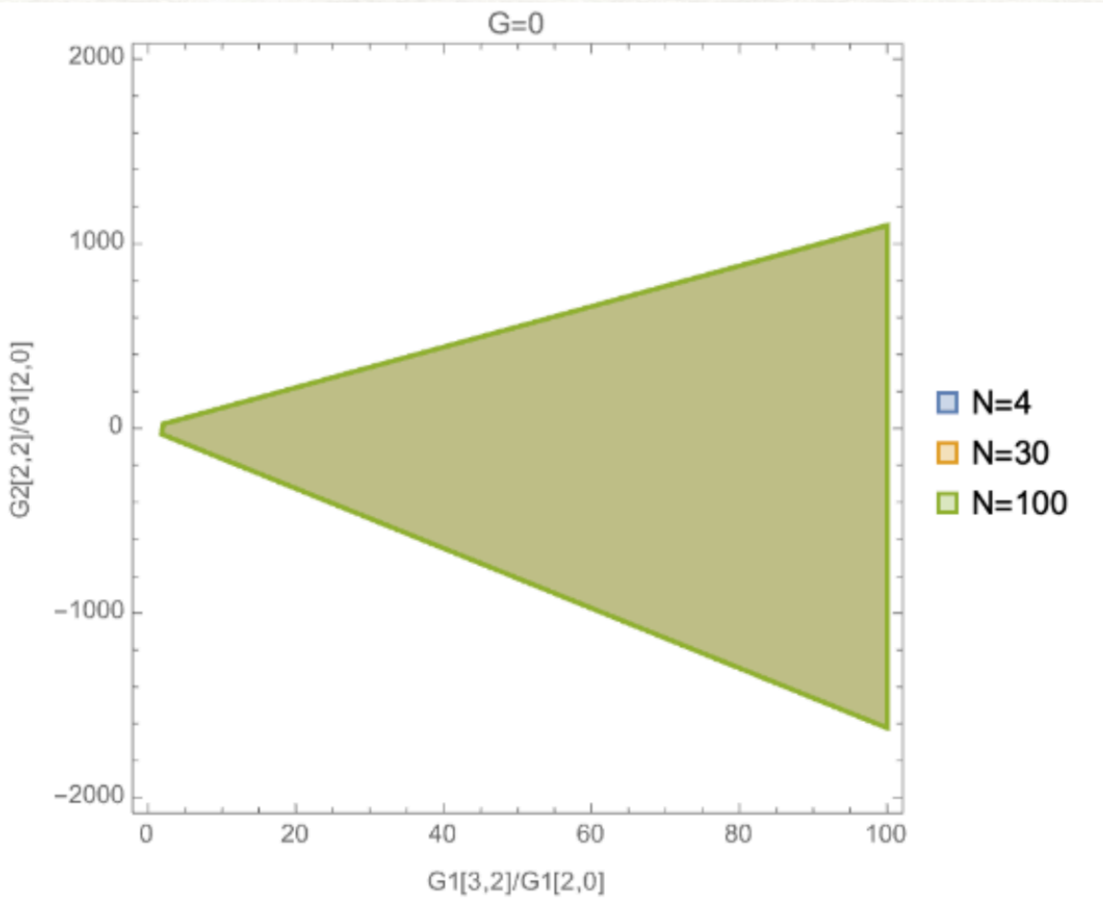
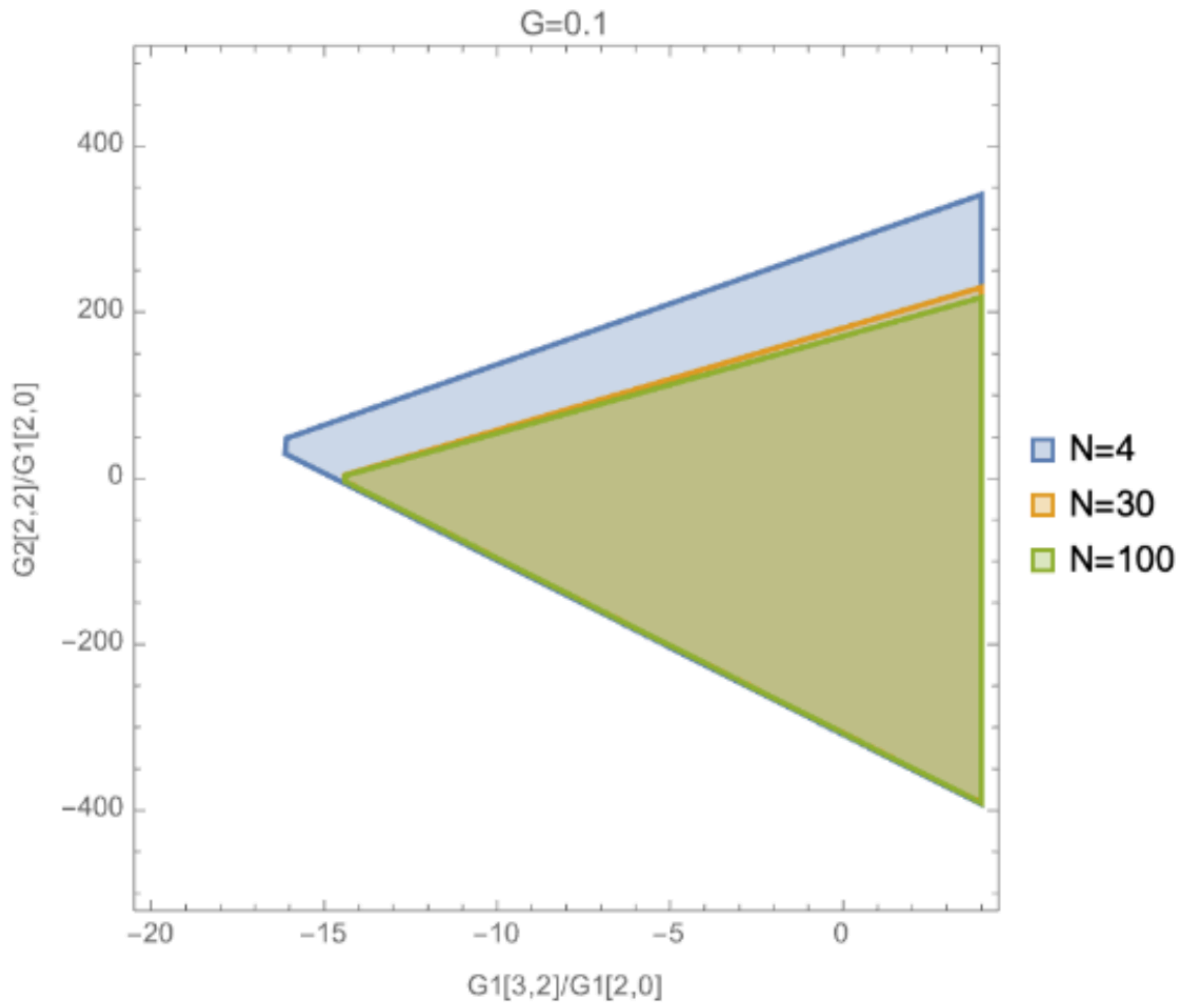


Figure 2. Adjoint representation, $G = 0$

We obtain two sided bounds that are
Independent of the rank

Let's turn on gravity



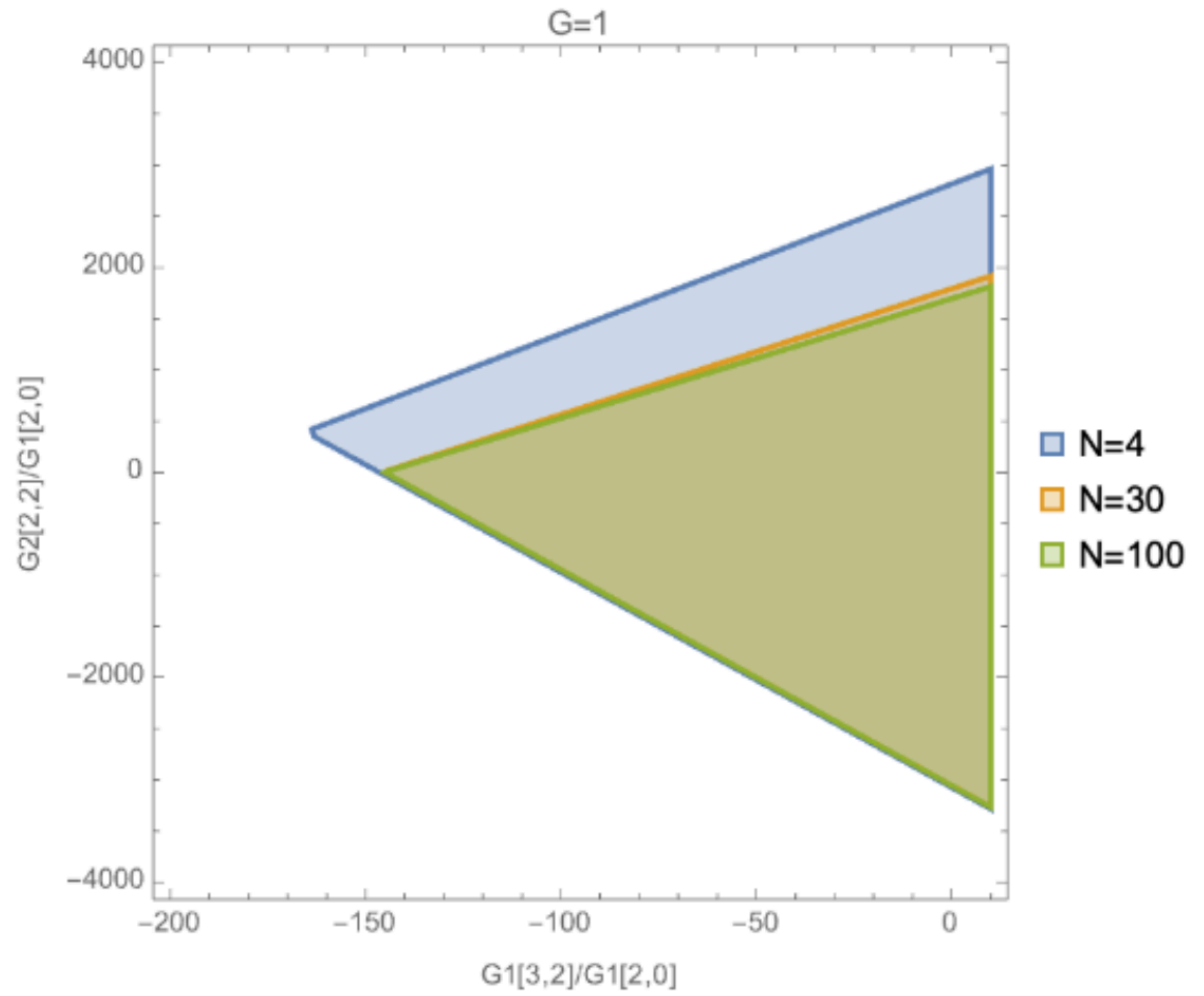


Figure 4. Adjoint representation, $G = 1$

We see N
dependence!

The dependence
Converges for large
N, an emergent
Universality

We can also consider supersymmetry. Consider maximal SUSY (16 super charges)

$$M^{abcd} = \delta^8(Q) f(s,t) \qquad \delta^8(Q) \sim s^2$$

This leads to zero subtraction dispersion relations for the couplings since

$$\lim_{s\to\infty} f(s,t) < s^0$$

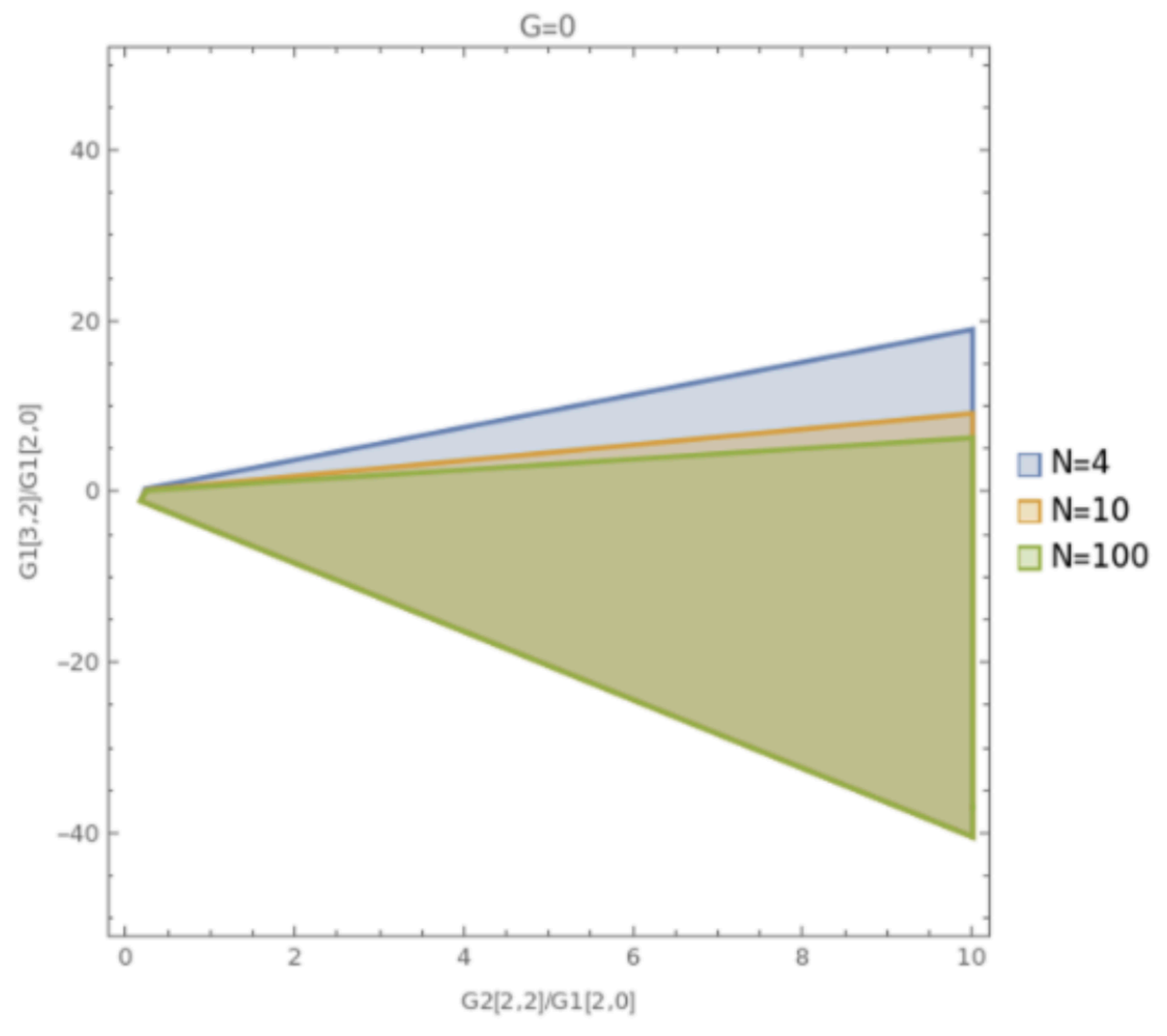


Figure 5. Adjoint representation, maximal SUSY, $G = 0$

We see N dependence even without Gravity !

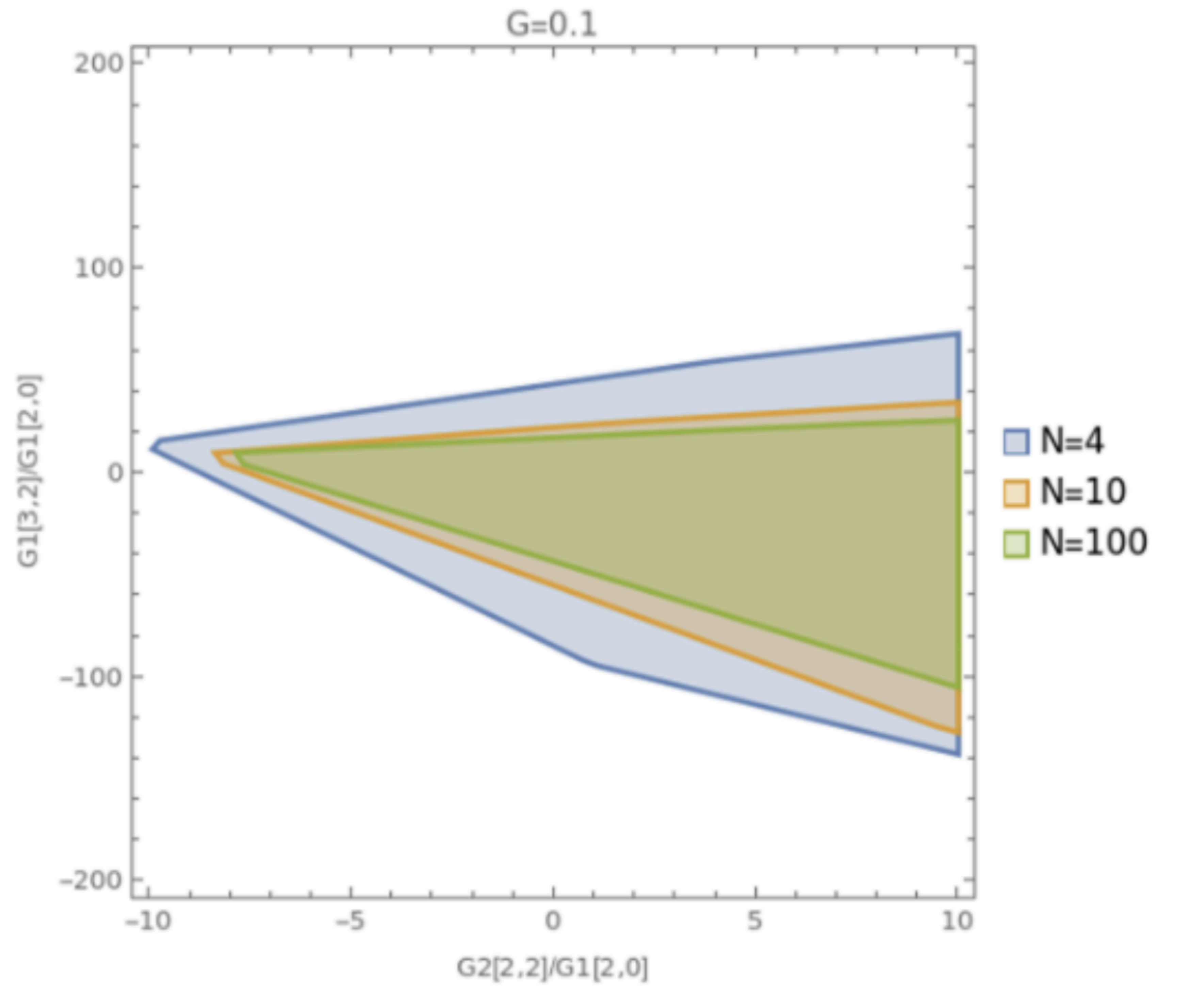


Figure 6. Adjoint representation, maximal SUSY, $G = 0.1$

Additional N dependence when gravity is turned on

Summary and Outlook

- We've seen that by combining color and gravity, the S-matrix bootstrap exhibits sensitivity to the rank of the group

- Imposing maximal susy also induces rank-dependent bounds

- Generalization to SU(N), higher dimensions can be done straightforwardly (critical dimensions ?)

- D=4 would require regularization, Hubble scale as IR scale

- So far there's no distinction between global or local symmetry, we are not sensitive to the gauge pole

Under the assumption that black hole physics can be reproduced at long distances implies 1SDR

Haring, Zhiboedov 2202.08280

$$\lim_{|s|\to\infty} \frac{T_{\psi_{a>d-4,b>\frac{d-3}{2}}}(s)}{|s|} = 0.$$

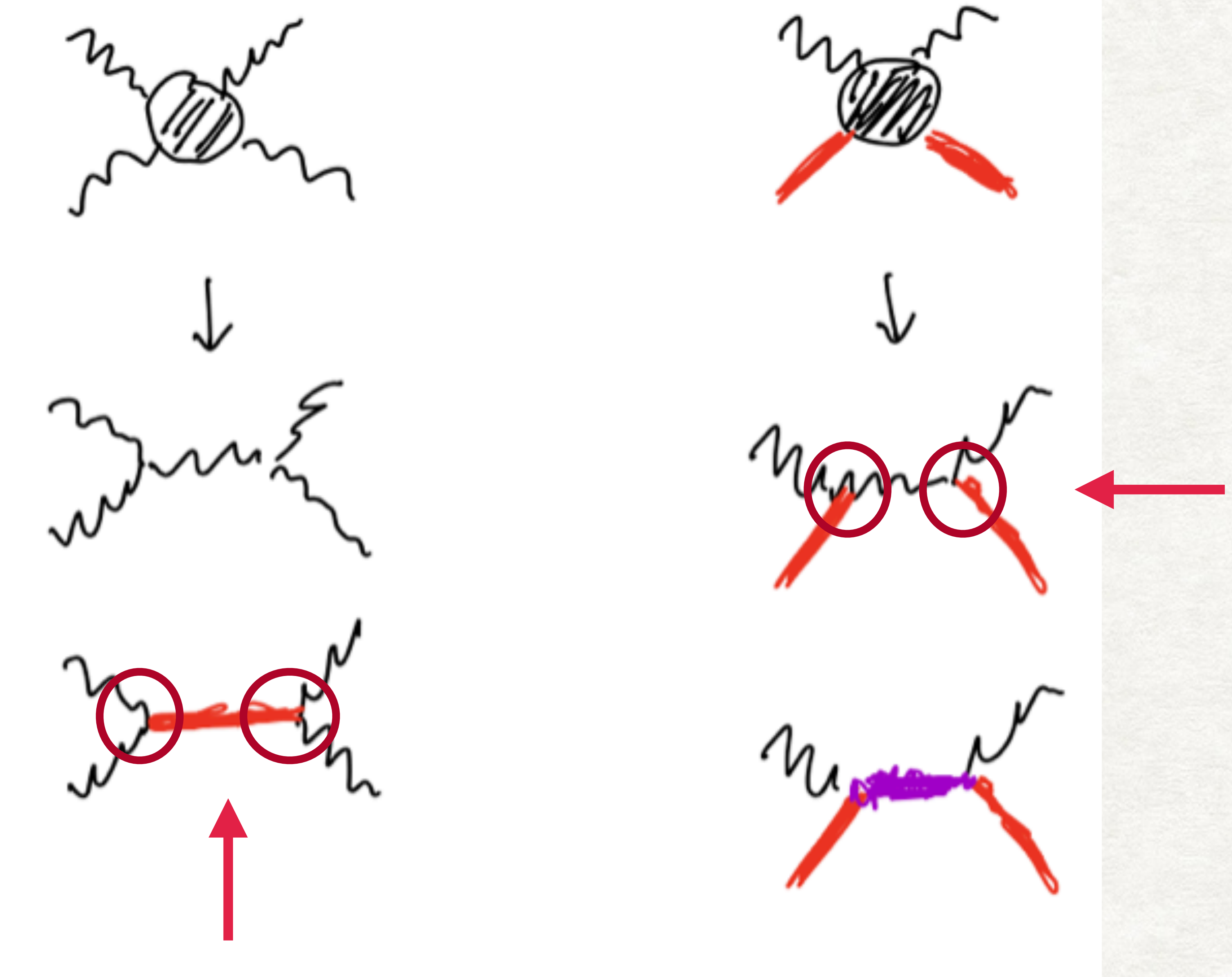
→ The dispersion relations will be sensitive to both gauge and gravity poles.

We can also consider the scattering of massless vectors

(Immediate) Future Directions: The spin-4 Compton

Consistent mixed (spin-4) amplitudes

ℓ	2	3	4	5	6	7	$\ldots$
$h=1, \tilde{\omega}_1^u(\ell)$	-	+	+	+	+	+	$\ldots$
$h=2, \tilde{\omega}_1^u(\ell)$			-	+	+	+	$\ldots$
$h=1, \tilde{\omega}_2^u(\ell)$	-	-	+	+	+	+	$\ldots$
$h=2, \tilde{\omega}_1^u(\ell)$			-	-	+	+	$\ldots$



$(g_4)^2$

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