Black Hole from Entropy Maximization

RIKEN iTHEMS

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arXiv 2309.00602

Sub-Refs:
Kawai-Matsuo-Yokokura 2013,
Kawai-Yokokura 2014, 2015, 2017, 2020, 2021,
Yokokura 2022,
Ho-Kawai-Liao-Yokokura 2023

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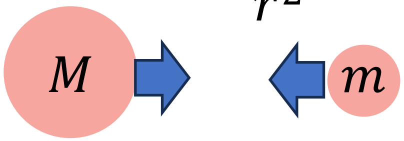
Gravity is universal.

$$F = \frac{GMm}{r^2}$$



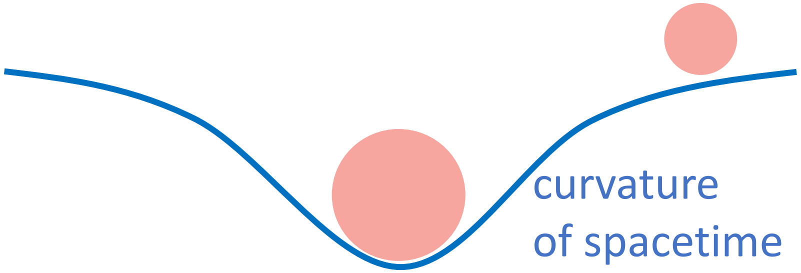
- Charge for gravity = (positive) energy

⇒Universal attractive force

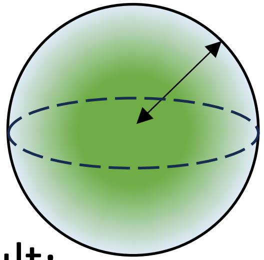
Einstein eq:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

curvature of spacetime energy-momentum density



- Accumulating the effect of gravity makes the bulk non-uniform.



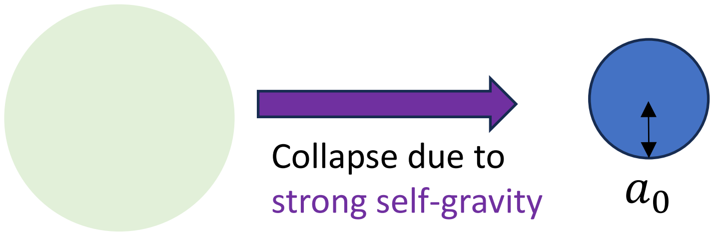
Cf: Charge for EM = +/- electric charge

⇒ The bulk can be neutral and uniform.

- An extreme and universal result:

In a strong gravity limit, any configuration becomes a “black hole”.

Any spherical configuration with mass M

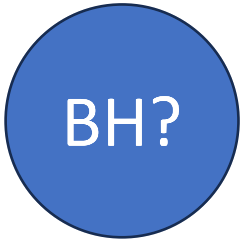


Collapse due to strong self-gravity

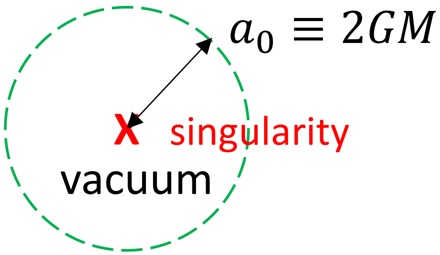
a_0

“black hole” with size $a_0 \equiv 2GM$ (Schwarzschild radius)

What is a black hole in quantum theory?



- Classical definition of black holes = existence of horizons



However,…

- **No** observational data showing the existence of horizons yet.
- Black-hole **entropy** $\boldsymbol{S = \frac{A}{4\hbar G}} = \log \Omega$ (Bekenstein-Hawking formula)

  ⇒ black hole = essentially quantum object consisting of (still unknown) d.o.f.
- In quantum gravity, spacetime should fluctuate.

  ⇒ The classical geometric definition must be modified/replaced somehow.

⇒A possible approach is to consider

What is the quantum definition/characterization of black holes?

But there should be many candidates…

Maximization of entropy

[Yokokura 2023]

- A candidate quantum definition is

Black hole maximizes thermodynamic entropy for a given surface area.

expectation value of

BH = S_{max}

[Dvali-Gomez 2013,
Oriti-Pranzetti-Sindoni 2016]

- Motivations

(1) Thermodynamic entropy is quantum:

surface area A

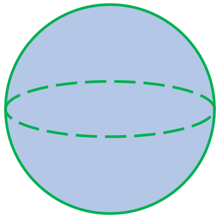
$$S = \log\,\Omega\,, \qquad \Omega = \#\text{ of } \{|\psi\rangle \text{ consistent with } (\cancel{E, V})\}$$

(2) Bekenstein-Hawking formula saturates the holographic entropy bound:

(still conjecture)
$$S[L] \leq \frac{A(B)}{4l_p^2}$$

[Bousso 1999]

Area of B: $A(B)$



(3) Any configuration with mass $\frac{R}{2G}$ in a strong-gravity limit becomes a BH with size R.

⇒BH = a macroscopic state (phase) with maximum entropy according to 2nd law?

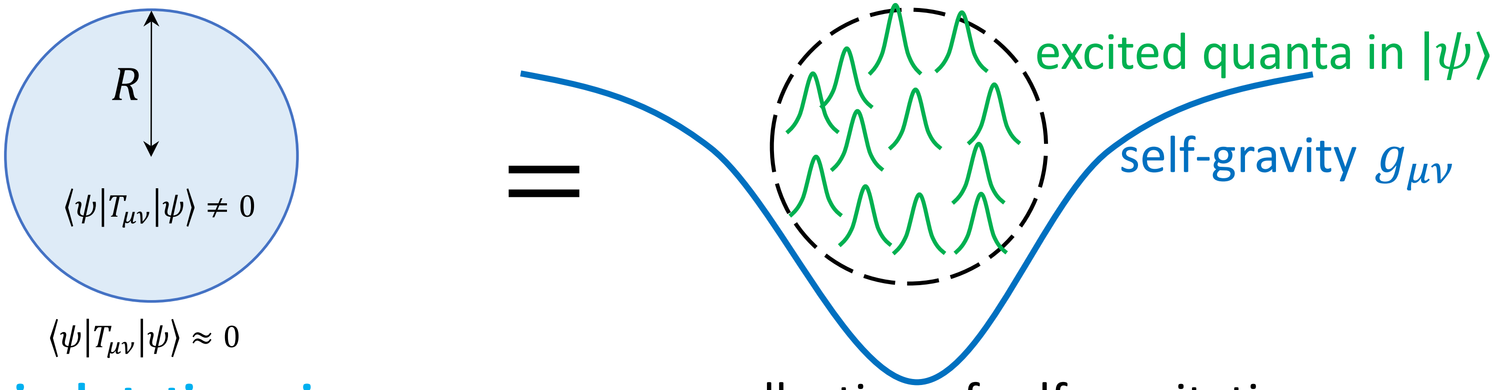
- This should be valid in quantum theory, but…

Semi-classical Einstein eq.

- *As a first trial*, we consider the 4D semi-classical Einstein eq with many matter fields in a self-consistent manner:

$$\boxed{G_{\mu\nu} = 8\pi G \langle\psi|T_{\mu\nu}|\psi\rangle}$$

$g_{\mu\nu}$:classical metric
ϕ_i:quantum operators



spherical static region
with a surface area $4\pi R^2$

a collection of self-gravitating
many excited quanta in $(|\psi\rangle, g_{\mu\nu})$

metric

$$ds^2 = \begin{cases} -\left(1-\frac{a_0}{r}\right)dt^2 + \frac{1}{1-\frac{a_0}{r}}dr^2 + r^2 d\Omega^2 & \text{for } R \leq r \\ -\left(1-\frac{a(r)}{r}\right)e^{A(r)}dt^2 + \frac{1}{1-\frac{a(r)}{r}}dr^2 + r^2 d\Omega^2 & \text{for } r \leq R \end{cases}$$

$\frac{a(r)}{2G} \equiv m(r)$: quasi-local energy inside r

Semi-classical Einstein eq.

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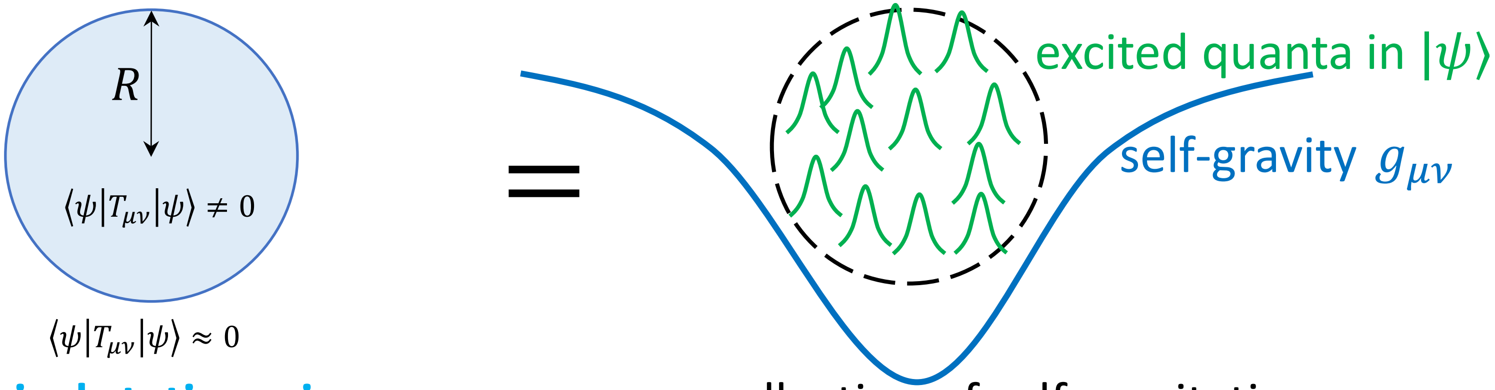


spherical static region
with a surface area $4\pi R^2$

a collection of self-gravitating
many excited quanta in $(|\psi\rangle, g_{\mu\nu})$

↓

thermodynamic entropy

$$S[g_{\mu\nu}, R) \underset{\text{Assume } \nabla_\mu \tilde{s}^\mu = 0}{=} \int_\Sigma d\Sigma_\mu\, \tilde{s}^\mu \underset{\text{spherical and static}}{=} \int_0^R dr \sqrt{g_{rr}(r)}\, s(r)$$

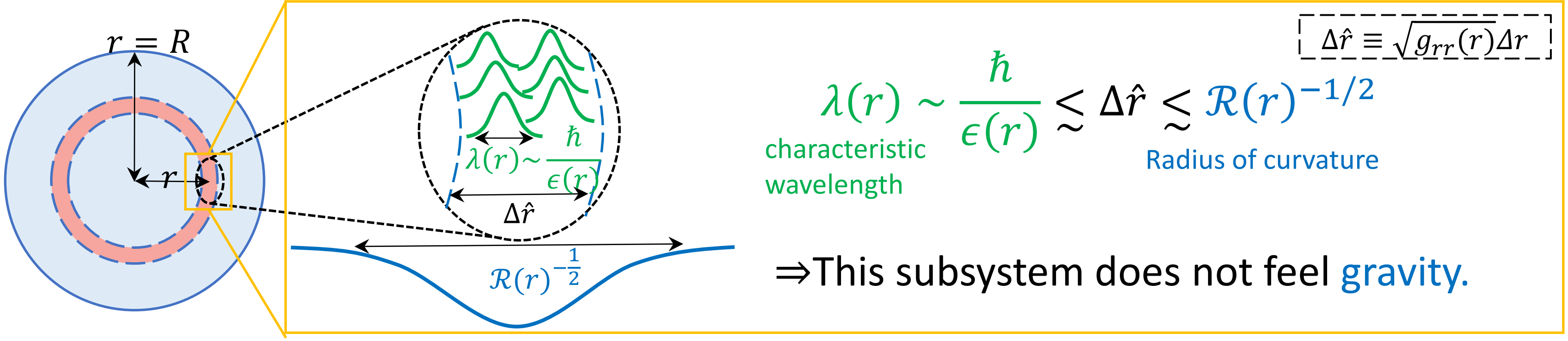
entropy density
per proper radial length

Non-uniform bulk structure by self-gravity $g_{\mu\nu}$
⇒Violate extensivity/intensivity
⇒ Self-gravity dependence of entropy

Step 1: Estimation of $S[g_{\mu\nu}; R)$

[Yokokura 2023]

- Focus on a small spherical subregion of the static configuration $(g_{\mu\nu}, |\psi\rangle)$.



$\Delta\hat{r} \equiv \sqrt{g_{rr}(r)}\Delta r$

$$\lambda(r) \sim \frac{\hbar}{\epsilon(r)} \lesssim \Delta\hat{r} \lesssim \mathcal{R}(r)^{-1/2}$$

characteristic wavelength

Radius of curvature

⇒This subsystem does not feel gravity.

- Consider a typical state $|\psi\rangle$ for $g_{\mu\nu}$ s.t.

[Goldstein et al 2006, Reimann 2007, more]

$$\langle\psi| - T^t_t(r)|\psi\rangle = \langle -T^t_t(r)\rangle_{thermal}, \qquad \epsilon(r) \sim T_{loc}(r)$$

local temperature

entropy density per proper radial length

$$\Rightarrow \quad s(r) \underset{\text{highly excited}}{\sim} \frac{4\pi r^2 \langle\psi| - T^t_t(r)|\psi\rangle}{T_{loc}(r)} \underset{\text{Einstein eq}}{=} \frac{r^2(-G^t_t(r))}{2G T_{loc}(r)}$$

- Thus, we can estimate

$$S[g_{\mu\nu}; R) = \int_0^R dr \sqrt{g_{rr}(r)}\, s(r) \sim \int_0^R dr \left[1 - \frac{2G}{r}\int_0^r dr'\, T_{loc}(r')s(r')\right]^{-\frac{1}{2}} s(r)$$

⇒ A larger $T_{loc}(r)$ gives a larger S (for $s(r)$, an increasing function of $T_{loc}(r)$).

$n = O(1) \gg 1$

- For the semi-classical description to be valid, we must have $\epsilon(r) \leq \epsilon_{max} \sim \frac{m_p}{\sqrt{n}}$

Step2: Upper bound from static condition

[Yokokura 2023]

- A static spacetime has a timelike Killing vector globally. [Mars-Senovilla 2003]

  ⇒No trapped surface (horizon) exists.

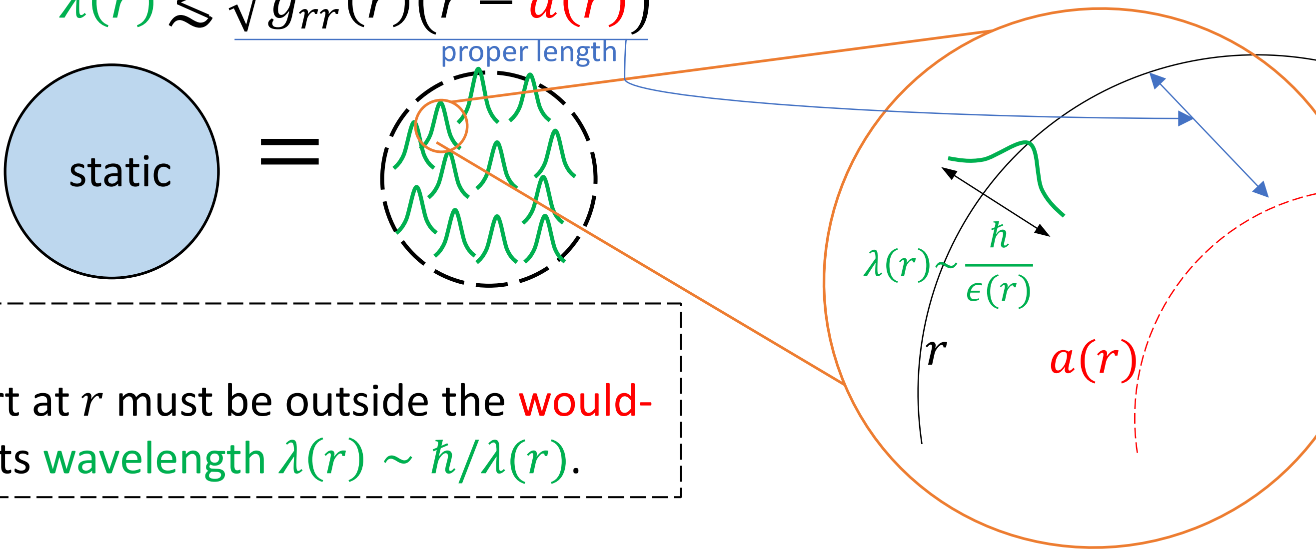
Ex. $k = \partial_t$ in Schwarzschild metric

$$k^2 = -\left(1 - \frac{a_0}{r}\right)$$

- Semi-classical condition for no trapped surface: [Sorkin-Wald-Zhang 1981]

$\frac{a(r)}{2G} \equiv m(r)$:energy inside r

$$\lambda(r) \lesssim \underbrace{\sqrt{g_{rr}(r)}\left(r - a(r)\right)}_{\text{proper length}}$$



<u>Meaning</u>:
A quantum composing a part at r must be outside the would-be horizon $a(r)$ at least by its wavelength $\lambda(r) \sim \hbar/\lambda(r)$.

- Using $s(r) \sim \frac{r^2(-G^t_t(r))}{2G\epsilon(r)}$, we reach the upper bound:

$$\boxed{S \lesssim \frac{1}{l_p^2}\int_0^R dr\, r\partial_r a(r)}$$

Step3: Saturating condition

[Yokokura 2023]

- To get the saturating configuration, we solve

$$\lambda(r) \sim \sqrt{g_{rr}(r)}\big(r - a(r)\big) \text{ for } \epsilon(r) = \epsilon_{max} \sim \frac{m_p}{\sqrt{n}}$$ at each r

and use

- consistency with local thermodynamics: $\langle T_r^r \rangle, \langle T_\theta^\theta \rangle \geq 0$

- consistency with semi-classical approximation: $\langle T_r^r \rangle, \langle T_\theta^\theta \rangle \leq O(1)$

- These lead to uniquely the entropy-maximizing spacetime: for $O(\sqrt{n} l_p) \leq r \leq R$

$$ds^2 = -\frac{\eta^2 \sigma}{2r^2} e^{-\frac{R^2 - r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2.$$

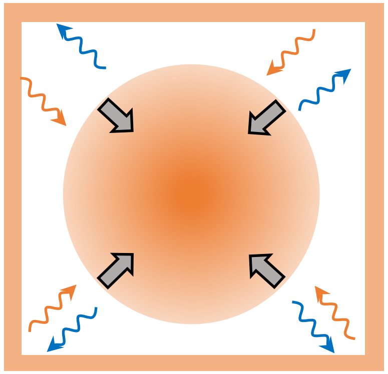
- Satisfies $G_{\mu\nu} = 8\pi G \langle \psi | T_{\mu\nu} | \psi \rangle$ with $n(\gg 1)$ scalar fields non-perturbatively in $\hbar$ for

$$\sigma = \frac{n l_p^2}{120\pi\eta^2}, \qquad 1 \leq \eta < 2.$$

- This can be obtained in various manners and should be robust.

[Kawai-Matsuo-Yokokura 2013,
Kawai-Yokokura 2014, 2015, 2017,
2020, 2021,
Yokokura 2022,
Ho-Kawai-Liao-Yokokura 2023]

(In particular, formation process in a heat bath.)



Heat baht at T_H

Semi-classical gravity condensate

- The entropy-maximizing configuration

[Yokokura 2023]

$$ds^2 = -\frac{2\sigma}{r^2} e^{-\frac{R^2-r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2$$

(←Locally $AdS_2 \times S^2$)

represents that self-gravitating quanta condensates into a dense configuration.

⇒ ***Semi-classical gravity condensate** is the black hole.*

This pressure and self-gravity are balanced!

Finite!

- Curvatures $\mathcal{R} = O\left(\frac{1}{nl_p^2}\right) \ll O\left(\frac{1}{l_p^2}\right)$

⇒**No singularity**

Quantum pressure

[Kawai-Yokokura 2020] $\langle T^\theta_\theta \rangle = \frac{1}{16\pi G\sigma\eta^2}$

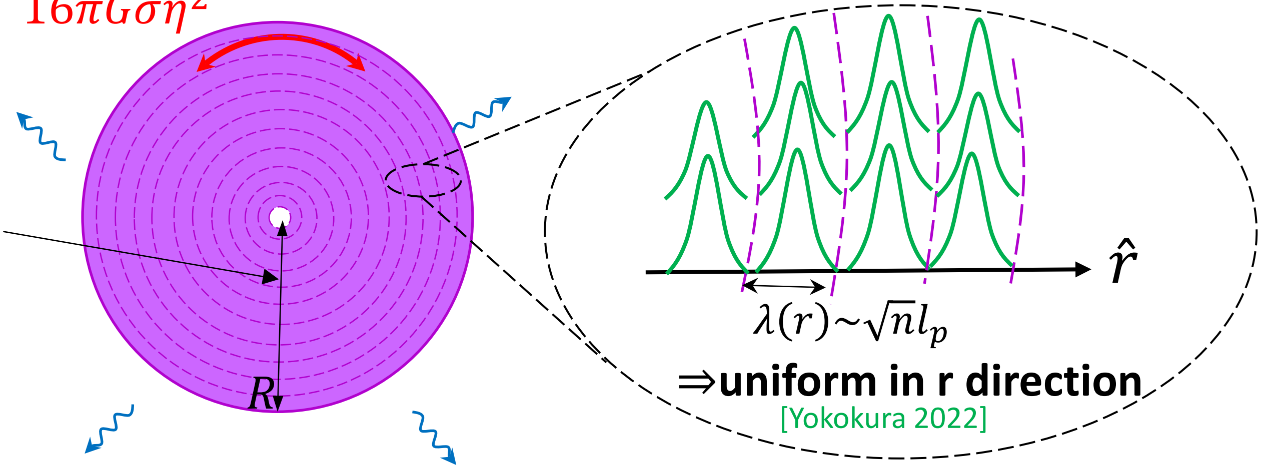
vacuum fluctuation

Surface $(a_0 \equiv 2GM)$

$$R = a_0 + \frac{\sigma\eta^2}{2a_0} (> a_0)$$

⇒No horizon but surface

[Yokokura 2022]



⇒**uniform in r direction**

[Yokokura 2022]

Hawking-like radiation occurs

$$T = \frac{\hbar}{4\pi a_0(u)}$$

[Kawai-Matsuo-Yokokura 2013]

Maximum entropy S_{max}

[Yokokura 2022,2023]

- The excited quanta behave like a local thermal state at

$$T_{loc}(r) = \frac{\hbar}{2\pi\sqrt{2\sigma\eta^2}}. \quad \sim \frac{m_p}{\sqrt{n}}$$

How to get this?
- Unruh temperature
- Particle creation

- In this local equilibrium, we have

$$T_{loc}s = \rho_{1d} + p_{1d}, \qquad p_{1d} = \frac{2-\eta}{\eta}\rho_{1d}.$$

1D Gibbs relation

equation of states

$$\rho_{1d} = 4\pi r^2(-\langle T_t^t\rangle), \quad p_{1d} = 4\pi r^2\langle T_r^r\rangle$$

$$-\langle T_t^t\rangle = \frac{1}{8\pi G r^2}, \quad \langle T_r^r\rangle = \frac{2-\eta}{\eta}(-\langle T_t^t\rangle)$$

- Then, we can evaluate the entropy density: $s(r) = \frac{2\pi\sqrt{2\sigma}}{l_p^2}$

⇒Integrating it over the volume produces the Bekenstein-Hawking formula:

$$S_{max} = \int_0^R dr\sqrt{g_{rr}(r)}\,s(r) = \int_0^R dr\sqrt{\frac{r^2}{2\sigma}}\,\frac{2\pi\sqrt{2\sigma}}{l_p^2} \underset{\uparrow}{=} \frac{A}{4l_p^2}$$

$A \equiv 4\pi R^2 \approx 4\pi a_0^2$

σ cancels out!

Note: The self-gravity changes the volume law to the area law.

- This derives the Bousso bound for thermodynamic entropy: $S \le S_{max} = \frac{A}{4l_p^2}$.

Conclusions

- We have considered a candidate for quantum characterization of BH:

Black hole maximizes thermodynamic entropy for a given surface area.

- Studying this for spherical static cases in $G_{\mu\nu} = 8\pi G \langle\psi|T_{\mu\nu}|\psi\rangle$ under

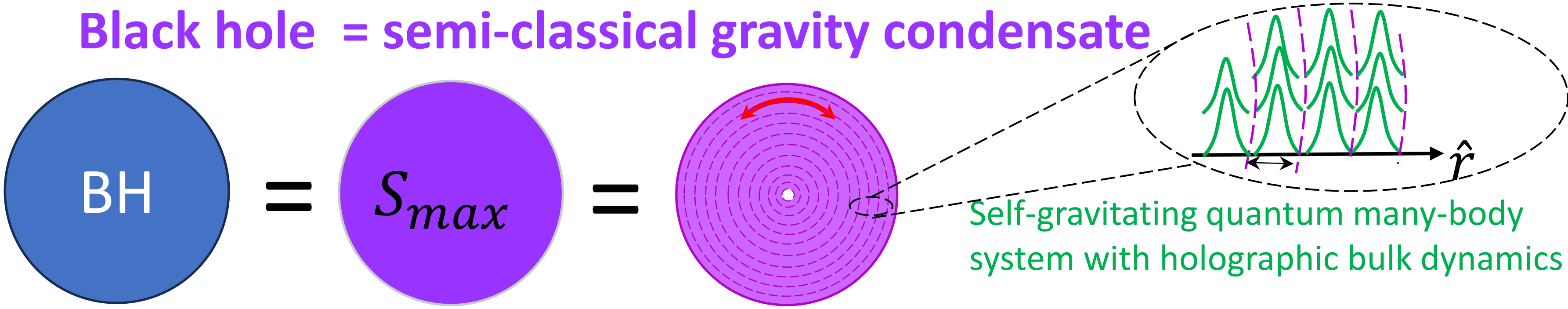
(1) Local spherical typicality: $\langle\psi| - T^t_t(r)|\psi\rangle = \langle -T^t_t(r)\rangle_{thermal},\ \epsilon(r) \sim T_{loc}(r), \langle T^i_j\rangle \geq 0$

(2) Semi-classicality: $\epsilon(r) \leq \frac{m_p}{\sqrt{n}},\ \langle T^\mu_\nu\rangle \leq O(1)$

(3) Static condition: $\lambda(r) \leq \sqrt{g_{rr}(r)}\,(r - a(r))$,

we found uniquely

Black hole = semi-classical gravity condensate



Self-gravitating quantum many-body system with holographic bulk dynamics

$$ds^2 = -\frac{\sigma\eta^2}{2r^2} e^{-\frac{R^2-r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2$$

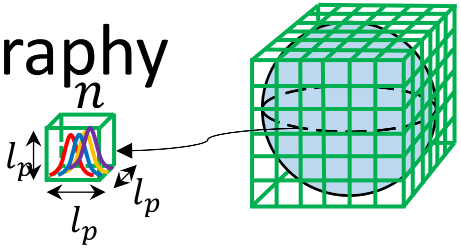
- 4D non-perturbative solution for $\hbar$

$$S \leq S_{max} = \frac{A}{4l_p^2}$$

-Derive Bousso bound for thermodynamic entropy

Future prospects（1/2）

- 1. Role of self-gravity in holography



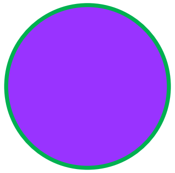
  - $s^{3d}(r) \sim \frac{\sqrt{n}}{l_p r^2} \ll s^{3d}_{naive}(r) \sim \frac{n}{l_p^3}$
  - S-wave excitations can explain $S = \frac{A}{4 l_p^2}$.

[Kawai- Yokokura 2020] (But the full 4D fluctuation is needed for $g_{\mu\nu}$.)

⇒***Self-gravity suppresses excitations of local d.o.f. in the bulk?***

⇒Origin of holography?

- 2. Gravity-condensate phase



- The radial direction is uniform radially.

⇒The gravity condensate is a thermodynamic phase?

Thermal wavelength　Mean classical-particle distance

In a (non-rel) material, $\lambda_T = \frac{\hbar}{\sqrt{mT}} \sim \rho_N^{-1/3}$ ⇒Quantum effects govern the system.

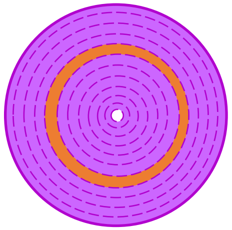
In the gravity condensate, $\lambda(r) \sim \frac{\hbar}{T_{loc}(r)} \sim \mathcal{R}(r)^{-\frac{1}{2}}$. ⇒Quantum gravitational phase?

$\sim \sqrt{n} l_p$

⇒ Can we make an effective theory/a quantum many-body model?

Future prospects（2/2）

• 3. Similarities to other gravity-condensate in QG



- Model in Group Field Theory [Oriti-Pranzetti-Sindoni 2016]
  ⇒ radially uniform
- Model in gravitons [Dvali-Gomez 2013]
  ⇒ Their picture appears in each spherical subsystem of ours.
  ⇒ It has

$$N_{matter}(r) \sim N_{graviton}(r) \sim n \ (\gg 1)$$

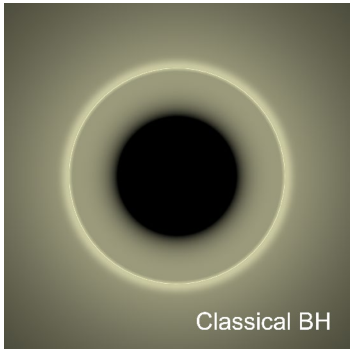
⇒It represents semi-classically a mixture of matter quanta and gravity quanta?

• 4. Phenomenology

-Imaging (“BH shadow”) the gravity condensate.
⇒ - consistent with current observations
 - tiny characteristic difference

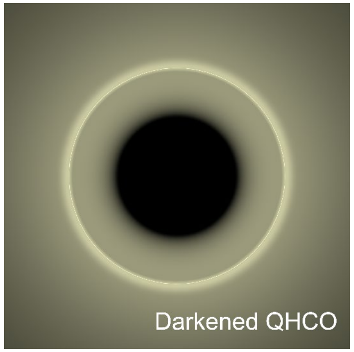
- How about GW?

[Chen-Yokokura 2024]



Classical Schwarzschild BH

$\approx$



Our gravity condensate

Thank you very much!!