Corner Symmetries
Local Holography
Carrollian Physics

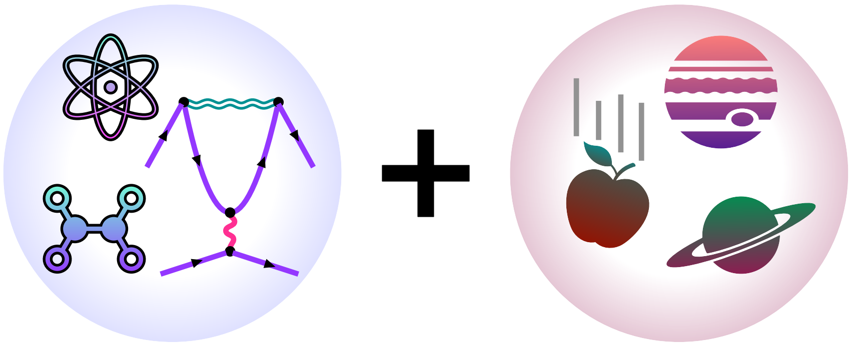
Puttarak JAI-AKSON

iTHEMS

NCTS-iTHEMS Workshop on Matters to Spacetime: Symmetries and Geometry

August 27, 2024

Quantum Gravity-ish Questions



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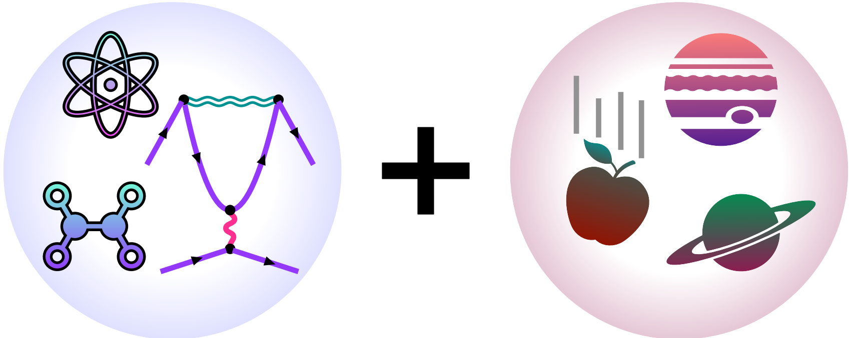
It gives rise to numerous questions and calls for changes in perspective.

Quantum Gravity-ish Questions



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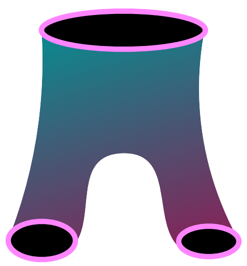
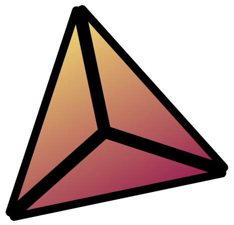
Top-down questions

- What is the most fundamental thing in the universe ? strings, loops, ... ?
- Is spacetime fundamentally quantum ?
- How to recover classical gravity ?
- Experimental tests ?





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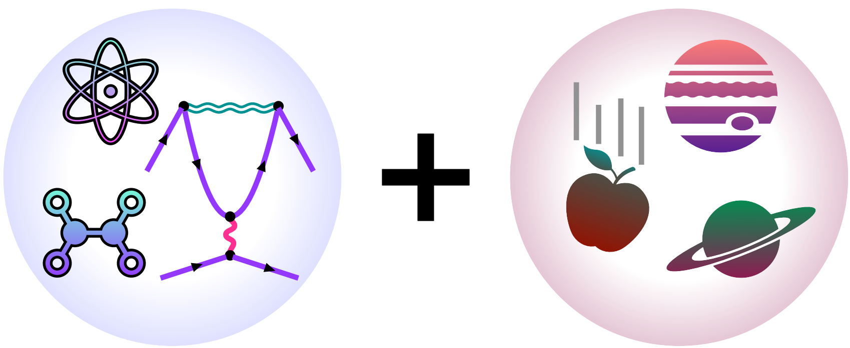

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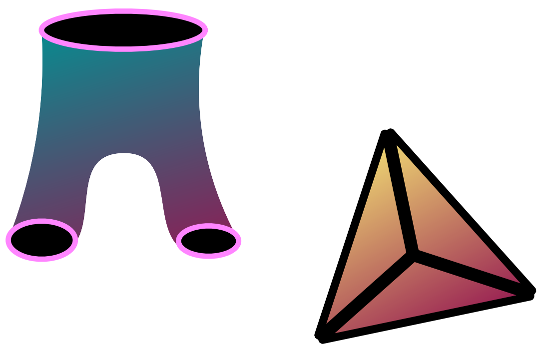

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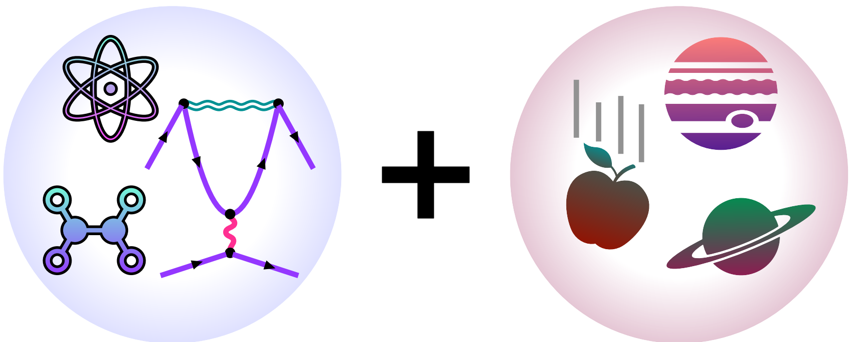

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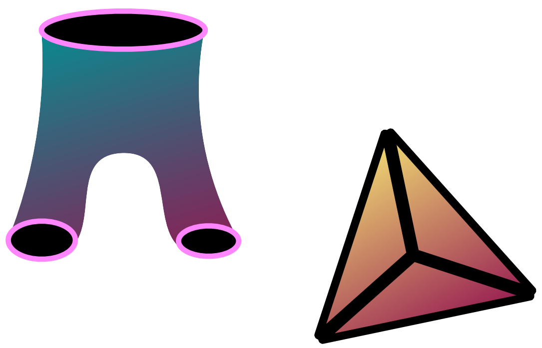

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What are symmetries of gravity & How to characterize sub-systems of gravity ?

Sub-systems of Gravity

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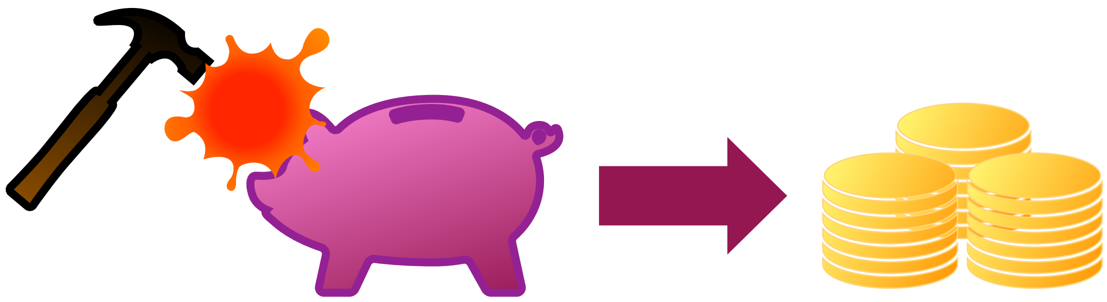
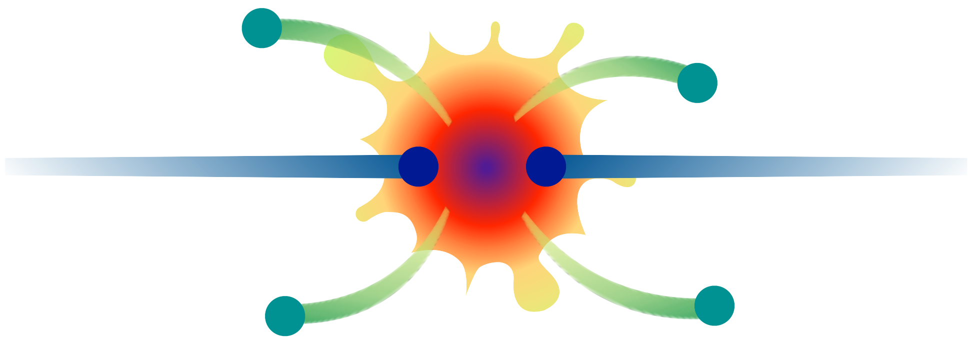
What do we normally do to know constituents of something? **Just break it !**





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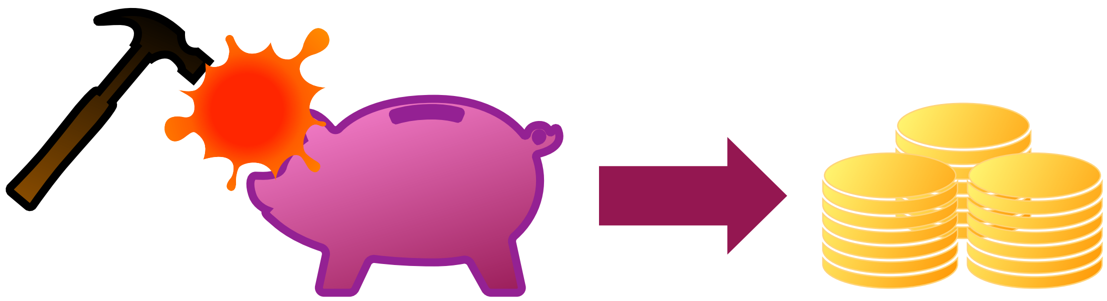

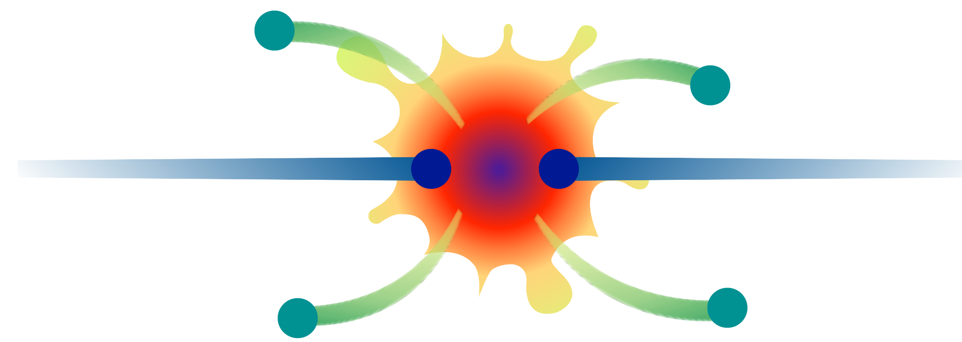

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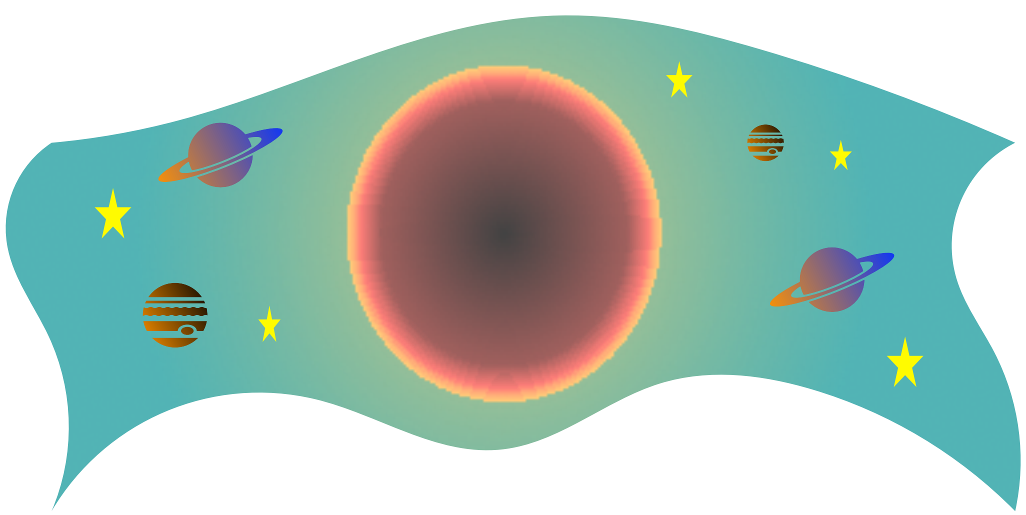

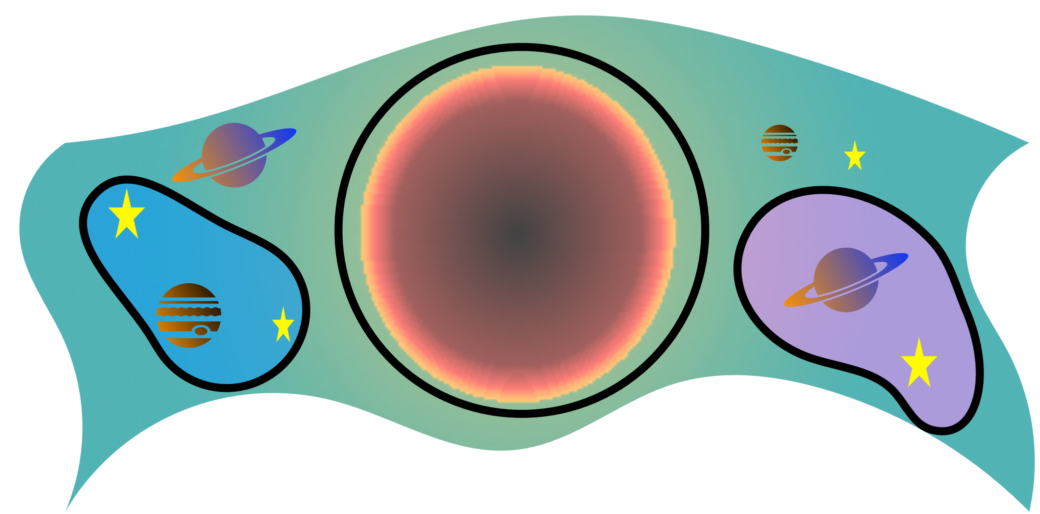

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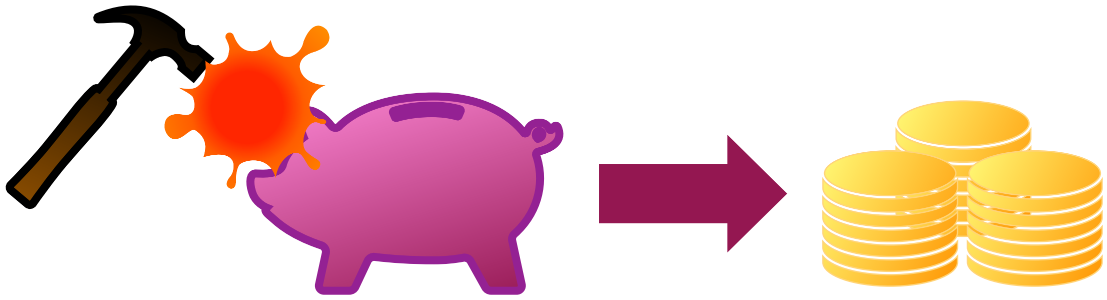

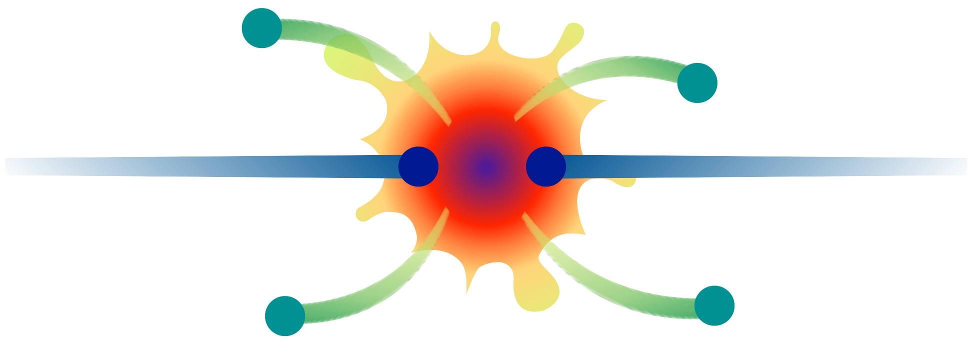

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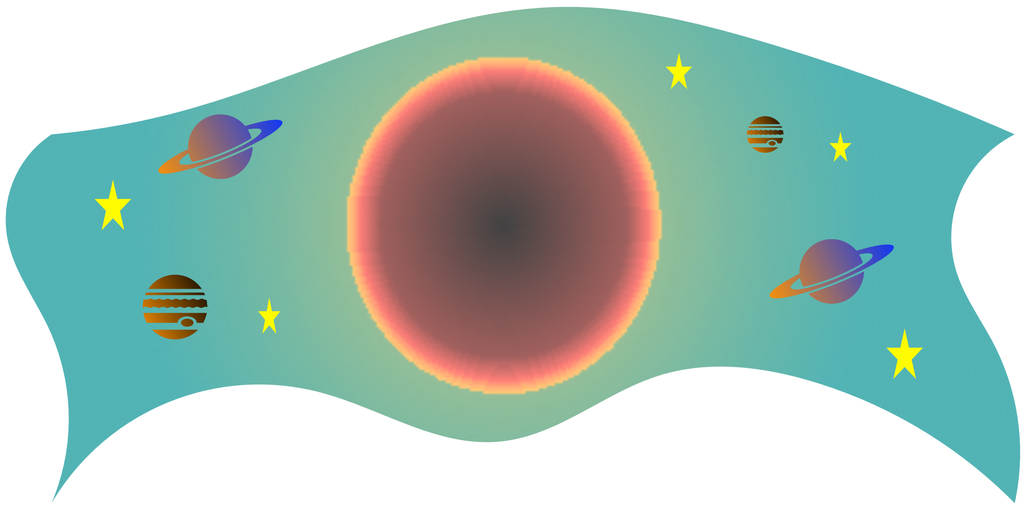

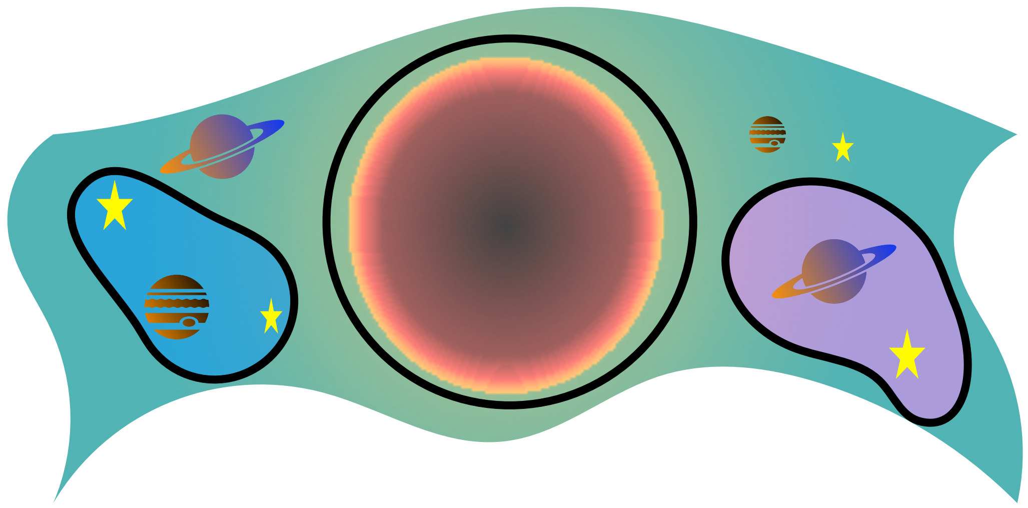

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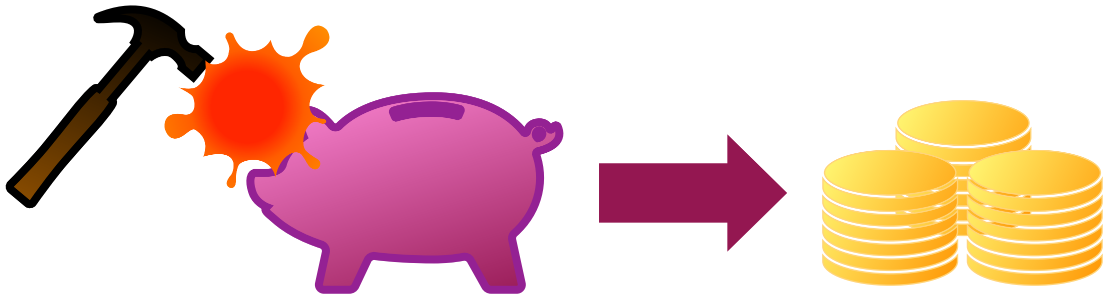

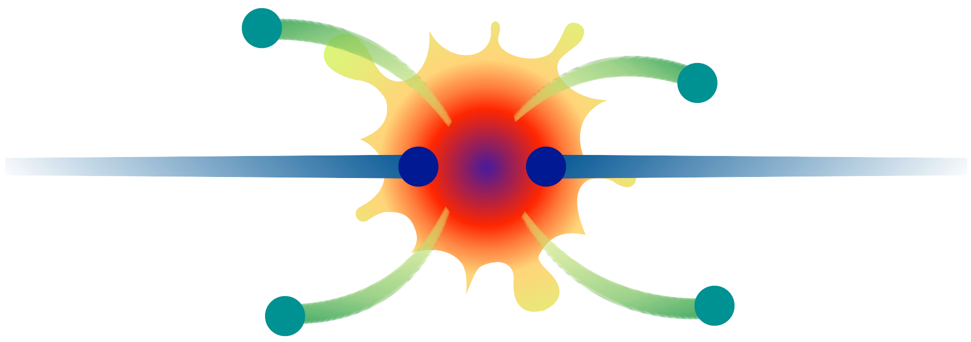

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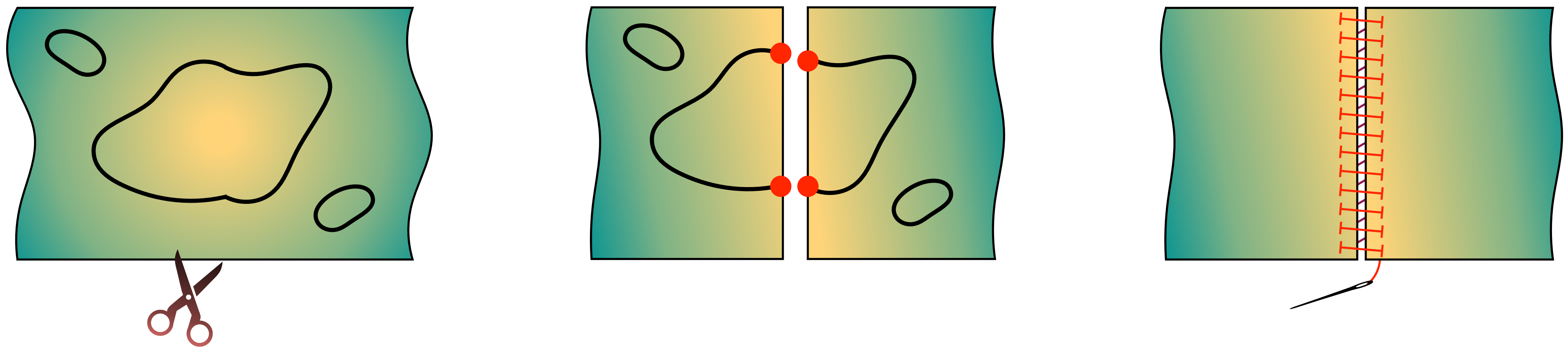

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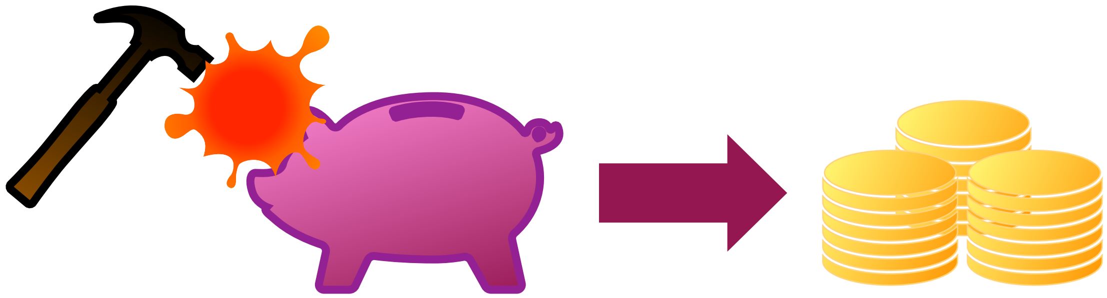

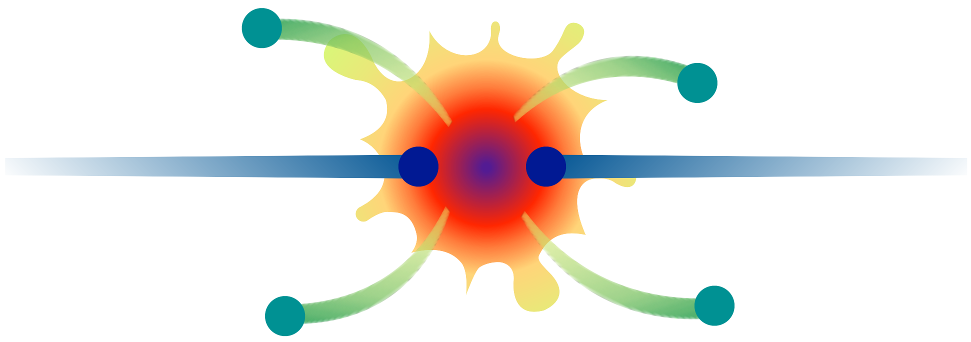

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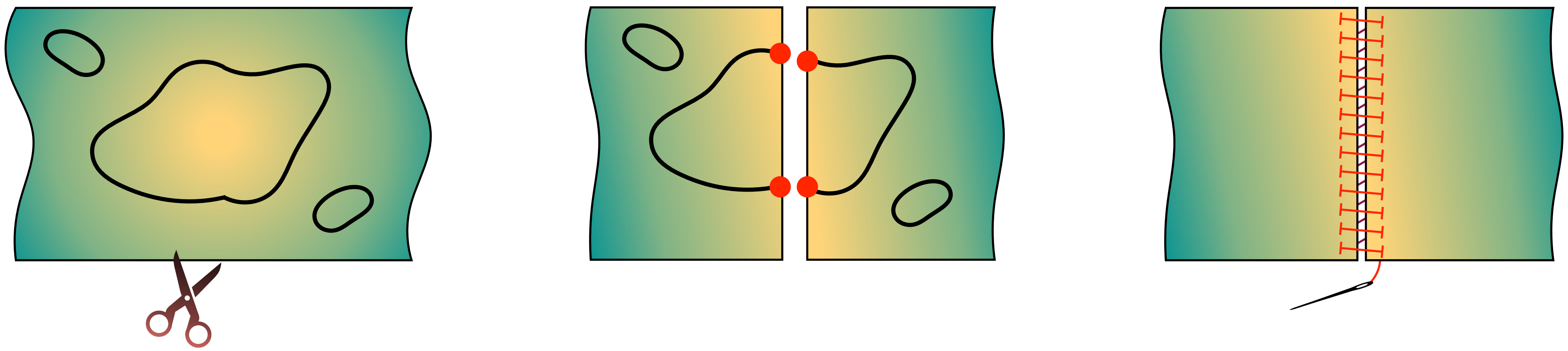

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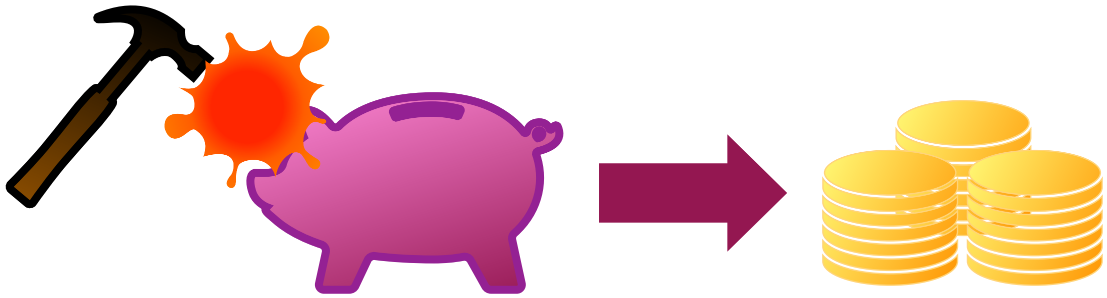

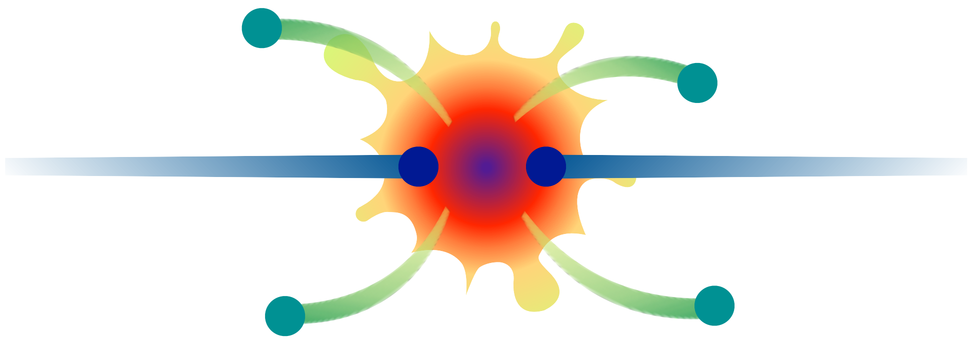

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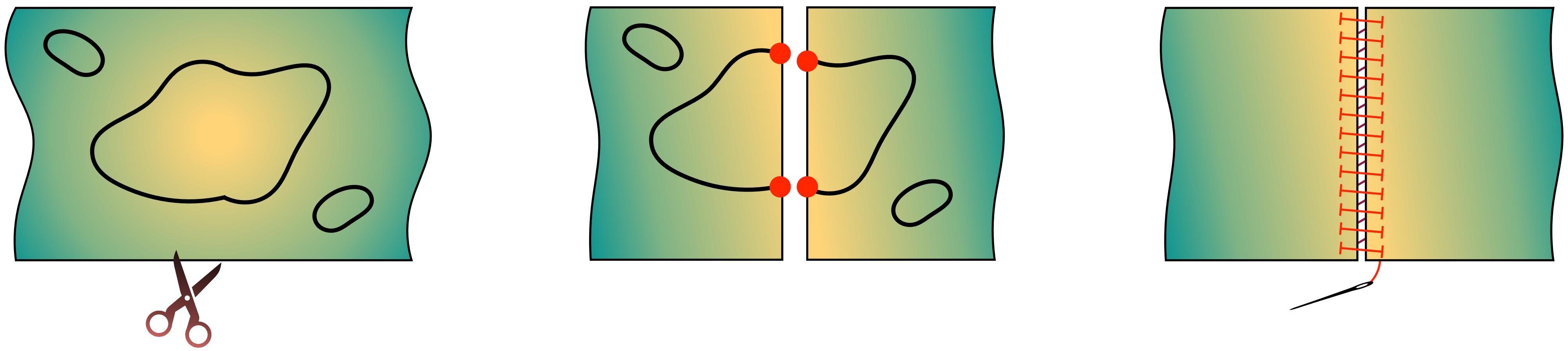

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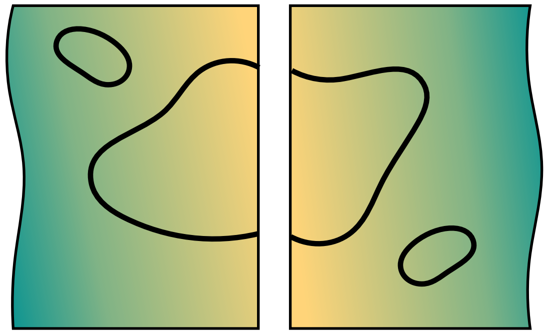
- Physical Phase & Hilbert space does not admit tensor-product factorization $\mathscr{H}_{1\cup 2} \neq \mathscr{H}_1 \otimes \mathscr{H}_2$
- Usually remedied by extending the Hilbert space with d.o.f. living along the cut, **edge modes**

Holographies and Symmetries



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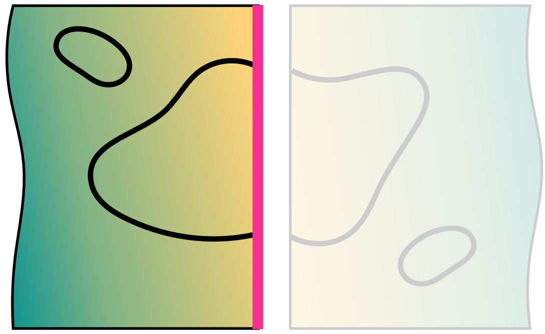

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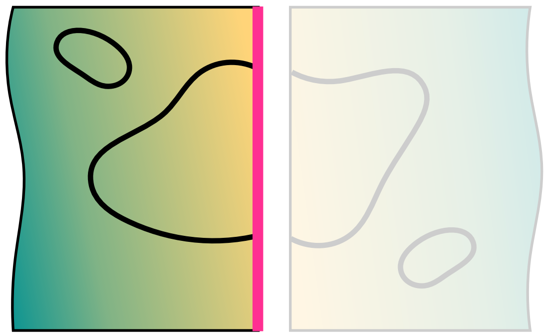

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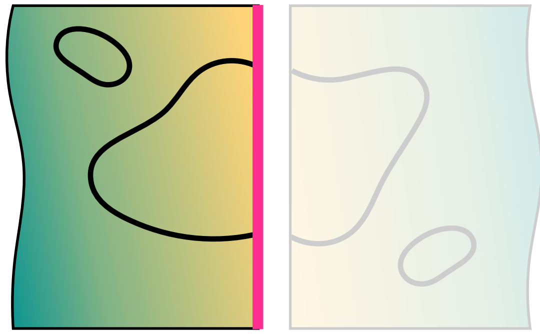

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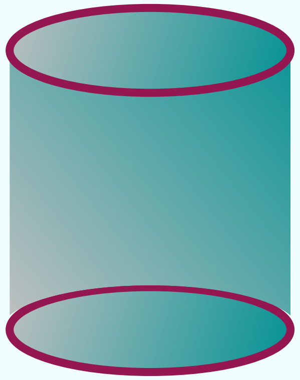
Holography: Information of spacetime is encoded in some lower-dimensional objects

AdS/CFT holography

- Timelike boundary
- Conformal field theory (CFT)
- Codim-1 holography



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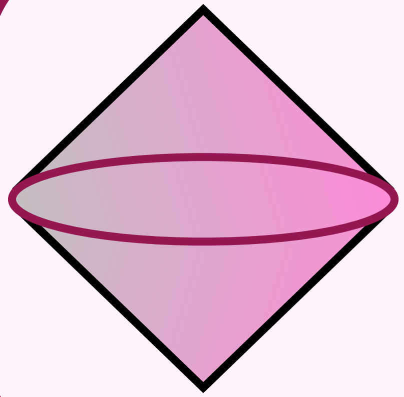
Celestial holography



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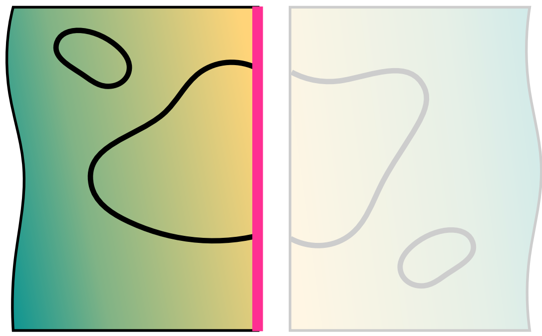

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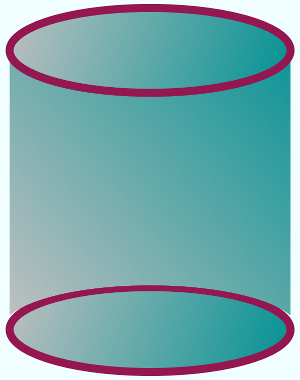

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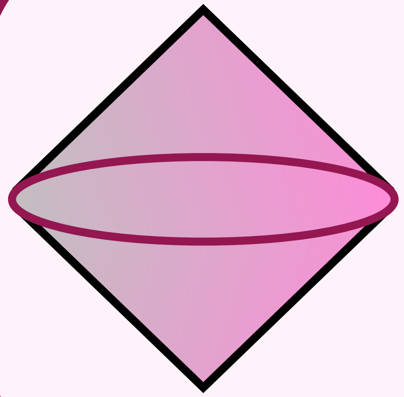

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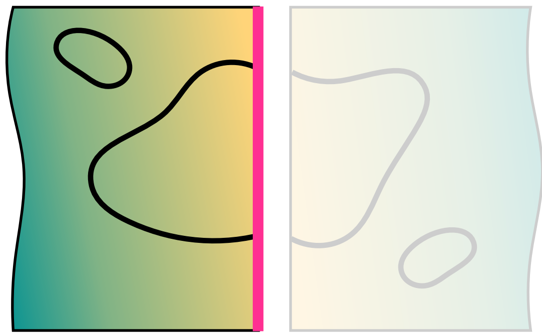

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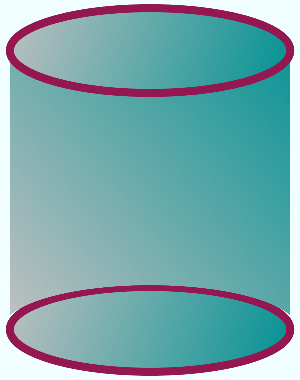

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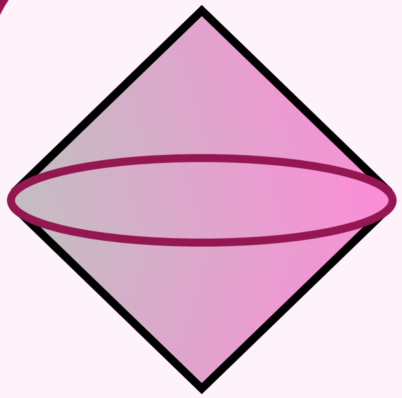

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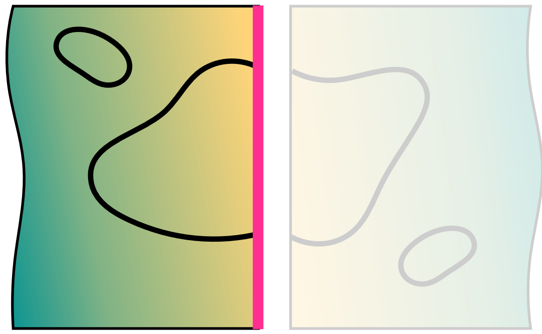

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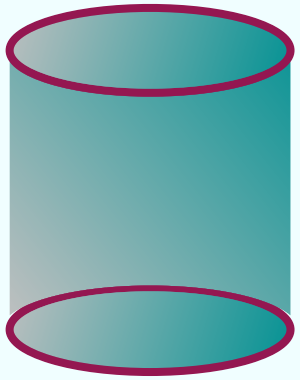

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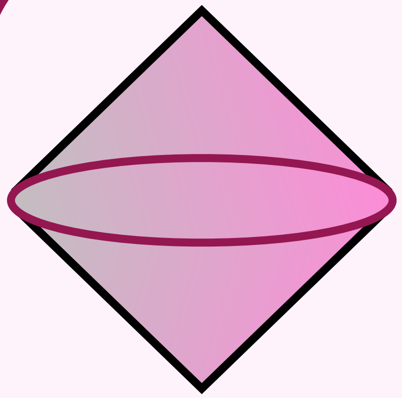

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- Without boundaries, diffeomorphism ($x \to x'(x)$) is **gauge redundancy**
- With boundaries, parts of diffeomorphism turn physical with associated Noether charges supported at **corners** (codim-2)
- Utilization of the Noether's 2nd theorem

Symmetries of Gravity

Covariant Phase Space Formalism (modern language of Noether's theorems)

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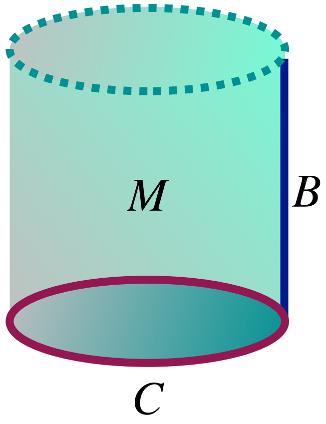
Starting with a spacetime M with a boundary B and a theory of gravity

Lagrangian $L[g]$

Identifying symmetries

Diffeomorphism ξ

$$g \to g + \delta_\xi g$$



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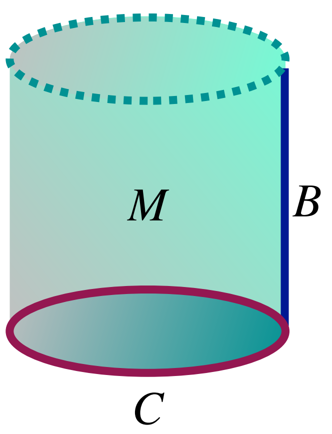
Extracting phase space structure **covariantly**

$$\delta L = E + \mathrm{d}\theta$$

Equation of motion: $E = 0$

Symplectic 2-form

$$\Omega[g, \delta g] = \int_B \delta\theta$$



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$$(\Omega, \xi) \Longrightarrow \mathcal{Q}_\xi = \int_C q_\xi$$

as well as their algebra

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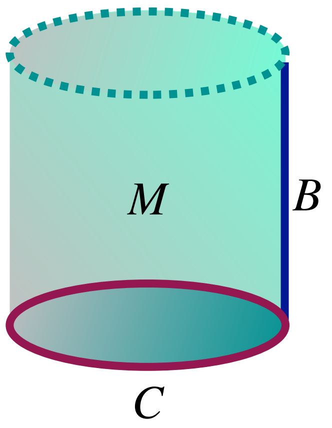

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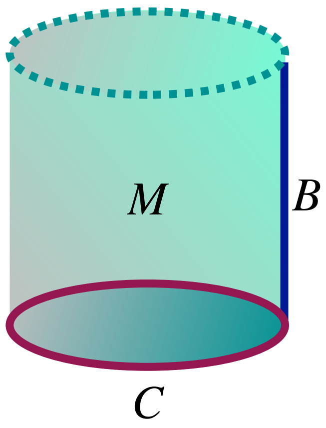

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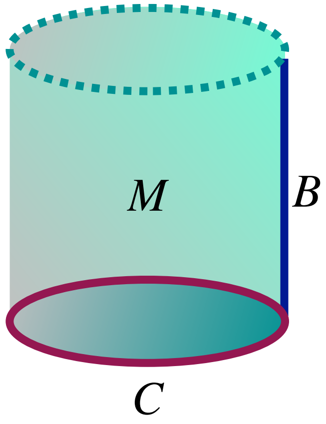

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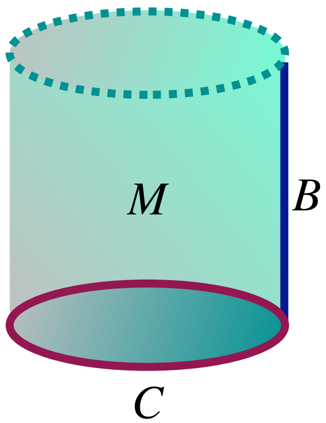

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- Non-integrable due to dissipation (very hard to have a closed system in gravity)
- Renormalization at infinities

Many techniques were invented to deal with these issues.

Symmetries of Gravity

Covariant Phase Space Formalism (modern language of Noether's theorems)

Starting with a spacetime M with a boundary B and a theory of gravity

Lagrangian $L[g]$

Identifying symmetries

Diffeomorphism ξ

$$g \to g + \delta_\xi g$$

Extracting phase space structure **covariantly**

$$\delta L = E + \mathrm{d}\theta$$

Equation of motion: $E = 0$

Symplectic 2-form

$$\Omega[g, \delta g] = \int_B \delta\theta$$

Given the symplectic structure & symmetries, Noether charges can be computed

$$(\Omega, \xi) \Longrightarrow \mathcal{Q}_\xi = \int_C q_\xi$$

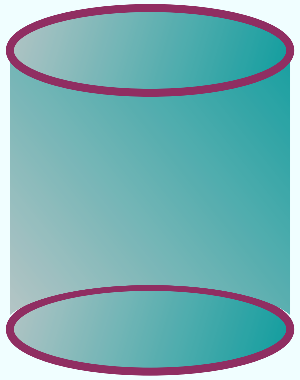
as well as their algebra

$$\{\mathcal{Q}_{\xi_1}, \mathcal{Q}_{\xi_2}\} = \mathcal{Q}_{[\xi_1,\xi_2]}$$

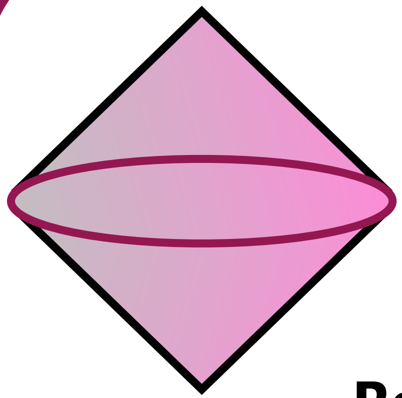
Usually, symmetry group is not maximal and depends on the type of boundaries (bdy. conditions)

AdS/CFT holography

- Timelike boundary
- Conformal field theory (CFT)
- Codim-1 holography



Conformal group



Celestial holography

- Null boundary
- CFT at celestial sphere
- Codim-2 holography

Bondi-Metzner-Sachs (BMS) group

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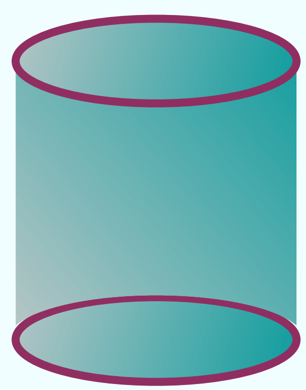

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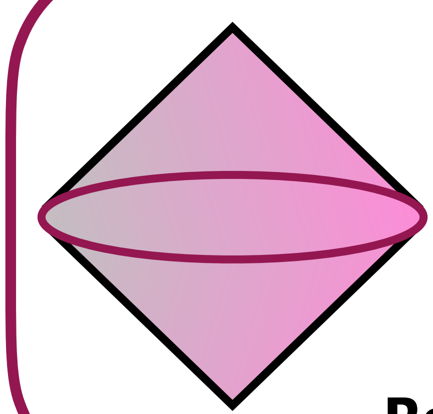

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What is the maximal symmetry group that is independent of boundary conditions ?

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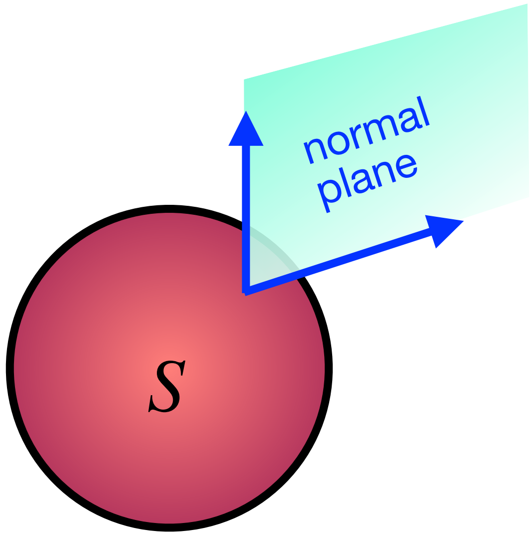
Universal Corner Symmetries (Ciambelli & Leigh)



The corner symmetry group associated to isolated corner sphere is

$$\mathrm{UCS} = \mathrm{Diff}(S) \ltimes \mathrm{GL}(2,\mathbb{R})^S \ltimes (\mathbb{R}^2)^S$$

Deformation of S

Transformation of normal plane to S

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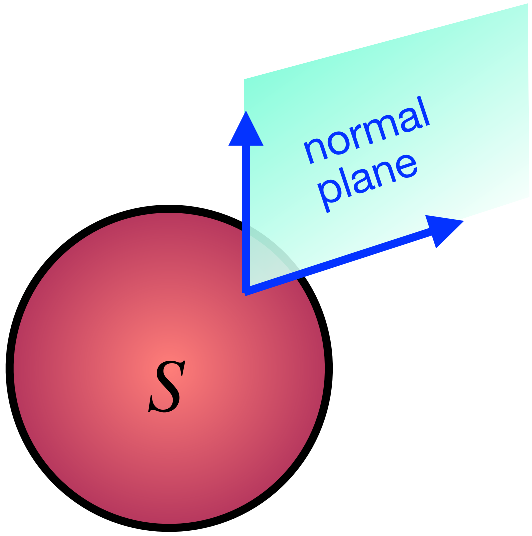

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kinematical **dynamical**

Local Holography (Corner proposal for QG)

(Freidel, Donnelly, Ciambelli, Leigh, …)

A bottom-up proposal to understand some aspects of QG using symmetries as a guiding principle

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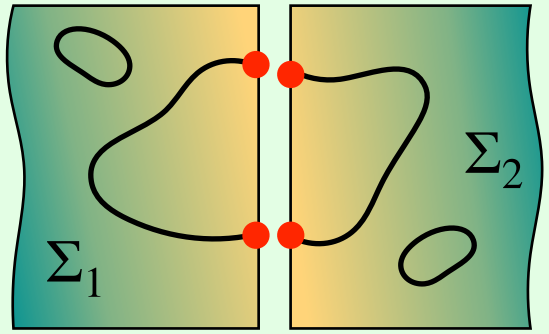

- **Extended Hilbert Space:** The Hilbert space must carry non-trivial rep. of corner symmetries (additional label arising from corner charges)
- Physical Hilbert space is a subspace

$\mathcal{H}_{\text{phys}} \subset \mathcal{H}^{\text{ext}}_{\Sigma_1} \otimes \mathcal{H}^{\text{ext}}_{\Sigma_2}$ containing states $(\mathcal{Q}(\Sigma_1) \otimes \mathbb{I} + \mathbb{I} \otimes \mathcal{Q}(\Sigma_2))\,|\psi\rangle_{\text{phys}} = 0$

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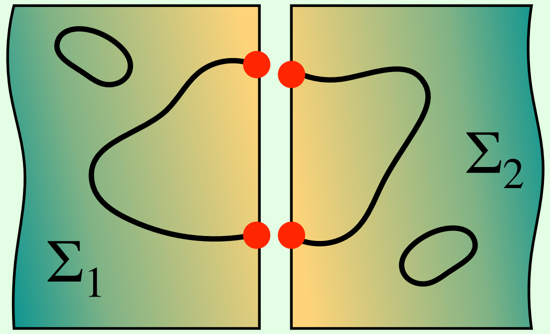

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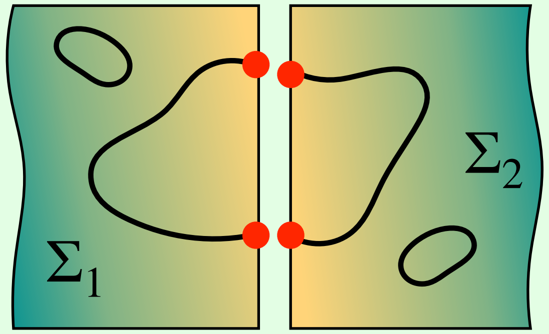

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Null Surface & Carrollian Physics

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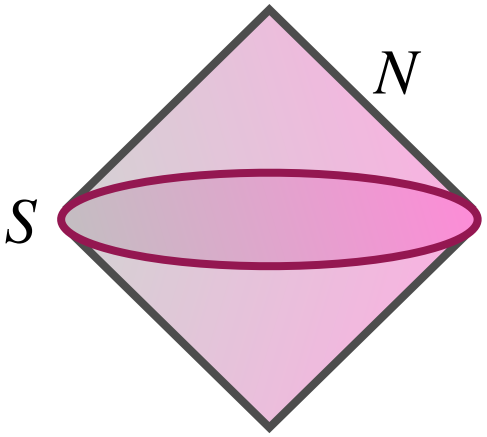
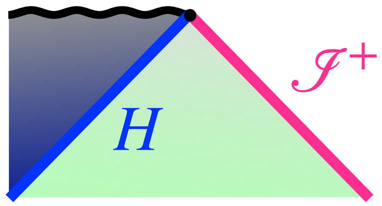
Null surface N are crucial for the understanding of *black holes* and *flat spacetimes*





- Black hole horizons and the boundary of asymptotically flat spacetimes are null
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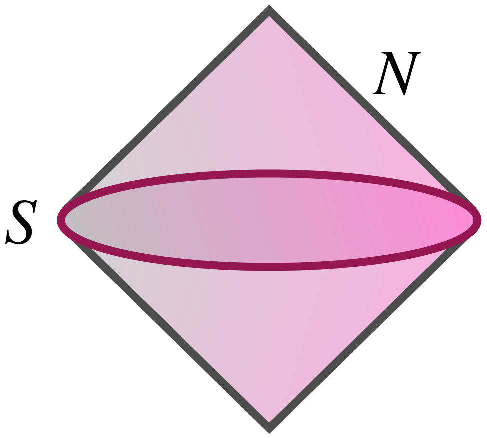

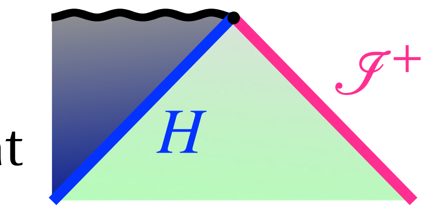

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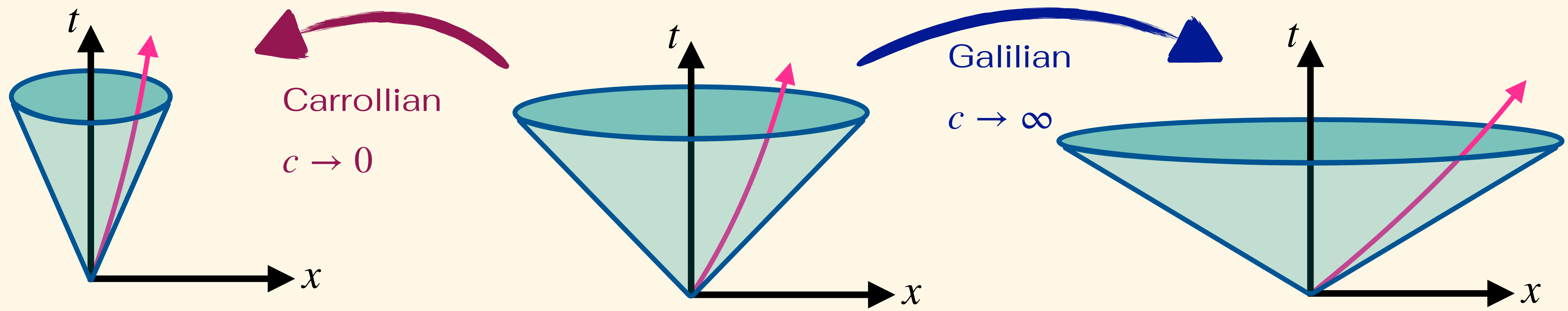
Carrollian limit a lá Levy-Leblond and Sen Gupta (1965)



Relative time | Absolute space

$t \to t'(t, x) \qquad x \to x'(x)$

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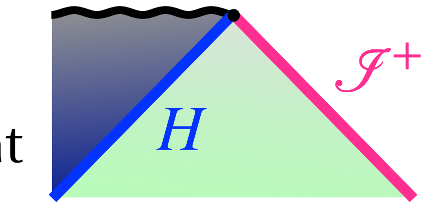

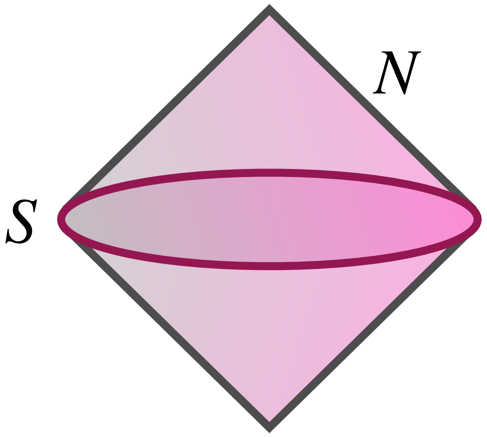

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Diffeomorphism preserving the geometry of the surface is BMSW symmetry

$$\text{BMSW} = \text{Diff}(S) \ltimes \mathbb{R}^S \ltimes \mathbb{R}^S$$

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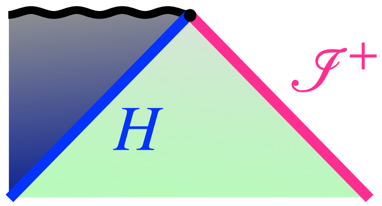

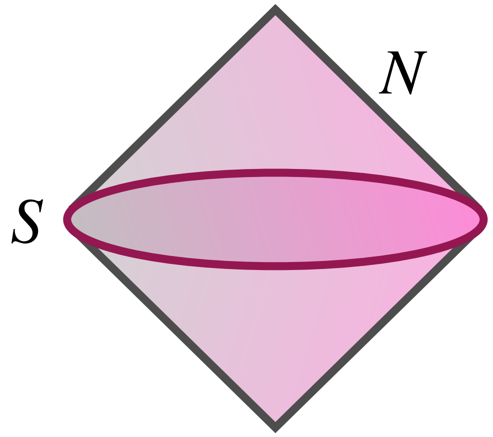

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Dynamics and phase space of gravity on N are Carrollian geometro-hydrodynamics

Gravity d.o.f = Fluid quantities
Einstein equations = Hydrodynamics

$(G_{ab} = T_{ab})\ell^b$ → $D_b \mathrm{T}_a{}^b = \mathrm{F}_a$

Raychaudhuri & Damour

Energy and momentum conservations for Carrollian fluid

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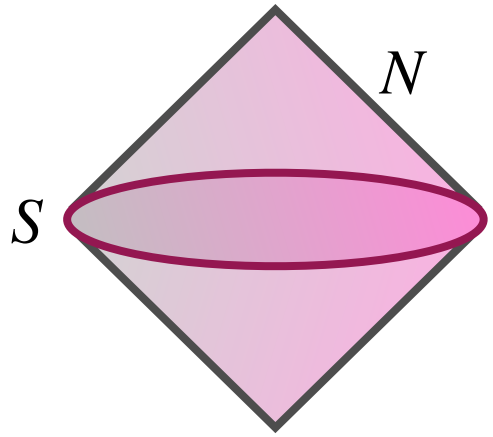

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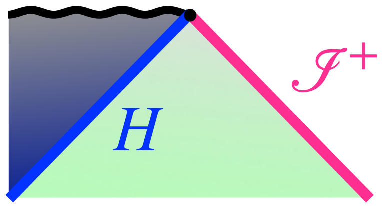

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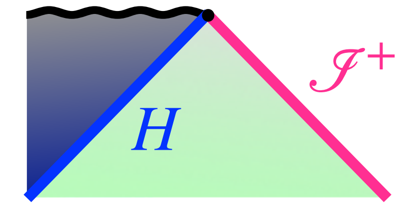

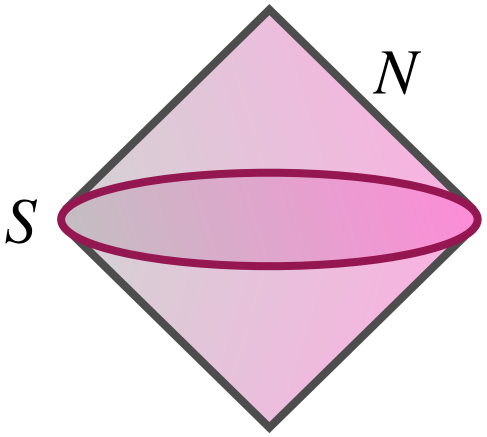

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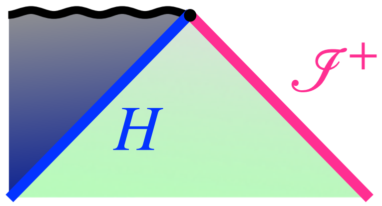

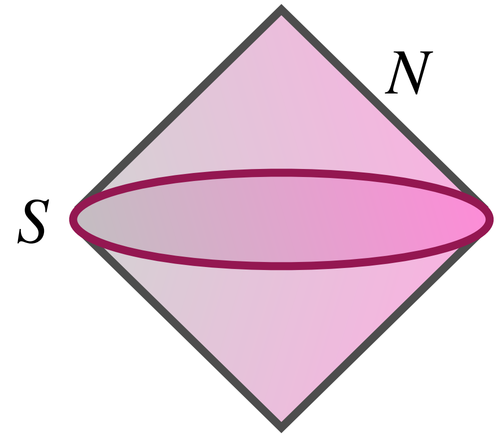

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Fluid route for black hole physics and gravity, both classical and quantum

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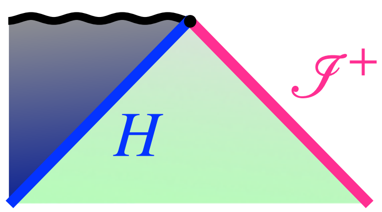

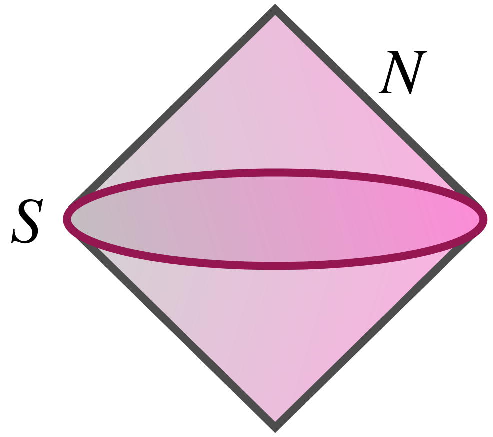

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- Geometry and symmetry of a generic null surface is Carrollian

Ruled Carrollian structure: (ℓ^a, k_a, q_{ab}) capture intrinsic geometry of a null surface

Diffeomorphism preserving the geometry of the surface is BMSW symmetry

$$\text{BMSW} = \text{Diff}(S) \ltimes \mathbb{R}^S \ltimes \mathbb{R}^S$$

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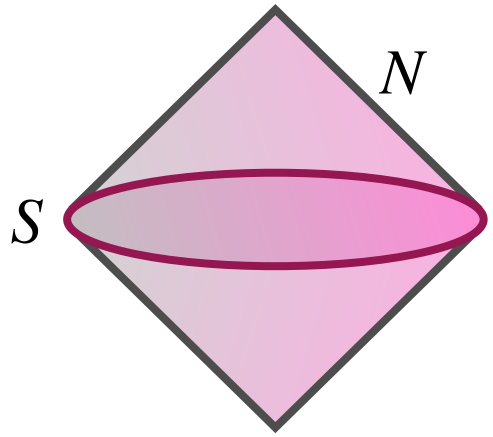
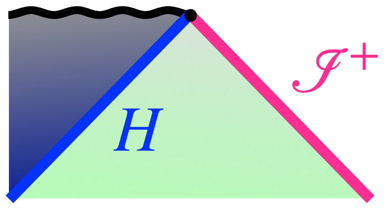
Null Surface & Carrollian Physics

Null surface N are crucial for the understanding of *black holes* and *flat spacetimes*





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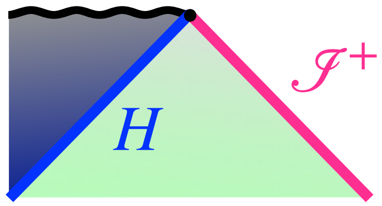

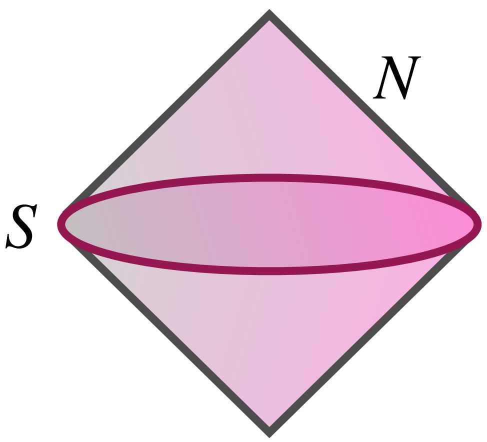

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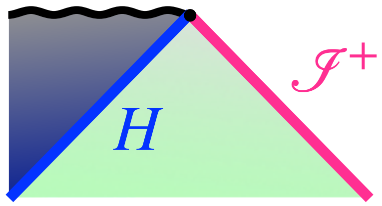

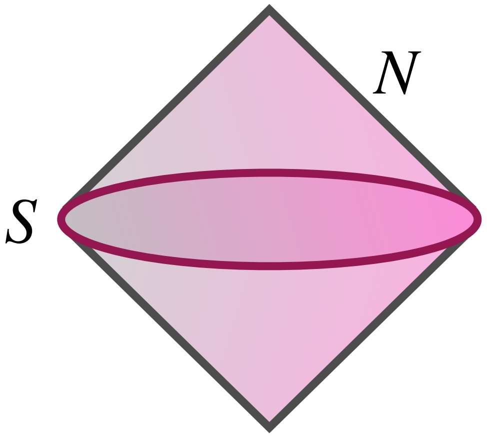

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  - ➡ Positive + discrete = molecular representation, **embadons**, a cutoff to central charges

Gravitational Edge Modes

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We saw that diffeomorphism that does not vanish at boundaries (large gauge transformations) turns physical and, by virtue of Noether's theorems, has non-zero charge

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- Adding a new field χ, **edge modes**, to the symplectic structure $\Omega[g, \delta g]$. This new field keep track of the variation of the embedding of the boundary.

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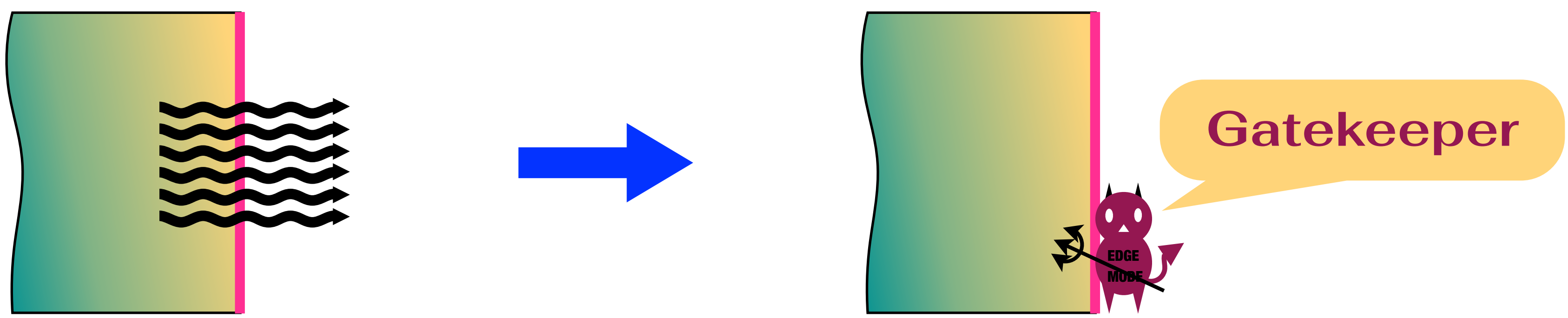

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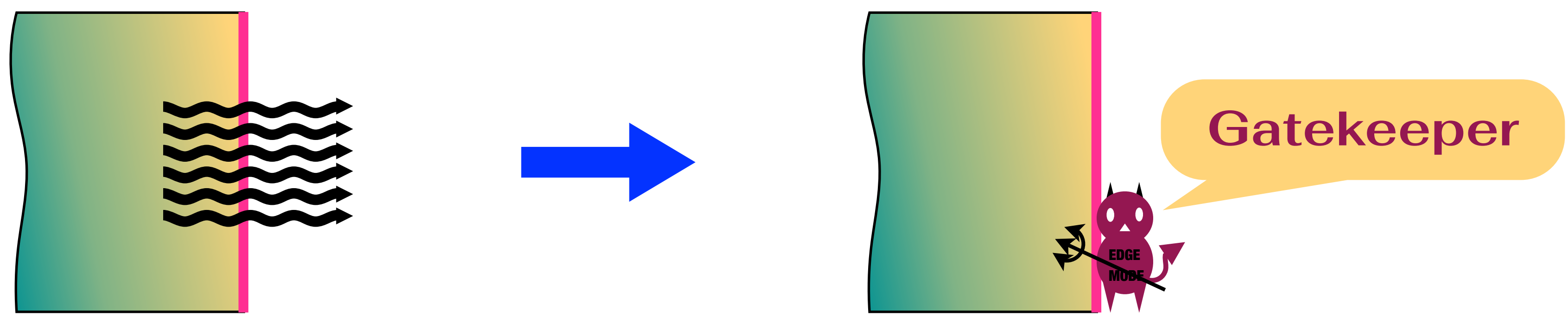

This is at the phase space level. What about the action ? Will edge mode has its own dynamics ?

For internal gauge theories (like electromagnetism and Chern-Simons), edge modes can have their own dynamics (boundary action) (Blommaert-Mertens-Verschelde, Geiller-PJ ...)

The situation involving gravity is still awaiting exploration

Upshot

- **Diffeomorphism**, once thought as being gauge, **is physical in the presence of spacetime boundaries.** It is associated, through the Noether's 2nd theorem, is the charge that represents physical quantities (energy, momentum, area, …)

- **The charges are corner integrals** and they form an algebra of corner symmetry group or its subgroup

- One can regard this as an emergence of new excitations living at boundaries, **edge modes**

- This then turns the geometry problem to the algebra problem, readily for quantization

- This procedure may help us understand some aspect of quantum gravity

- The case of **null boundary** is particularly interesting as it reveals a deep connection with **Carrollian physics**

Thank you!