1 The Term Structure of Interest Rates

- The **term structure of interest rate** is the relationship between the **term to maturity** and the **yield to maturity** (YTM) for bonds similar in all respects except their maturities. The **yield curve** plots the relationship between interest rates (long-term rates and short-term rates) and terms to maturity.

- Why are we interested in studying the term structure? The shape of the yield curve is important because it contains information about the future course of interest rates and the economic perspectives (Estrella and Mishkin (1996)).

  – Evidence shows that the term spread (long-term rate - short-term rate) is a very significant indicator in predicting changes in output. When the economic perspective is optimistic, the yield curve tends to be positively sloped, i.e., the **term spread** rises.

– A typical term spread is the difference between the interest rates on the ten-year Treasury bond and the three-month Treasury bill.

- Since the purpose of term structure analysis is to understand the differences in bond yields that arise strictly from differences in maturity, the bonds used in a yield curve should be similar in their risk level and should have the same characteristics. But it is difficult to find a pool of bonds that meets all of those conditions. Thus, we focus on the term structure of Treasury securities, because Treasury securities are lowest in default risk, the Treasury yield curve can be considered as the benchmark yield curve.

1.1 Forward Rates

- **Forward rates** are the interest rates that cover future time periods implied by currently available **spot rates**. A spot rate is a yield prevailing on a bond for immediate purchase. Given a set of spot rates, it is possible to calculate forward rates for any intervening time period.

- For convenience, we first consider **zero-coupon bonds** (like T-bills).
  - $r_{0,5}$ is the **spot rate** for the period beginning at time 0 and maturing at time 5. The present is always time 0, so a bond yield covering any time span beginning at time 0 is a spot rate.
  - If the time covered by a particular rate begins after time 0, it is a forward rate. For example, $f_{2,5}$ (rather than $r_{2,5}$) is the **forward rate**

beginning two years from now and extending three years to time 5, and $r^e_{2,5}$ is the expected interest rate on a three-year bond to cover the period from time 2 to time 5.

- Taking a five-year period of investment horizon as an example. Note that we observe the spot rates, but not the forward rates:

Spot Rate	$r_{0,1}$	$r_{0,2}$	$r_{0,3}$	$r_{0,4}$	$r_{0,5}$
Yield	0.08	0.088	0.09	0.093	0.095
Maturity	1 year	2 years	3 years	4 years	5 years

- Plotting these spot rates gives us the yield curve. This example gives us an upward-sloping yield curve because the yield increase with the maturity of the bond.

- These spot rates also imply a set of forward rates to cover periods ranging from time 1 to time 5. The forward rates can be calculated over the five years on the **assumption** that all of the following strategies would earn the same returns over the five-year period (Notice that this is not a prediction of returns, but it is an assumption used to calculate forward rates):
  (1) Buy the five-year bond and hold it to maturity.
  (2) Buy a one -year bond and when it matures, buy another one-year bond, following this procedure for the entire five years.
  (3) Buy a two-year bond and when it matures, buy a three-year bond and hold it to maturity.

- We can express these three strategies as follows:

(1) Hold one five-year bond for five years: Total Return $= \left(1 + r_{0,5}\right)^5$.

(2) Hold a sequence of one-year bonds: Total Return $= \left(1 + r_{0,1}\right)\left(1 + f_{1,2}\right)$ $\left(1 + f_{2,3}\right)\left(1 + f_{3,4}\right)\left(1 + f_{4,5}\right)$.

(3) Hold a two-year bond followed by a three-year bond: Total Return $=$ $\left(1 + r_{0,2}\right)^2\left(1 + f_{2,5}\right)^3$.

- Given the $\boldsymbol{assumption}$ that all of the above 3 strategies earn the same returns over the 3-year period, we have:

$$
\begin{aligned}
(1 + r_{0,5})^5 &= \left(1 + f_{0,1}\right)\left(1 + f_{1,2}\right)\left(1 + f_{2,3}\right)\left(1 + f_{3,4}\right)\left(1 + f_{4,5}\right) \\
&= \left(1 + r_{0,2}\right)^2\left(1 + f_{2,5}\right)^3.
\end{aligned}
$$

- How to calculate forward rates? Consider the third strategy. Since we know $r_{0,2}$ and $r_{0,5}$ at time 0, we can calculate the forward rate $f_{2,5}$ which covers the time span from time 2 to time 5:

$$(1 + r_{0,5})^5 = \left(1 + r_{0,2}\right)^2 \left(1 + f_{2,5}\right)^3 .$$

Using the spot rates given above, we have $f_{2,5} = 9.97\%$. The forward rate, implied by this set of spot rates, to cover the period from year 2 to year 5, is 9.97%.

- Notice that nothing has been said so far about how the forward rates are to be interpreted. **We can consider the forward rate to be an estimate of the expected future rate. Different theories of the term structure interpret forward rates in different ways.**

1.2 Theories of the Term Structure

1.2.1 The Expectations Theory

- The expectations theory states that forward rates are unbiased estimators of future interest rates, or Forward Rates = Expected Future Spot Rates ($f = r^e$).

- In terms of the previous example, where it was calculated that the forward rate for a three-period bond to cover from time 2 to time 5 was 9.97%, the expectations theory would say that 9.97% is a good estimate of the spot rate that will prevail on a three-year bond beginning two years from now.

- This theory has strong practical implications. If it is true, the observable term structure contains predictions of future interest rates. Suppose the bond market is well developed and populated with many participants who **do not have any particular preference about the maturity of instruments**, and they are considering investment in either the first strategy or the third strategy.

- If the investor planning to hold the two-year bond at 8.8%, what is the required rate demanded by the investor in the next 3 years for a three-year bond, in order for the investor to be indifferent from holding a 5-year bond?

$$1.095^5 = 1.088^2 \left(1 + r^e_{2,5}\right)^3 .$$

Thus, $r^e_{2,5} = 9.97\% = f_{2,5}$.

– If, for some reason, the investors expected a higher future interest rate, say, $r^e_{2,5} = 11\%$, then we have $(1+r_{0,5})^5 < \left(1+r_{0,2}\right)^2 \left(1+r^e_{2,5}\right)^3$. Thus, investors demand more 2-year bonds, and less 5-year bonds. Thus,

$$Q_{05} \downarrow, \quad Q_{02} \uparrow \qquad \Longrightarrow r_{05} \uparrow, \quad r_{02} \downarrow .$$

These investors would stop switching from the five-year to the two-year bond only when the expected returns from the two strategies were equal, eliminating any incentive to switch from one to the other. Thus, we will have a new set of r_{05} and r_{02}, and a new implied forward rate.

- In general, if the investment horizon is N periods, we have calculated the following forward rates, $f_{1,2}$, $f_{2,3}$, $...f_{N-1,N}$, then

$$(1+r_{0,N})^N = \left(1+r_{0,1}\right)\left(1+f_{1,2}\right)...\left(1+f_{N-1,N}\right),$$

that is,

$$(1 + r_{0,N})^N = \left(1 + r_{0,1}\right)\left(1 + r^e_{1,2}\right) \ldots \left(1 + r^e_{N-1,N}\right).$$

Then, we can approximately express the long-term rate as the average all future expected short-term rates:

$$r_{0,N} = \frac{r_{0,1} + r^e_{1,2} \cdots + r^e_{N-1,N}}{N}.$$

- For example, when $N = 2$, we have

$$r_{0,2} = \frac{r_{0,1} + r^e_{1,2}}{2}. \tag{1}$$

- From the view of the expectations hypothesis, an upward-sloped yield curve means that the public expect that the future (short-term) interest rates will rise. To see this, by (1), an upward-sloped yield curve implies $r_{0,2} > r_{0,1}$ (long-term rate is greater than the short-term rate). The equation (1) can hold only if $r^e_{1,2} > r_{0,1}$.

1.2.2 The Liquidity Premium Theory

- Liquidity premium theory asserts that **bondholders greatly prefer to hold short-term bonds** rather than long-term bonds. Short-term bonds have less interest rate risk than long-term bonds, because their prices change less for a given changes in interest rates. Thus, investors are willing to pay more for short-term bonds than for long-term bonds. In other words, investors are willing to hold long-term bonds only if they are paid a higher interest rate. The extra interest rate they request is the **liquidity premium**.

- For example, assume that the investors demand an extra 0.2% of yield in order for them to hold 5-year bonds, rather than 2-year bonds. This means that the yield 9.5% of the 5-year bond contains a liquidity premium 0.2%. Thus, given $r_{0,5}$ and $r_{0,2}$, we must have

$$(1.095 - 0.002)^5 = 1.088^2 \left(1 + r^e_{2,5}\right)^3 .$$

Thus, we have $r^e_{2,5} = 9.635\% < 9.97\% = f_{2,5}$. This also means

$$(1 + r_{0,5})^5 > \left(1 + r_{0,2}\right)^2 \left(1 + r^e_{2,5}\right)^3 .$$

Thus, using forward rates to estimate future spot rates of interest gives estimates that are too high due to the presence of liquidity premium.

- In general,

$$r_{0,N} = \frac{r_{0,1} + r^e_{1,2} \cdots + r^e_{N-1,N}}{N} + l_N,$$

where l_N is the liquidity premium for the N-year bond. We assume $l_2 < l_3 < \ldots < l_N$, so that the yield curve will be upward sloping even when there is no change in the expected short-term rates. For $N = 2$,

$$r_{0,2} = \frac{r_{0,1} + r^e_{1,2}}{2} + l_2.$$

Clearly, even if $r^e_{1,2} = r_{0,1}$, we have $r_{0,2} > r_{0,1}$, and thus the yield curve is upward sloping.

- Q: Given a 1-year bond $r_{0,1} = 8\%$ and the public expects $r^e_{1,2} = 10\%$, will the investor be happy to buy a 2-year bond with $r_{0,2} = 9\%$?

1.2.3 The Market Segmentation (Preferred Habitat) Theory

- Unlike the pure expectations theory and the liquidity premium theory, the market segmentation theory is not expressly stated in terms of forward rates. Rather, the market segmentation theory takes a more **institutional approach**.

- According to this theory, the bond market is dominated by large financial institutions which have strong maturity preferences stemming from the kind of business each of them pursues. Commercial banks, for example, have relatively short-term liabilities in the form of demand deposits and certificates of deposit. As a consequence, they prefer to invest in relatively short-term bonds. Life insurance companies, by contrast, have their liabilities due far in the future upon the death of policyholders. Thus, life insurance companies prefer long-term bonds.

- These preferences of different types of financial institutions stem from the nature of their businesses and a desire to match the maturity of their assets and liabilities in order to control risk. Because of these preferences, the institutions tend to trade bonds only in their respective maturity ranges. The desire of these different institutions to participate only in certain maturity segments of the bond market leads directly to the **segmented markets hypothesis**: The yield curve is determined by the interplay of supply and demand factors in different segments of the maturity spectrum of the bond market.

1.3 How the Three Theories Explain Different Observed Yield Curves

- Suppose the yield curve is upward sloping, according to the expectations theory, this means that short-term interest rates are expected to rise. With a flat yield curve, all forward rates equal the current short-term spot rate, so the expectations theory interprets this as the market's belief that interest rates will remain constant. For a downward-sloping yield curve, the expectations theory stresses the fact that forward rates will be smaller the farther they are in the future and interprets this as the market's belief that short-term interest rates are expected to fall.

- Assume the market expects short-term interest rates to be constant. The liquidity premium would then force a long-term bond to pay a higher yield.

According to the liquidity premium theory, this means that the yield curve will be sloping slightly upward even when short-term rates are expected to remain constant. For this reason, many people believe that an upward sloping yield curve is the normal shape of the yield curve. When the liquidity premium is larger and the market expects a higher interest rate, we will observe a steeper upward-sloping yield curve.

For a downward-sloping yield curve, the liquidity premium theory argues that the market must be expecting a drop in interest rates sufficiently large to offset the effect of the liquidity premium.