計量經濟學導論暨實習期中考參考解答

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一、選擇題，45%。

1 (a)

2 (b)

3 (b)

4 (c)

5 (b)

6 (c)

7 (b)

8 (b)

9 (d)

10 (b)

11 (d)

12 (d)

13 (d)

14 (c)

15 (a)

二、非選擇題，55%。

1a Demand equals to supply, $\beta_0 + \beta_1 P + u = \gamma_0 + \gamma_1 P + v$, yields

$$\begin{aligned} P &= \frac{\beta_0 - \gamma_0}{\gamma_1 - \beta_1} + \frac{u - v}{\gamma_1 - \beta_1}, \\ Q &= \beta_0 + \beta_1 \left(\frac{\beta_0 - \gamma_0}{\gamma_1 - \beta_1} + \frac{u - v}{\gamma_1 - \beta_1} \right) + u \\ &= \frac{\beta_0 \gamma_1 - \beta_1 \gamma_0}{\gamma_1 - \beta_1} + \frac{\gamma_1 u - \beta_1 v}{\gamma_1 - \beta_1}. \end{aligned}$$

1b The means of P and Q are

$$\begin{aligned} \mathrm{E}(P) &= \frac{\beta_0 - \gamma_0}{\gamma_1 - \beta_1}, \\ \mathrm{E}(Q) &= \frac{\beta_0 \gamma_1 - \beta_1 \gamma_0}{\gamma_1 - \beta_1}. \end{aligned}$$

1c The variance and covariance are

$$\begin{aligned}
\mathrm{Var}(P) &= \mathrm{E}([P-\mathrm{E}(P)]^2) = \frac{\sigma_u^2+\sigma_v^2}{(\gamma_1-\beta_1)^2}, \\
\mathrm{Var}(Q) &= \mathrm{E}([Q-\mathrm{E}(Q)]^2) = \frac{\gamma_1^2\sigma_u^2+\beta_1^2\sigma_v^2}{(\gamma_1-\beta_1)^2}, \\
\mathrm{Cov}(P,Q) &= \mathrm{E}([P-\mathrm{E}(P)][Q-\mathrm{E}(Q)]) = \frac{\gamma_1\sigma_u^2+\beta_1\sigma_v^2}{(\gamma_1-\beta_1)^2}.
\end{aligned}$$

1d

$$\begin{aligned}
\hat\beta_1 \xrightarrow{p} \frac{\mathrm{Cov}(P,Q)}{\mathrm{Var}(P)} &= \frac{\gamma_1\sigma_u^2+\beta_1\sigma_v^2}{\sigma_u^2+\sigma_v^2} \\
&= \beta_1 + \frac{(\gamma_1-\beta_1)\sigma_u^2}{\sigma_u^2+\sigma_v^2} > \beta_1 \text{ because } \gamma_1>0, \beta_1<0.
\end{aligned}$$

The estimated slope is too large.

2a

$$\begin{aligned}
Y_i &= \beta_0+\beta_1X_i+u_i \\
&= \beta_0+\beta_1(\tilde X_i - w_i)+u_i \\
&= \beta_0+\beta_1\tilde X_i + (-\beta_1 w_i + u_i) \\
&\equiv \beta_0+\beta_1\tilde X_i + v_i
\end{aligned}$$

where $v_i = -\beta_1 w_i + u_i$. The probability limit of $\hat\beta_1$ is

$$\begin{aligned}
\hat\beta_1 &\xrightarrow{p} \beta_1 + \frac{\mathrm{Cov}(\tilde X_i, v_i)}{\mathrm{Var}(\tilde X_i)} \\
&= \beta_1 + \frac{\mathrm{Cov}(X_i+w_i, -\beta_1 w_i + u_i)}{\mathrm{Var}(\tilde X_i)} \\
&= \beta_1 + \frac{-\beta_1\sigma_w^2}{\sigma_X^2+\sigma_w^2} = \frac{\sigma_X^2}{\sigma_X^2+\sigma_w^2}\beta_1
\end{aligned}$$

2b

$$\begin{aligned}
Y_{it} &= \beta_0+\beta_1X_{it}+u_{it} = \beta_0+\beta_1(\tilde X_{i,t}-w_{i,t})+u_{i,t} \\
&= \beta_0+\beta_1\tilde X_{i,t}+(u_{i,t}-\beta_1 w_{i,t}) \\
&\equiv \beta_0+\beta_1\tilde X_{i,t}+v_{i,t},\ v_{i,t}\equiv(u_{i,t}-\beta_1 w_{i,t}) \\
Y_{i,t}-Y_{i,t-1} &= \beta_1(\tilde X_{i,t}-\tilde X_{i,t-1})+(v_{i,t}-v_{i,t-1})
\end{aligned}$$

Thus,

$$
\begin{aligned}
\hat{\beta}_1^{FD} &\xrightarrow{p} \frac{\mathrm{Cov}(\tilde{X}_{i,t}-\tilde{X}_{i,t-1}, Y_{i,t}-Y_{i,t-1})}{\mathrm{Var}(\tilde{X}_{i,t}-\tilde{X}_{i,t-1})} \\
&= \beta_1 + \frac{\mathrm{Cov}(\tilde{X}_{i,t}-\tilde{X}_{i,t-1}, v_{i,t}-v_{i,t-1})}{\mathrm{Var}(\tilde{X}_{i,t}-\tilde{X}_{i,t-1})} \\
&= \beta_1 + \frac{\mathrm{Cov}(X_{it}-X_{i,t-1}+w_{i,t}-w_{i,t-1}, u_{i,t}-u_{i,t-1}-\beta_1(w_{i,t}-w_{i,t-1}))}{\mathrm{Var}(X_{it}-X_{i,t-1}+w_{i,t}-w_{i,t-1})} \\
&= \beta_1 + \frac{-\beta_1\mathrm{Var}(w_{i,t}-w_{i,t-1})}{\mathrm{Var}(X_{i,t}-X_{i,t-1})+\mathrm{Var}(w_{i,t}-w_{i,t-1})+2\mathrm{Cov}(X_{i,t}-X_{i,t-1}, w_{i,t}-w_{i,t-1})} \\
&= \beta_1 + \frac{-\beta_1 2\sigma_w^2}{2\sigma_X^2 - 2\mathrm{Cov}(X_{i,t}, X_{i.t-1}) + 2\sigma_w^2 + 0} \\
&= \beta_1 - \frac{\beta_1\sigma_w^2}{\sigma_X^2 - \rho_X\sigma_X^2 + \sigma_w^2} \\
&= \beta_1 - \frac{\sigma_w^2}{(1-\rho_X)\sigma_X^2+\sigma_w^2}\beta_1 = \frac{(1-\rho_X)\sigma_X^2}{(1-\rho_X)\sigma_X^2+\sigma_w^2}\beta_1
\end{aligned}
$$

2c The bias of $\hat{\beta}_1^{FD}$ is $\frac{\sigma_w^2}{(1-\rho_X)\sigma_X^2+\sigma_w^2}\beta_1$, and the bias of $\hat{\beta}_1$ in (a) is $\frac{\sigma_w^2}{\sigma_X^2+\sigma_w^2}\beta_1$. Since $\frac{\sigma_w^2}{(1-\rho_X)\sigma_X^2+\sigma_w^2}\beta_1 > \frac{\sigma_w^2}{\sigma_X^2+\sigma_w^2}\beta_1$ as long as $\rho_X > 0$, the bias of $\hat{\beta}_1^{FD}$ is greater than the bias of $\hat{\beta}_1$ in (a).

3a The coefficient of -0.368 means that, holding constant the control variables, having a "shall-carry" law results in a reduction in the violent crime rate of 37%. This is a very large effect in a real-world sense.

3b Severity of punishment (1) arguable affects the violent crime rate and (2) could be correlated with shall-carry laws. If so, severity of punishment satisfy the conditions for omitted variables bias.

3c The OLS estimate on *shall* overstates the effect of having a shall-carry, which is, in part, picking up the effect of tough laws. In other words, the coefficient on *shall* over-estimates the effect of "**shall-issue**" laws.

3d When state fixed-effects are included in the model, the effect of *shall* on violent crime is reduced to -0.046 or -4.6%, and is not significant at 5% level. Evidently there was important omitted variable bias in the regression in (a).

3e When fixed time effects are added, the estimate on *shall* is further reduced to -0.028 or -2.8% and is not significant. The time fixed-effects are jointly statistically significant (F-statistic is 21.62 with p-value<0.0001).

3f The standard error of *shall* is 0.0407 when the autocorrelation of error terms within a state is corrected, which is larger than the standard error without considering autocorrelation (0.0172). The OLS standard errors without clustering usually understate the true uncertainty when there is autocorrelation in error terms within a state.

3g The most credible results are those with both state and time fixed effects. There is no statistically significant evidence that concealed weapons laws have any effect on violent crime rates.