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Estimating the Implicit Market Model from Option Prices

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The main novelty of this paper is to develop a preference-free option pricing formula which involves index return volatility and the alpha, beta, and firm-specific risk of underlying stock returns by formulating option payoffs with the market model in the multivariate risk-neutral valuation framework. We thus estimate the option implicit market model, namely the forward-looking alpha, beta, and firm-specific risk, by calibrating equity and index option prices. Empirical illustration indicates that our model calibrates equity options accurately, the CAPM

「政策與管理意涵」

如何預測未來期間之 CAPM 或是市場模型中的 alpha、beta、與公司特定風險，除了是重要投資議題之外，金融機構對於基金和投資組合的風險管理也仰賴這些參數的預測值。而本文以選擇權價格估計隱性市場模型所得到之 alpha、beta、公司特定風險的前瞻性預測，繼承選擇權能反映未來情況的特性，可提供市場上交易與風險管理者更有效的參數預測值。

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holds with option-implied estimates, and option-implied estimates are more effective than regression-based historical estimates in predicting alpha, beta, firm-specific risk, and stock returns (conditional on the contemporaneous index returns) in a future horizon.

Key Words: Forward-looking alpha; Forward-looking beta; Forward-looking firm-specific risk; Option implicit market model; Risk-neutral valuation relationship.

JEL classification: G12, G13

1. Introduction

Since the seminal work on the market model (single-index model) of Sharpe (1963), Fama (1968), and Jensen (1968), beta and firm-specific risk have become standard risk measurements for equity investments; while alpha is widely used for measuring abnormal returns and testing for the validity of the capital asset pricing model (CAPM). However, the levels of alpha, beta, and firm-specific risk of a stock are unobservable and traditionally estimated from a backward viewpoint, that is, using historical excess returns on the individual stock and the market portfolio to estimate them based on a regression model. Using this method to predict (or extrapolate) future alpha, beta, and firm-specific risk could yield poor performance, especially when the entire financial market is in a crisis or when individual firms are involved in mergers or acquisitions, starting large-scale projects, or adjusting their capital structures, since historical returns may not provide adequate information to estimate future alpha, beta, and firm-specific risk. Thus, it is important to obtain reasonable and feasible estimates of alpha, beta, and firm-specific risk on a forward-looking basis.

To achieve this goal, this paper develops a methodology to estimate the market model on an *ex ante* basis by deriving an option pricing model through combining the framework of multivariate risk-neutral valuation relationships (RNVRs) with the market model, thus introducing the market return and the firm-specific risk of an individual stock as two underlying stochastic variables by which to determine option prices. The resulting option pricing model involves the observable risk-free interest rate, the volatility of the market return, and the alpha, beta, and firm-specific risk of the underlying stock asset. Based on the proposed model, one can estimate alpha, beta, and firm-specific risk for subsequent periods of time by calibrating the whole range of index and equity options with different strike prices and times to maturity on each trading day. In addition to exploiting the forward-looking information in option prices, we expect that this option implicit market model, analogous to the market model, can bridge the theoretical CAPM to pragmatic practitioners by providing reliable estimations of alpha, beta, and firm-specific risk.

Regarding alpha and firm-specific risk, to our knowledge, few if any attempts have been made to derive option-implied estimates in the literature. Beta estimation using information on option prices has drawn more attention. To approximate future beta, the forward-looking feature of option-implied volatilities or correlations has been intuitively exploited. For example, French, Groth, and Kolari (1983) combine the implied volatilities from option prices and the historical correlation of individual equity and index returns to estimate beta. Siegel (1995) calibrates the correlation between individual equity and index returns and thus the equity beta from option prices by taking advantage of the exchange option pricing formula in Margrabe (1978). However, the exchange option between the individual stock and market returns is not generally available for most individual stocks.

After those attempts, empirical studies such as Bates (1998), Buraschi and Jackwerth (2001), Dennis and Mayhew (2002), and Bakshi, Kapadia, and Madan (2003) have demonstrated the existence of systematic risk factors in option prices. These findings motivate researches to estimate beta from option prices. Husmann and Stephan (2007) propose an option pricing formula for equity options based on the CAPM in an incomplete market, so that the correlation between the individual stock and stock index and thus the beta value can be estimated from the market prices of individual equity options. Chen, Kim, and Panda (2009) derive a counterpart of the Black and Scholes (1973) formula under the physical probability measure by linking the future returns of the underlying stock and the option contract with a linear single-factor model. With this option

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pricing formula, they calibrate implied returns of the individual stock and the stock index from market option prices and derive the option-implied beta by regressing the series of the implied returns of the individual stock on the series of the implied returns of the stock index.

Instead of developing explicit option pricing formulas, Buss and Vilkov (2012) and Chang, Christoffersen, Jacobs, and Vainberg (2012) propose model-free approaches to estimate option-implied beta under the risk-neutral measure. Chang, Christoffersen, Jacobs, and Vainberg (2012) exploit the market model to express beta in terms of the variance and skewness of the returns of the individual stock and the stock index. They next utilize the option-implied variance and skewness of Bakshi, Kapadia, and Madan (2003) to derive the option-implied beta. Buss and Vilkov (2012) express the implied variance of a stock index as a function of the weights, implied volatility, and pairwise correlations of the component stocks, as the return of the stock index is, by definition, the weighted average of the returns of the component stocks. Based on the above expression, they propose a parametric relation for the pairwise correlations under the risk-neutral and physical probabilities measures. Then, equipped with the physical pairwise correlations estimated from historical data and the option-implied volatilities, they derive the risk-neutral pairwise correlations and thus the option-implied beta for all components of a stock index.

One of the cores of the proposed model is to develop an option pricing model by utilizing the RNVR framework in Stapleton and Subrahmanyam (1984) and Câmara (2005). The RNVR framework was first developed by Rubinstein (1976) and Brennan (1979) for derivatives pricing with only one underlying variable. Stapleton and Subrahmanyam (1984) extend the Brennan (1979) model to allow for multiple underlying variables. Câmara (2003) provides a generalized RNVR framework to encompass a family of transformed normal distributions for the underlying asset price to price derivatives. Similar to the extension in

Stapleton and Subrahmanyam (1984), Câmara (2005) expands Câmara (2003) to a multivariate setting. The RNVR framework is a useful technique in asset pricing, especially for derivatives whose payoffs are determined by one or several underlying variables, whether tradable or non-tradable. In addition, the RNVR framework is based on the market equilibrium rather than on the no-arbitrage argument under the Black-Scholes framework. Therefore, the RNVR framework is highly general for asset pricing, especially when additional assumptions are imposed on pricing models.

There are several merits of the proposed estimation method. First, in contrast to previous papers that focus on estimating only option-implied beta, this paper exploits the market model to formulate the return of the underlying asset in the RNVR option pricing framework. As a result, our option implicit market model, which is the only comparable model with the market model in the literature, not only assesses beta but also alpha and firm-specific risk. Second, unlike the option pricing formulas in Husmann and Stephan (2007) and Chen, Kim, and Panda (2009), our option pricing formula, by taking advantage of the RNVR, contains the observable risk-free interest rate rather than the expected returns for a stock index and its component stocks, which theoretically vary according to the risk preference of investors and are notoriously difficult to estimate accurately. We also argue that one of the major purposes of estimating beta is to predict expected stock returns. If one can estimate directly the expected returns for a stock index and its component stocks, there remains little need to further estimate beta. Third, our option-implied alpha, beta, and firm-specific risk are unadulterated parameters which are specified in the option payoff function under the physical measure, whereas the approaches in Buss and Vilkov (2012) and Chang, Christoffersen, Jacobs, and Vainberg (2012) must address the transformation or mapping of the values of statistic quantities under the risk-neutral and physical measures. We note that market participants care about alphas, betas, and firm-specific risks in the real world (i.e., under the physical measure), but it remains difficult to discern errors caused by unclear relationships between parameter values under the risk-neutral and physical measures.

To illustrate the practical implications of our option implicit market model, we conduct an empirical examination based on the Dow-Jones Industrial Average (DJIA) component stocks. Using the market prices of high-liquidity option contracts on DJIA component stocks and the S&P 500 stock index, we discover that the calibration of our model for equity options performs quite accurately, which means it is appropriate to introduce the market model into the option pricing model. The empirical results demonstrate that option-implied alphas, betas, and firm-specific risks are satisfactory estimates for forecasting future alphas, betas, and firm-specific risks, respectively. In addition, the insignificant option-implied alphas support the validity of the CAPM. Moreover, the option-implied estimates are more capable than the historical estimates of predicting conditional stock returns on the contemporaneous index returns in a future horizon.

The remainder of this paper is organized as follows. Section 2 briefs the framework of multivariate RNVR, and Section 3 proposes our novel option pricing model involving alpha, beta, and firm-specific risk. The calibration process for the empirical illustration is introduced in Section 4, and the empirical analyses are discussed in Section 5. Section 6 presents a robustness test. Section 7 concludes the paper and suggests future research.

2. General Multivariate Risk-Neutral Valuation Relationship (RNVR)

Given multiple underlying processes in a one-period economy, the standard pricing formula for derivatives considered by Brennan (1979), Stapleton and

Subrahmanyam (1984), and Câmara (2003, 2005) is

$$V = R_f^{-1} E^P\big(C(\mathbf{X})Z(\mathbf{X})\big), \tag{1}$$

where V is the price of a derivative contract today, R_f is the gross return of the risk-free asset for the examined period $[0, T]$, and $E^P(\cdot)$ stands for the expectation operator under the physical probability measure. The variable $\mathbf{X}$ is a vector of payoffs of n underlying processes at T and its transpose is denoted by $\mathbf{X}^{\mathbf{T}} \equiv [X_1\ X_2\ \cdots\ X_n]$, $C(\mathbf{X})$ is the payoff function specified in any derivative contract, and $Z(\mathbf{X})$ is the asset-specific pricing kernel defined as

$$Z(\mathbf{X}) = \frac{E^P\big(U'(w_T)|\mathbf{X}\big)}{E^P\big(U'(w_T)\big)}, \tag{2}$$

where $U'(w_T)$ is the marginal utility function of the end-of-period wealth of the representative agent.

Theorem 1 (Multivariate RNVR with joint transformed normal distributions). *Suppose that the marginal utility function has the exponential form of* $U'(w_T) = \exp\big(\gamma g(w_T)\big)$, *where* γ *is a constant risk preference parameter and* w_T *denotes the period-end wealth, and it follows a transformed normal distribution,*[1] *i.e.,*

$$g(w_T) \sim ND(\mu_w, \sigma_w^2),$$

where μ_w *and* σ_w^2 *are the mean and variance of the normal distribution of* $g(w_T)$, *respectively, and* $g(\cdot)$ *is a strictly monotonic and differentiable function.*[2] *Suppose further that the underlying assets follow a multivariate transformed normal distribution at* T:

1 The definition of a transformed normal random variable Y is that it can be expressed as $Y = f^{-1}(Z\sigma + \mu)$, where Z follows the standard normal distribution, denoted as $Z \sim ND(0,1)$, and f is a strictly monotonic and differentiable function.

2 Note that different assumptions for $g(\cdot)$ can represent risk attitudes such as increasing, constant, and decreasing proportional risk aversion or constant absolute risk aversion. However, Câmara (2003) argues that the exact functional form of $g(\cdot)$ is not critical as long as $g(w_T)$ is normally distributed; thus $U'(w_T)$ follows a lognormal normal distribution. For details refer to Câmara (2003).

$$\mathbf{h}(\mathbf{X}) = [h_1(X_1)\ h_2(X_2)\ \cdots\ h_n(X_n)]^{\mathrm{T}} \sim ND(\boldsymbol{\mu}, \boldsymbol{\Sigma}),$$

where $\boldsymbol{\mu}$ *and* $\boldsymbol{\Sigma}$ *are the mean vector and the variance-covariance matrix, respectively, of the underlying multivariate normal distribution, and the* h_i*'s are arbitrary strictly monotonic and differentiable functions. Equipped with the joint transformed normal distribution for* $U'(w_T)$ *and* $\mathbf{X}$*, a probability density function of* $\mathbf{X}$ *without preference parameters can be derived and thus the price of the derivative can be written as the risk-neutral pricing equation*

$$V = R_f^{-1} E^Q\big(C(\mathbf{X})\big), \tag{3}$$

where $E^Q(\cdot)$ *denotes the expectation operator under the risk-neutral probability density function of* $\mathbf{X}$*, where the location parameter is a function of* R_f*.*

***Proof*:** This theorem inherits the RNVR framework from Câmara (2003) and Stapleton and Subrahmanyam (1984). The detailed proof can be found in these two papers.

Consequently, when pricing derivative contracts, there is no need to estimate the future expected returns of $\mathbf{X}$, since the R_f's with respect to different times to maturity are observable today. Furthermore, the above RNVR-based result is consistent with the classic option pricing theory, in the sense that derivative contracts can be evaluated in the risk-neutral world, in which the expected returns of all assets are equal to the riskless return.

3. Option Pricing Model Involving alpha, beta, and Firm-Specific Risk

For the case of pricing European equity options, we consider two points of time, where today is time 0 and the maturity date of the option is time T. Moreover, with the market model, the option payoff can be formulated as a function of the alpha, beta, and firm-specific risk. In particular, the payoff at time

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T of a European call option,[3] with the two underlying state variables—the gross return of the market portfolio (M_T) and the firm-specific risk component (e_T)—is denoted as

$$C(M_T, e_T) \equiv \max(S_T - K, 0) = S_0 \max\left(\frac{S_T - K}{S_0}, 0\right) = S_0 \max(Y_T - k, 0)$$
$$= S_0 \max\left(\left[\alpha T + (1-\beta)R_f + \beta M_T + e_T\right]R_q^{-1} - k, 0\right), \quad (4)$$

where S_T is the stock price at time T, K is the strike price, S_0 is the stock price at time 0, $Y_T \equiv S_T/S_0$ represents the gross return on the underlying stock from time 0 to time T, and k is defined as K/S_0. In addition, the last equation transforms the payoff of the equity call option into a multivariate function of the abnormal return, market return, and the firm-specific risk component according to the market model in Jensen (1968), where α is the annualized abnormal return for the underlying stock, R_f and M_T denote the gross returns of the risk-free asset and the market portfolio, respectively, from time 0 to time T, and e_T represents the firm-specific risk component. According to the market model, the excess returns of the underlying asset $Y_T - R_f$ can be expressed as $\alpha T + \beta(M_T - R_f) + e_T$. Therefore, we substitute $\alpha T + (1-\beta)R_f + \beta M_T + e_T$ for Y_T to yield Equation (4). Moreover, multiplying by $R_q^{-1} \equiv e^{-qT}$ in Equation (4), where q is the per-annum continuous compounded dividend yield, reflects the decline of the stock price due to dividend payments.

3.1 Pricing Kernel and Distributions of Underlying Variables and Aggregate Wealth

Following the common assumptions of the market model, the diversifiable firm-specific risk component associated with the underlying individual stock is assumed to follow a normal distribution, that is, $e_T \sim ND(0, \sigma_e^2 T)$, where the

3 It is straightforward to derive the option pricing formula for equity put options by simply considering the payoff of the put option in Equation (4) and following the same procedure described later.

expected return on the firm-specific risk component is zero and σ_e represents the corresponding annual volatility for the firm-specific risk component. Hereafter, this paper employs σ_e to represent the magnitude of the firm-specific risk. Furthermore, the gross return on the market portfolio is assumed to follow a lognormal distribution, that is, $\ln M_T \sim ND(\mu_m T, \sigma_m^2 T)$, where μ_m is the annualized expected return of the market portfolio and σ_m^2 is the annual variance of the market return. We note that although the market model or the Black-Scholes model adopts the assumptions of the (log-)normal distributions, which may be inconsistent with the real conditions, they are still the most popular models for both practitioners and academics. Although it is not necessary, this paper still inherits their (log-)normal distributions assumptions for two reasons. The first is to ensure that our model is as comparable with the market model as possible. Rather than attempting to outperform the market model with different distribution assumptions, we seek to retain the framework of the market model but exploit option price information to obtain forward-looking alpha, beta, and firm-specific risk. The second reason is that the (log-)normal distribution assumptions can allow the proposed option pricing formula to incorporate the Black-Scholes formula as a special case. This characteristic ensures the superiority of our market-model option pricing formula over the classical Black-Scholes formula in calibrating option prices.

Furthermore, following the RNVR in Theorem 1, we assume that the end-of-period wealth of the representative agent w_T, the gross return on the market portfolio M_T, and the firm-specific risk component e_T follow a tri-variate transformed normal distribution as follows:

$$\begin{bmatrix} g(w_T) \\ h_1(M_T) = \ln(M_T) \\ h_2(e_T) = e_T \end{bmatrix} \sim ND\left(\begin{bmatrix} \mu_w \\ \mu_m T \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_w^2 & \rho_{wm}\sigma_w\sigma_m\sqrt{T} & \rho_{we}\sigma_w\sigma_e\sqrt{T} \\ \rho_{wm}\sigma_w\sigma_m\sqrt{T} & \sigma_m^2 T & 0 \\ \rho_{we}\sigma_w\sigma_e\sqrt{T} & 0 & \sigma_e^2 T \end{bmatrix} \right),$$

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where the firm-specific risk component is assumed to be independent of the return on the market portfolio according to the market model, and $g(w_T)$ is assumed to be correlated to $\ln(M_T)$ and e_T with the coefficients ρ_{wm} and ρ_{we}, respectively.

Since the representative agent's marginal utility follows $U'(w_T) = \exp\left(\gamma g(w_T)\right)$, which is lognormally distributed with mean $\gamma\mu_w$ and variance $\gamma^2\sigma_w^2$, the mean of this lognormal random variable is

$$E^P\left(U'(w_T)\right) = \exp\left(\gamma\mu_w + \frac{1}{2}\gamma^2\sigma_w^2\right).$$

Furthermore, the conditional distribution of marginal utility is derived as

$$(\ln U'(w_T)|M_T, e_T) \sim ND(\gamma\mu_w + \gamma\rho_{wm}\frac{\sigma_w}{\sigma_m\sqrt{T}}(\ln M_T - \mu_m T) + \gamma\rho_{we}\frac{\sigma_w}{\sigma_e\sqrt{T}}(e_T - 0),$$

$$\gamma^2\sigma_w^2(1 - \rho_{wm}^2 - \rho_{we}^2)),$$

and its conditional mean is obtained as

$$E^P(U'(w_T)|M_T, e_T) = \exp\left(\gamma\mu_w + \gamma\rho_{wm}\frac{\sigma_w}{\sigma_m\sqrt{T}}(\ln M_T - \mu_m T) + \gamma\rho_{we}\frac{\sigma_w}{\sigma_e\sqrt{T}}(e_T - 0)\right.$$

$$\left. + \frac{1}{2}\gamma^2\sigma_w^2(1 - \rho_{wm}^2 - \rho_{we}^2)\right).$$

Finally, following Equation (2), we obtain the asset-specific pricing kernel $Z(M_T, e_T)$ as

$$Z(M_T, e_T) = \exp\left(\gamma\rho_{wm}\frac{\sigma_w}{\sigma_m\sqrt{T}}(\ln M_T - \mu_m T) + \gamma\rho_{we}\frac{\sigma_w}{\sigma_e\sqrt{T}}(e_T - 0)\right.$$

$$\left. - \frac{1}{2}\gamma^2\rho_{wm}^2\sigma_w^2 - \frac{1}{2}\gamma^2\rho_{we}^2\sigma_w^2\right). \quad (5)$$

3.2 Risk-Neutral Valuation Relationships

Since the log-return on the market portfolio ($h_1(M_T) = \ln(M_T)$) and the firm-specific risk ($h_2(e_T) = e_T$) follow a bivariate transformed normal distribution, and they are by definition independent of each other, we obtain their joint probability density function as

$$\phi^P(M_T, e_T) = \frac{1}{\sqrt{2\pi}\sigma_m\sqrt{T}M_T} \exp\left(-\frac{1}{2\sigma_m^2 T}(\ln M_T - \mu_m T)^2\right) \cdot \frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}(e_T - 0)^2\right). \tag{6}$$

Given the density of $\phi^P(M_T, e_T)$ in Equation (6) and the asset-specific pricing kernel $Z(M_T, e_T)$ in Equation (5), the option pricing formula based on Equation (1) is derived as

$$V = R_f^{-1} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{C(M_T, e_T)}{\sqrt{2\pi}\sigma_m\sqrt{T}M_T} \exp\left(-\frac{1}{2\sigma_m^2 T}\left(\ln M_T - \left(\mu_m T + \gamma\rho_{wm}\sigma_w\sigma_m\sqrt{T}\right)\right)^2\right) \cdot \frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}\left(e_T - \left(0 + \gamma\rho_{we}\sigma_w\sigma_e\sqrt{T}\right)\right)^2\right) dM_T de_T. \tag{7}$$

Proposition 1. (The prices of market portfolio return and firm-specific risk and the RNVR substitution). *By definition, the current prices P_m and P_e of the underlying variables M_T and e_T, which are the returns on the market portfolio and the firm-specific risk component for the individual stock, respectively, are one and zero. P_m is one because market participants can earn M_T by investing one dollar in the market portfolio today. Following the assumption in CAPM, only the systematic risk generates the risk premium, while the firm-specific risk is not priced, i.e., P_e is zero. Hence the current prices of the underlying variables are linked with the means of $\ln M_T$ and e_T in Equation (7) as*

$$\begin{bmatrix} P_m \\ P_e \end{bmatrix} R_f = \begin{bmatrix} 1 \\ 0 \end{bmatrix} R_f = \begin{bmatrix} \exp\left(\mu_m T + \gamma\rho_{wm}\sigma_w\sigma_m\sqrt{T} + \frac{1}{2}\sigma_m^2 T\right) \\ 0 + \gamma\rho_{we}\sigma_w\sigma_e\sqrt{T} \end{bmatrix}. \tag{8}$$

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We then obtain the following relations:

$$\begin{bmatrix} \mu_m T + \gamma \rho_{wm} \sigma_w \sigma_m \sqrt{T} \\ \gamma \rho_{we} \sigma_w \sigma_e \sqrt{T} \end{bmatrix} = \begin{bmatrix} \ln R_f - \frac{1}{2} \sigma_m^2 T \\ 0 \end{bmatrix}. \qquad (9)$$

***Proof*:** See Appendix 1.

Substituting relations (9) into Equation (7), the option value is given by

$$V = R_f^{-1} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{C(M_T, e_T)}{\sqrt{2\pi}\sigma_m \sqrt{T} M_T} \exp\left(-\frac{1}{2\sigma_m^2 T}\left[\ln M_T - \left(\ln R_f - \frac{1}{2}\sigma_m^2 T\right)\right]^2\right) \cdot$$

$$\frac{1}{\sqrt{2\pi}\sigma_e \sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}(e_T - 0)^2\right) dM_T de_T \equiv R_f^{-1} E^Q\big(C(M_T, e_T)\big), \qquad (10)$$

where the expectation operator $E^Q(\cdot)$ is defined with the following risk-neutral probability density function:

$$\phi^Q(M_T, e_T) = \frac{1}{\sqrt{2\pi}\sigma_m \sqrt{T} M_T} \exp\left(-\frac{1}{2\sigma_m^2 T}\left[\ln M_T - \left(\ln R_f - \frac{1}{2}\sigma_m^2 T\right)\right]^2\right) \cdot$$

$$\frac{1}{\sqrt{2\pi}\sigma_e \sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}(e_T - 0)^2\right). \qquad (11)$$

That is, equipped with the RNVR condition in Equation (9), we express the option price with the present value of its expected payoff as if we were in the risk-neutral world. In addition, note that with this RNVR, only the means of M_T and e_T are changed; the values of α, β, and σ_e of $C(M_T, e_T)$ in Equation (4) are unaffected. In other words, the values of α, β, and σ_e calibrated based on the proposed option pricing formula are exactly those desired by investors in the real world.

Finally, by substituting the payoff function in Equation (4) into Equation (10) and defining $Z_m = \ln(M_T)$ and $Z_e = \alpha T + e_T$, we write the European call option price as

$$V = R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_0 \max\left(\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right] R_q^{-1} - k, 0\right) \cdot$$

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$$\frac{1}{\sqrt{2\pi}\sigma_m\sqrt{T}}\exp\left(-\frac{1}{2\sigma_m^2T}(Z_m-\mu_m^*)^2\right)\frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}}\exp\left(-\frac{1}{2\sigma_e^2T}(Z_e-\mu_e^*)^2\right)dZ_m dZ_e$$

$$= R_f^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_0 \max\left(\left[(1-\beta)R_f+\beta e^{Z_m}+Z_e\right]R_q^{-1}-k,0\right)\phi^*(Z_m)\phi^*(Z_e)$$

$$dZ_m dZ_e, \quad (12)$$

where $\mu_m^* = \ln R_f - \frac{1}{2}\sigma_m^2 T$, $\mu_e^* = \alpha T$, and the second equation is derived by defining $\phi^*(Z_m)$ and $\phi^*(Z_e)$ to be the normal probability density functions for $Z_m \sim ND(\mu_m^*, \sigma_m^2 T)$ and $Z_e \sim ND(\mu_e^*, \sigma_e^2 T)$, respectively.

***Remark*:** Based on the same framework, we further discover that the abnormal return αT can be interpreted as the future value of the price of the firm-specific risk if we assume a nonzero value of P_e in Equation (8). To obtain this conclusion, we first use the market model without the abnormal return in the option payoff (4), i.e.,

$$\hat{C}(M_T, e_T) = S_0 \max\left(\left[(1-\beta)R_f+\beta M_T+e_T\right]R_q^{-1}-k,0\right). \quad (4')$$

Next, Equation (8) is rewritten given P_e to be nonzero:

$$\begin{bmatrix}1\\P_e\end{bmatrix}R_f = \begin{bmatrix}\exp\left(\mu_m T+\gamma\rho_{wm}\sigma_w\sigma_m\sqrt{T}+\frac{1}{2}\sigma_m^2T\right)\\0+\gamma\rho_{we}\sigma_w\sigma_e\sqrt{T}\end{bmatrix}. \quad (8')$$

Following the risk-neutral condition in Equation (9) results in

$$\begin{bmatrix}\mu_m T+\gamma\rho_{wm}\sigma_w\sigma_m\sqrt{T}\\\gamma\rho_{we}\sigma_w\sigma_e\sqrt{T}\end{bmatrix} = \begin{bmatrix}\ln R_f-\frac{1}{2}\sigma_m^2T\\P_eR_f\end{bmatrix}, \quad (9')$$

and substituting Equations (4′) and (9′) into Equation (7) yields

$$V = R_f^{-1}\int_{-\infty}^{\infty}\int_0^{\infty}\frac{\hat{C}(M_T,e_T)}{\sqrt{2\pi}\sigma_m\sqrt{T}M_T}\exp\left(-\frac{1}{2\sigma_m^2T}\left[\ln M_T-\left(\ln R_f-\frac{1}{2}\sigma_m^2T\right)\right]^2\right)\cdot$$

$$\frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}}\exp\left(-\frac{1}{2\sigma_e^2T}\left(e_T-P_eR_f\right)^2\right)dM_T de_T$$

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$$= R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_0 \max\left(\left[(1-\beta)R_f + \beta e^{Z_m} + \hat{Z}_e\right] R_q^{-1} - k, 0\right) \cdot$$

$$\frac{1}{\sqrt{2\pi}\sigma_m\sqrt{T}} \exp\left(-\frac{1}{2\sigma_m^2 T}(Z_m - \mu_m^*)^2\right) \frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}\left(\hat{Z}_e - \hat{\mu}_e^*\right)^2\right) dZ_m d\hat{Z}_e, \quad (12')$$

where $\hat{Z}_e$ is defined as e_T and $\hat{Z}_e \sim ND(\hat{\mu}_e^*, \sigma_e^2 T)$ given $\hat{\mu}_e^* = P_e R_f$. By comparing Equations (12) and (12′), one can conclude that they are identical if $\alpha T = P_e R_f$.

According to the above result, we are able to discuss a possible relationship between the abnormal return and the price of the firm-specific risk. For the market model, since the product of beta and the market risk premium captures the compensation for bearing the market risk, then alpha can be intuitively viewed as the compensation (and thus the price) for bearing the firm-specific risk. The above derivation exactly reflects this inference. In addition, this finding and explanation accommodate not only the market model but also the CAPM. Since the CAPM assumes that the firm-specific risk is not priced, i.e., $P_e = 0$, there should be no abnormal return α in the CAPM. As for the market model, a nonzero alpha is allowed, so the firm-specific risk may be or may be not priced depending on the regressed alpha value being nonzero or zero. Although the abnormal return α or equivalently the nonzero price P_e for the firm-specific risk is introduced in our option pricing model, we do not seek to disprove the CAPM. On the contrary, our model, just like the market model, can be utilized to attest the CAPM by examining the significance of the calibrated α based on option prices.

Lastly, regarding index options, since the firm-specific risks of individual stocks are fully diversified, there is no role for e_T and thus α should be ignored. Together with the definition of $\beta = 1$ for the market portfolio, the payoff of an index call option in Equation (4) reduces to $S_0 \max\left(M_T R_q^{-1} - k, 0\right)$, where M_T is the only remaining random variable. Later we will show that our option pricing formula reduces to the Black-Scholes formula given $\alpha = 0$, $\beta = 1$, and e_T fixed at 0.

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3.3 Deriving the Pricing Formula for European Calls

To develop the option pricing formula, we evaluate $V = R_f^{-1}E^Q\big(C(M_T, e_T)\big)$ or equivalently Eq. (12) and propose the following Theorem 2.

Theorem 2 (Market-model option pricing formula). *To deal with the max function in the payoff of the European call option, the option pricing formulas for β being positive, zero, or negative are considered separately. When $\beta > 0$,*

$$V = S_0 e^{-rT}\left[\left(e^{(r-q)T} - k + e^{-qT}\alpha T\right)N(M_1) + e^{-qT}\sigma_e\sqrt{T}n(-M_1)\right] +$$
$$S_0 e^{-rT}\int_{-\infty}^{e^{qT}k-(1-\beta)e^{rT}}\Big\{\{[(1-\beta)e^{rT} + Z_e]e^{-qT} - k\}N(D_2) +$$
$$\beta e^{(r-q)T}N(D_1)\Big\}\phi^*(Z_e)\,dZ_e; \tag{13}$$

when $\beta = 0$,

$$V = S_0 e^{-rT}\left[\left(e^{(r-q)T} - k + e^{-qT}\alpha T\right)N(M_2) + e^{-qT}\sigma_e\sqrt{T}n(-M_2)\right]; \tag{14}$$

and when $\beta < 0$,

$$V = S_0 e^{-rT}\int_{e^{qT}k-(1-\beta)e^{rT}}^{\infty}\Big\{\{[(1-\beta)e^{rT} + Z_e]e^{-qT} - k\}N(-D_2) +$$
$$\beta e^{(r-q)T}N(-D_1)\Big\}\phi^*(Z_e)\,dZ_e, \tag{15}$$

where

$$M_1 = \frac{\alpha T + (1-\beta)e^{rT} - e^{qT}k}{\sigma_e\sqrt{T}},\ \ M_2 = \frac{\alpha T + e^{rT} - e^{qT}k}{\sigma_e\sqrt{T}} = M_1 + \frac{\beta e^{rT}}{\sigma_e\sqrt{T}},$$

$$D_1 = \frac{-\ln\left(\frac{e^{qT}k-(1-\beta)e^{rT}-Z_e}{\beta}\right) + \left(rT + \frac{\sigma_m^2 T}{2}\right)}{\sigma_m\sqrt{T}},$$

$$D_2 = \frac{-\ln\left(\frac{e^{qT}k-(1-\beta)e^{rT}-Z_e}{\beta}\right) + \left(rT - \frac{\sigma_m^2 T}{2}\right)}{\sigma_m\sqrt{T}} = D_1 - \sigma_m\sqrt{T},$$

and $N(\cdot)$ and $n(\cdot)$ denote the cumulative distribution function and the

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probability density function of the standard normal distribution, respectively. In addition, the definitions of $R_f \equiv e^{rT}$ *and* $R_q \equiv e^{qT}$ *are introduced to further simplify the pricing formulas.*

***Proof*:** See Appendix 1.

Corollary 1 (Market-model option pricing for the market portfolio). *Option pricing on the market portfolio converges to the Black-Scholes formula. In pricing index options, one can set* $\beta = 1$ (*by definition*), $\alpha = 0$, *and* $\sigma_e = 0$ (*to reflect that in equilibrium, as the total firm-specific risk is fully diversified out, it is not necessary to consider the abnormal return*) *in Equations* (*4*) *and* (*13*). *Thus, the pricing formula for the market index call option is obtained as*

$$V = S_0 e^{-qT} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r-q)T + \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right) - K e^{-rT} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r-q)T - \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right). \quad (16)$$

***Proof*:** See Appendix 1.

The final implementation issue associated with Equation (13) (Equation (15)) is that its second (first) term should be evaluated with numerical techniques. Inspired by Andricopoulos, Widdicks, Duck, and Newton (2003), who develop option valuation models using quadrature methods, we use the Gaussian quadrature (GQ) method[4] to compute the integral over Z_e in Equations (13) and (15) to obtain a semi-analytical solution. Our computer program employs 600 points in the GQ method such that the generated option prices converge within 10^{-5}, which guarantees that option values generated through our program converge to their continuous limits. Moreover, our computer program is fast, taking less than 0.1 seconds to compute each option price.

4 The GQ method is a numerical method for approximating an integral of any function. As the convergence rate of the GQ method is theoretically faster than the rectangular rule, trapezoid rule, and Simpson's rule, we employ it to evaluate the integral in the proposed option pricing formulas numerically. For background knowledge about the GQ method, see Press, Teukolsky, Vetterling, and Flannery (1992).

4. Calibration for Option-Implied alpha, beta, and Firm-specific Risk

During the calibration process, we estimate σ_m in advance from the prices of the S&P 500 index options (SPX) on each trading day. Equipped with the value of σ_m and the prices of the equity options, we next calibrate the values of α, β, and σ_e for each individual stock on that trading day.

4.1 Calibration of Option-Implied $\sigma_m(t,T)$

It is commonly believed that the volatilities of financial assets are not constant for different times to maturity. Therefore, on each date t, we derive the whole term structure of the implied $\sigma_m(t,T)$ rather than a constant σ_m by respectively minimizing the option pricing errors of SPX index options for a different T:

$$\min_{\sigma_m(t,T)} \sum_{K_T} \left[\frac{\widehat{M}\big(T,K_T,S_m(t),R_f(t,T),\sigma_m(t,T),q_m(t)|\alpha=0,\beta=1,\sigma_e=0\big)-M_{T,K_T}}{M_{T,K_T}}\right]^2, \tag{17}$$

where M_{T,K_T} and $\widehat{M}(\cdot)$ denote the market prices and theoretical values generated by our option pricing model for SPX index options, respectively; $S_m(t)$ denotes the current index level of the S&P 500 index; K_T denotes the available strike prices of the SPX index options given a maturity date T; $R_f(t,T)$ is the gross risk-free rate from the current date t to the maturity date T. Note that when evaluating SPX index options with Equation (16), the dividend yield for each trading date t, $q_m(t)$, is estimated as the annualized average difference between the daily total returns and daily closing-price returns of the prior 252 trading days (about one calendar year).

4.2 Calibration of Option-Implied α, β, and σ_e

In calibrating $\alpha(t)$, $\beta(t)$, and $\sigma_e(t)$ for each date t, we minimize the squares of differences between the market and theoretical prices of the equity options across different strike prices and times to maturity, given $\sigma_m(t,T)$, at this time point; that is,

$$\min_{\alpha(t),\beta(t),\sigma_e(t)} \sum_T \sum_{K_T} \left[\frac{\hat{V}\big(T,K_T,S(t),R_f(t,T),\alpha(t),\beta(t),\sigma_e(t),q(t)|\sigma_m(t,T)\big)-V_{T,K_T}}{V_{T,K_T}}\right]^2, \tag{18}$$

where V_{T,K_T} and $\hat{V}(\cdot)$ denote the market prices and theoretical call prices of equity options generated by our model, $S(t)$ denotes the current stock price for the underlying stock on date t, T represents the different maturity dates of the equity option on date t, and K_T represents the different strike prices of equity options corresponding to a maturity date T. The dividend yield for each date t, $q(t)$, is estimated as the ratio of the sum of the cash dividends over the average stock price in the previous one calendar year (approximated by 252 trading days). Note that in order to obtain meaningful results for the three variables in the minimization problem in Equation (18), we limit ourselves to those trading dates with at least three qualified option prices.

There are a few miscellaneous details regarding solving Equation (18). First, we use the spline interpolation to generate $\sigma_m(t,T)$ so that its horizon exactly matches the time to maturity of the equity option contracts examined. Second, to solve the least squares problem in Equation (18), we combine a grid search method and a nonlinear least squares optimization procedure provided in Matlab.

We adopt a grid search method for $\beta(t)$[5] with increments of 0.01 in a proper range;[6] and for each value of $\beta(t)$ examined, the function **lsqnonlin** in Matlab is utilized to find the optimal values of $\alpha(t)$ and $\sigma_e(t)$ to minimize the least squares relative errors between the market and theoretical option prices. Finally, among all the examined values of $\beta(t)$, we choose that value which generates the smallest least squares errors. With the above process, we obtain the optimal $\alpha(t)$, $\beta(t)$, and $\sigma_e(t)$ on each date t. Appendix 2 presents an example illustrating the above calibration process to obtain the option-implied $\alpha(t)$, $\beta(t)$, and $\sigma_e(t)$ for a trading day.

5. Empirical Illustration

The aim of the empirical examination in this study is to illustrate the practical usefulness of the option implicit market model. A complete intensive empirical study on an extensive large-scale dataset is left as future work.

5.1 Data Description

Our empirical sample includes SPX index options and individual equity options; underlying each of these are the 21 component stocks of the S&P 500 index. Note that the chosen 21 blue-chip stocks are listed in the Dow Jones

5 Since the market model implies that the total variance can be decomposed into the systematic variance related to β^2 and the firm-specific variance, both positively increasing or negatively decreasing β values contribute to a larger total variance and thus a higher option price. Consequently, when solving Eq. (18), sometimes either a positive or a negative β value can generate similar option values. To avoid this problem, we identify a proper range for β according to its historical possible range and adopt the grid search for β.

6 For each date t, we derive the average value and standard deviation of the daily historical betas (calculated based on a rolling 21-trading-day window) over the prior 42 trading days to construct a proper range distribution of $\beta(t)$ for the grid search method. The upper (lower) bound of the range is set as the average value plus (minus) six times the standard deviation. Moreover, if the range of six standard deviations is too narrow (< 0.7) or too wide (> 1.2), the value 0.7 or 1.2 is used to replace the six standard deviations to generate a proper range for $\beta(t)$.

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Industrial Average (DJIA) index continuously from 2008 to 2013[7] because they have larger trading volumes and sufficient liquidity for their equity (options) trading than other S&P 500 component stocks. We examine the period of 2008-2013 for the following reason: since option traders are typically regarded as skilled or professional traders who would thus be expected to behave more rationally during the crisis period (2008-2010), our option implicit market model can test this argument by examining whether the performance of the option-implied estimates of α, β, and σ_e in this abnormal period is superior than that in a normal period, such as the three years following the crisis. To ensure a fair comparison, the same time length (three years) is considered for the abnormal period as well as the normal period.

The daily data for the option prices are collected from the OptionMetrics database. Only call options are considered,[8] and the middle prices of the bid and ask quotes are used as the market prices for each option contract. Since the component equity options traded on the Chicago Board Options Exchange are American-style options, we convert these American option prices into their European counterparts using the binomial tree model of Cox, Ross, and Rubinstein (1979). The continuously compounding Treasury zero rates with different days to maturity provided by the OptionMetrics database are employed to approximate risk-free interest rates.

7 For the historical changes in the composition of DJIA, see https://en.wikipedia.org/wiki/Historical_components_of_the_Dow_Jones_Industrial_Average. From 2008 to 2013, 23 stocks are continuously listed in the DJIA. However, as some data is missing in publicly-accessible databases, Coca Cola and DuPont are not included in our empirical sample.

8 Since the novelty of this paper is to propose the option implicit market model to provide forward-looking estimations of α, β, and σ_e based solely on the closing prices of the equity and index options on a trading day, our empirical studies play an auxiliary role on illustrating how to use this model given empirical data and showing the potential of this model to generate reliable estimates for subsequent horizons. Therefore, we follow Bakshi, Cao, and Chen (1997), Eraker (2004), Christoffersen, Heston, and Jacobs (2009), and Yun (2011) to achieve our goals based only on call options.

In our empirical sample, the option contracts are screened according to the following criteria: (1) we examine both equity and index option contracts with moneyness (K/S_0) in [0.75, 1.25]; (2) we filter out the market prices smaller than and equal to the minimum tick, \$0.25, because often the true values of these options are expected to be lower than \$0.25; (3) we eliminate the option contracts if their implied volatilities based on the Black-Scholes model do not exist; and (4) we discard the option market prices resulting in obvious arbitrage opportunities based on the butterfly or bull/bear spreads (ignoring transaction costs). Finally, we estimate forward-looking α, β, and σ_e for the subsequent 21 trading days (about one calendar month), since it is common for many fund managers and security traders to utilize one-month-ahead α, β, and σ_e in their investment decisions. To achieve this, we examine only equity options with times to maturity between 10 and 50 calendar days.[9] As for the SPX index options, we examine maturity date T between 2 and 180 calendar days for $\sigma_m(t,T)$ to guarantee that the interpolated $\sigma_m(t,T)$ with matched times to maturity for the examined equity options can be obtained. Table 1 reports the number of qualified call options for each stock in our empirical sample and the average days to maturity of the examined equity option contracts. The average times to maturity of the equity options in our sample are around 30 calendar days (approximately 21 trading days).

9 Another concern about excluding option quotes with longer times to maturity is their quality deficiency. First, those options with non-zero open interests concentrate on only one or two strike prices, which may skew option-implied estimates. Second, the sparsity of option contracts in the maturity dimension adds noise to the option-implied estimates. Take the BA (Boeing Company) stock options on January 2, 2008 for example. All available times to maturity on this day include 17, 45, 136, 227, 381, 745 calendar days. Our model utilizes the option contracts with 17 and 45 (227 and 381) days to maturity to calibrate option-implied estimates for the following one month (year). The differences between 227/381 and 365 are significantly larger than those between 17/45 and 30, which significantly affects the quality of the option-implied estimates.

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Table 1. Number of Quotes and Average Days to Maturity of Qualified Call Options, 2008-2013

Call options of S&P 500 components	Number of qualified quotes	Average time to maturity (number of calendar days)
American Express Company (AXP)	16,525	30.4
Boeing Company (BA)	19,795	28.6
Caterpillar Incorporated (CAT)	18,255	30.1
Walt Disney Company (DIS)	15,154	29.9
General Electric Company (GE)	9,856	29.9
Home Depot Incorporated (HD)	14,252	30.0
International Business Machines (IBM)	20,043	29.0
Intel Corporation (INTC)	8,975	31.8
Johnson & Johnson (JNJ)	9,030	29.5
JPMorgan Chase (JPM)	21,187	29.9
McDonald’s Corporation (MCD)	14,506	29.4
3M Company (MMM)	13,926	28.6
Merck & Company, Incorporated (MRK)	14,273	30.2
Microsoft Corporation (MSFT)	13,575	30.2
Pfizer Incorporated (PFE)	8,589	29.3
Procter & Gamble Company (PG)	14,014	28.6
AT&T Incorporated (T)	10,170	31.3
United Technologies Corporation (UTX)	13,728	29.0
Verizon Communications Inc. (VZ)	13,771	29.9
Wal-Mart Stores Incorporated (WMT)	13,080	28.9
Exxon Mobil Corporation (XOM)	12,295	30.2

This table reports the number of quotes and average days to maturity of the qualified call options of each examined stock in this paper.

5.2 Calibration Performance

Table 2 presents the calibration performance of our market-model option pricing model and the Black-Scholes model in terms of daily squared root of mean squared percentage errors (RMSEs). To obtain the RMSE of our model on each examined date, we calculate the square root of the ratio of the minimized values in Equation (18) over the number of examined option contracts on that date. As for the Black-Scholes model, we first obtain one implied volatility value $\sigma(t)$ by minimizing the squared percentage pricing errors of the Black-Scholes model for all examined option contracts on each trading day t, i.e.,

$$\min_{\sigma(t)} \sum_T \sum_{K_T} \left[\frac{\widehat{BS}\left(T, K_T, S(t), R_f(t,T), \sigma(t), q(t)\right) - V_{T,K_T}}{V_{T,K_T}} \right]^2,$$

where the definitions of all variables are the same as those in Equation (18), $\widehat{BS}\left(T, K_T, S(t), R_f(t,T), \sigma(t), q(t)\right)$ returns the theoretical Black-Scholes option value, and $r = \frac{\ln R_f(t,T)}{T-t}$ is employed as the input for the Black-Scholes model. Then the RMSE of the Black-Scholes model on each examined date is calculated as the square root of the ratio of the minimized values in the above equation over the number of examined option contracts on that date.

For each stock, the reported calibration error is computed as the average of the daily RMSEs in the examined period. Generally speaking, our market-model option pricing model generates option values which are very close to actual market prices. The average percentage errors are 2.27%, 3.00%, and 2.64% in 2008-2010, 2011-2013, and 2008-2013, respectively. A comparison of Panels (a) and (b) in Table 2 reveals that the calibration errors of our model are smaller than those of the Black-Scholes model in all cases except for the case of VZ in 2012. For the Black-Scholes model, the averages of calibration RMSEs across all stocks are 5.63%, 5.82%, and 5.72% in 2008-2010, 2010-2013, and 2008-2013, respectively. It is worth noting that the accurate fitting performance demonstrates the appropriateness of incorporating the market model with multivariate normal or lognormal distributions into the RNVR framework. Moreover, the excellent fitting performance ensures that our option implicit market model delivers reasonable estimates of alpha, beta, and firm-specific risk based on option prices.

Table 2. Calibration Performance of Market-Model Option Pricing Model and Black-Scholes Option Pricing Model

Panel (a) Average RMSEs in different periods based on proposed market-model option pricing model

Average RMSEs (%)	2008	2009	2010	2008-2010	2011	2012	2013	2011-2013	2008-2013
AXP	2.47	1.90	2.52	2.31	2.41	2.04	2.12	2.19	2.25
BA	3.33	1.98	1.77	2.36	2.67	3.99	4.39	3.68	3.03
CAT	4.19	1.73	2.07	2.66	3.43	4.23	4.58	4.08	3.37
DIS	2.15	2.01	2.66	2.28	2.25	3.44	3.65	3.11	2.71
GE	2.40	1.47	1.03	1.68	2.88	1.08	2.13	2.02	1.85
HD	1.48	2.22	3.29	2.35	1.99	4.71	2.82	3.16	2.76
IBM	4.10	2.97	2.44	3.17	3.34	4.52	11.26	6.38	4.77
INTC	2.37	2.41	1.87	2.22	2.13	2.55	2.87	2.51	2.36
JNJ	2.05	1.39	1.63	1.69	2.62	1.69	5.59	3.36	2.58
JPM	2.02	1.89	2.12	2.01	2.25	2.09	2.66	2.33	2.18
MCD	3.49	2.52	2.00	2.68	3.17	2.09	3.85	3.05	2.86
MMM	2.50	1.60	1.63	1.90	2.79	2.52	3.47	2.92	2.42
MRK	3.30	2.20	3.19	2.89	1.94	1.86	3.33	2.38	2.64
MSFT	1.90	1.15	1.73	1.59	2.08	1.90	3.29	2.43	2.00
PFE	1.92	0.96	2.08	1.58	1.23	1.19	1.65	1.37	1.46
PG	2.79	2.36	1.58	2.25	1.67	1.28	2.54	1.84	2.05
T	2.93	2.44	1.80	2.42	1.39	2.33	2.11	1.94	2.18
UTX	2.61	2.10	1.55	2.09	2.16	2.41	3.30	2.63	2.36
VZ	2.39	2.52	1.20	2.05	2.12	8.82	4.40	5.13	3.63
WMT	3.24	1.68	1.30	2.09	1.52	4.11	4.40	3.36	2.72
XOM	3.72	3.14	3.27	3.38	2.59	2.36	4.48	3.14	3.26
Average	2.73	2.03	2.03	2.27	2.32	2.91	3.76	3.00	2.64

Panel (b) Average RMSEs in different periods based on Black-Scholes option pricing model

Average RMSEs (%)	2008	2009	2010	2008-2010	2011	2012	2013	2011-2013	2008-2013
AXP	7.23	6.71	6.12	6.66	6.22	4.57	4.14	4.98	5.79
BA	5.25	5.26	5.38	5.30	6.56	5.97	5.69	6.07	5.69
CAT	6.01	5.46	5.63	5.70	6.60	6.15	4.96	5.90	5.80
DIS	4.86	4.90	5.13	4.96	6.09	5.67	5.50	5.75	5.37
GE	6.14	5.33	4.13	5.27	4.97	3.88	4.62	4.48	4.87
HD	5.53	5.90	5.66	5.70	5.94	4.91	4.99	5.28	5.49
IBM	8.28	6.63	5.10	6.67	7.37	7.47	12.47	9.11	7.89
INTC	5.64	5.11	4.97	5.24	4.78	5.09	5.33	5.06	5.15
JNJ	5.08	4.32	5.02	4.83	6.54	5.21	5.68	5.82	5.36
JPM	8.49	6.55	5.56	6.85	6.43	5.01	4.71	5.39	6.11
MCD	6.70	5.30	5.02	5.68	5.73	4.95	6.33	5.68	5.68
MMM	6.18	5.88	5.07	5.70	7.22	6.83	6.36	6.80	6.26
MRK	5.00	5.95	6.34	5.77	6.56	5.87	5.89	6.10	5.93

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MSFT	4.76	3.42	3.56	3.91	4.29	3.84	4.68	4.27	4.09
PFE	4.66	4.56	3.81	4.26	4.87	3.87	4.20	4.28	4.27
PG	6.44	4.87	5.20	5.51	5.96	4.97	5.91	5.61	5.56
T	6.17	7.00	7.30	6.80	6.77	6.91	6.02	6.56	6.68
UTX	5.98	5.59	4.89	5.49	7.31	6.32	6.66	6.77	6.14
VZ	6.38	6.69	8.57	7.20	8.75	6.70	5.82	7.07	7.13
WMT	6.25	4.26	4.05	4.87	4.84	4.41	5.35	4.87	4.87
XOM	7.01	5.66	4.83	5.85	6.72	5.71	6.41	6.28	6.07
Average	6.10	5.49	5.30	5.63	6.22	5.44	5.80	5.82	5.72

This table reports the calibration errors based on our market-model option pricing model (Panel (a)) and the Black-Scholes model (Panel (b)). The errors in each period are measured as the averages of the daily square roots of mean square errors (RMSEs). For our market-model option pricing model, RMSEs are calculated according to the minimization results in Equation (18). For the Black-Scholes model, we first calibrate one implied volatility value by minimizing the squared percentage pricing errors of the Black-Scholes model for all examined option contracts on a trading day and then calculate the corresponding RMSE on that trading day. The calibration errors of our market-model option pricing model are smaller than those of the Black-Scholes model in all cases except for the case of VZ in 2012.

Note that the small calibration errors also suggest that the commonly found volatility skew for equity options is partially explained by our market-model option pricing formula, since there is a tight relationship between an option price and its Black-Scholes implied volatility. In the illustrative example in Appendix 2, the implied volatilities of the option values generated by our option pricing formula and the market option prices exhibit similar skewed patterns across different strike prices. Although it is not our intention to develop an option pricing model to account for volatility skew, this desired byproduct enhances the rationality of the incorporation of the market model into the option pricing formula.

Several reasons are presented to explain the superiority of the market-model option pricing formula over the Black-Scholes formula in describing the option price behavior. First of all, we believe that calibrating prices of options with different strike prices could help to identify the volatility of the idiosyncratic term (i.e., the firm-specific risk); at the same time, alpha could be identified as the corresponding market price of risk of the firm-specific risk according to our analysis in Section 3.2. Concerning beta, the following two criteria could be implicitly matched when determining its value in the optimization process. First, beta and firm-specific risk co-determine the total (implied) variance of an individual stock, which equals the sum of the systematic risk ($\beta^2\sigma_m^2$) and the firm-specific risk (σ_e^2) according to the market model. Second, as shown in the proposed option pricing formula in Theorem 2, beta and alpha co-determine the in-the-money probability of options. Since the above two criteria are well known to be critical in evaluating option values, determining a proper value of beta could significantly enhance the calibration performance for option prices.

5.3 Empirical Results

This section reports the estimation results and the prediction performance of our option-implied α, β, and σ_e. Since our model obtains a set of option-implied estimates (applied in the subsequent one month) based on options prices with different strike prices for a single trading day, to examine this special feature, we here adopt a daily rather than monthly frequency for the empirical illustration. On each trading date, to analyze the prediction ability of our option-implied estimates, the comparison benchmarks of the α, β, and σ_e are obtained by performing the regression between the daily stock and index excess returns on the subsequent 21 trading days according to the market model.[10] In

10 In estimating the realized estimates on a trading day t, the excess daily returns on the subsequent 21 days (including the examined day t) are employed to estimate the realized α, β, and σ_e; i.e., we regress

$$r_{i,t+d} - \frac{r_{f,t+d}}{252} = \alpha_{real} + \beta_{real}\left(r_{m,t+d} - \frac{r_{f,t+d}}{252}\right) + \varepsilon, \text{ for } d = 0,1,\ldots,20,$$

where $r_{i,t}$ and $r_{m,t}$ are the one-day equity and index total returns for day t (collected from the Yahoo! Finance website and Datastream, respectively), $r_{f,t}$ is the prevailing one-month Treasury zero rate on day t (collected from OptionMetrics), and ε follows a normal distribution with zero mean and standard deviation σ_{real}. Given the estimates of $\hat{\alpha}_{real}$, $\hat{\beta}_{real}$, and $\hat{\sigma}_{real}$, the reported realized α, β, and σ_e on day t are $252\hat{\alpha}_{real}$, $\hat{\beta}_{real}$, and $\sqrt{252}\hat{\sigma}_{real}$, respectively. Similarly, based on the excess daily returns on the prior 21 days, the reported historical α, β, and σ_e on day t are respectively $252\hat{\alpha}_{hist}$, $\hat{\beta}_{hist}$, and $\sqrt{252}\hat{\sigma}_{hist}$, where $\hat{\alpha}_{hist}$, $\hat{\beta}_{hist}$, and $\hat{\sigma}_{hist}$ are obtained by regressing

$$r_{i,t-d} - \frac{r_{f,t-d}}{252} = \alpha_{hist} + \beta_{hist}\left(r_{m,t-d} - \frac{r_{f,t-d}}{252}\right) + \epsilon, \text{ for } d = 1,2,\ldots,21,$$

where ϵ follows a normal distribution with zero mean and standard deviation σ_{hist}. Note that $\hat{\alpha}_{real}$ and $\hat{\sigma}_{real}$ ($\hat{\alpha}_{hist}$ and $\hat{\sigma}_{hist}$) must to be annualized such that the realized and historical α and σ_e are comparable to the α and σ_e of our option-implicit market model specified in Equation (4).

this paper, we call these benchmarks the realized α, β, and σ_e.

In the literature, almost no attempts have been made to derive approximations for actual alpha and firm-specific risk, although there are two common comparison benchmarks to approximate the actual β for a subsequent period of time. One follows the definition of beta in the CAPM and the other applies the concept of realized (co)variance to generate the approximation of actual beta.[11,12] We emphasize that since our option implicit market model generates option-implied forecasts for not only β but also α and σ_e, it is not fair to compare individually our option-implied β with the above two comparison benchmarks for actual β or other papers which estimate only β for a subsequent period of time. To fully examine the proposed model, our choices are limited to evaluating the prediction performance by comparing our option-implied α, β, and σ_e with the realized α, β, and σ_e. In addition, the historical regression results of α, β, and σ_e for the prior 21 trading days are also employed as a competitor of our model for predicting the realized α, β, and σ_e.

5.3.1 Estimation Means and Standard Deviations and Tests of CAPM

Table 3 reports the means of daily option-implied, realized, and historical α, β, and σ_e for each stock in the period from 2008 to 2013. In addition, the corresponding t-statistic is also presented in parentheses under each mean value. Since overlapping samples of stock returns are employed to estimate realized and historical estimates, the Newey-West method is employed to calculate the standard errors of the means of realized and historical α and β. The mean and corresponding t-statistic in boldface indicate 5% significance.

11 For details on this realized (co)variance approach, refer to Andersen, Bollerslev, Diebold, and Wu (2006).

12 For example, Blume (1971), French, Groth, and Kolari (1983), Braun, Nelson, Sunier (1995), Armitage and Brzeszczynski (2011), and Bali, Engle, and Tang (2013) employ the first type of benchmark to approximate the actual beta; Buss and Vilkov (2012), Chang, Christoffersen, Jacobs, and Vainberg (2012), Baule, Korn, and Saβning (2016), Hollstein and Prokopczuk (2016), and Hollstein, Prokopczuk, and Simen (2019) employ the second type of benchmark to approximate the actual beta.

Table 3. Means of Option-implied, Realized, and Historical α, β, and σ_e, 2008-2013

Mean (t-statistic)	Option-implied alpha	Option-implied beta	Option-implied sigma_e	Realized alpha	Realized beta	Realized sigma_e	Historical alpha	Historical beta	Historical sigma_e
AXP	0.089	**1.382**	0.183	0.095	**1.373**	0.214	0.089	**1.387**	0.220
	(0.860)	**(4.141)**	(1.774)	(1.643)	**(9.104)**	(1.620)	(1.392)	**(8.756)**	(1.594)
BA	0.066	**1.100**	**0.182**	0.074	**1.027**	**0.203**	0.063	**1.033**	**0.204**
	(0.856)	**(4.959)**	**(2.363)**	(1.115)	**(24.652)**	**(2.421)**	(0.960)	**(23.653)**	**(2.443)**
CAT	0.063	**1.384**	**0.196**	-0.032	**1.324**	**0.204**	-0.038	**1.324**	**0.204**
	(0.622)	**(5.316)**	**(2.326)**	-(0.380)	**(16.331)**	**(2.153)**	-(0.454)	**(16.414)**	**(2.164)**
DIS	0.067	**1.156**	**0.149**	**0.094**	**1.036**	**0.165**	0.085	**1.031**	**0.164**
	(0.978)	**(5.284)**	**(2.324)**	**(2.014)**	**(26.782)**	**(2.274)**	(1.850)	**(25.398)**	**(2.296)**
GE	0.075	**1.323**	0.149	-0.123	**1.175**	0.192	-0.081	**1.182**	0.191
	(0.670)	**(4.727)**	(1.234)	-(1.693)	**(30.592)**	(1.333)	-(1.236)	**(30.255)**	(1.342)
HD	0.081	**1.061**	**0.165**	**0.199**	**0.913**	**0.186**	**0.207**	**0.917**	**0.189**
	(1.068)	**(3.716)**	**(1.991)**	**(3.067)**	**(26.672)**	**(2.232)**	**(3.158)**	**(27.018)**	**(2.239)**
IBM	0.057	**0.866**	**0.125**	0.087	**0.741**	**0.143**	0.090	**0.742**	**0.144**
	(0.893)	**(4.487)**	**(2.068)**	(1.655)	**(33.122)**	**(2.115)**	(1.392)	**(32.635)**	**(2.112)**
INTC	0.073	**1.170**	**0.181**	0.032	**1.009**	**0.195**	0.027	**1.010**	**0.195**
	(1.260)	**(3.992)**	**(2.631)**	(0.516)	**(18.546)**	**(2.628)**	(0.421)	**(16.994)**	**(2.589)**
JNJ	0.064	**0.634**	0.088	0.061	**0.526**	**0.107**	0.066	**0.520**	**0.105**
	(1.133)	**(2.615)**	(1.712)	(1.707)	**(12.571)**	**(2.079)**	(1.860)	**(14.106)**	**(2.038)**
JPM	0.093	**1.478**	0.203	-0.004	**1.556**	0.256	0.013	**1.571**	0.260
	(0.859)	**(4.783)**	(1.686)	-(0.037)	**(14.754)**	(1.393)	(0.133)	**(14.575)**	(1.389)
MCD	0.071	**0.625**	**0.124**	**0.126**	**0.524**	**0.139**	**0.122**	**0.519**	**0.142**
	(1.021)	**(2.974)**	**(2.687)**	**(2.321)**	**(23.214)**	**(2.405)**	**(2.327)**	**(22.974)**	**(2.399)**
MMM	0.072	**0.997**	0.102	0.034	**0.919**	0.126	0.027	**0.913**	0.126
	(1.114)	**(6.276)**	(1.722)	(0.895)	**(41.894)**	(1.846)	(0.753)	**(41.260)**	(1.853)
MRK	0.063	**0.844**	0.152	0.037	**0.674**	0.191	0.023	**0.683**	0.191
	(0.820)	**(3.118)**	(1.711)	(0.603)	**(24.643)**	(1.654)	(0.366)	**(26.316)**	(1.661)
MSFT	0.050	**1.044**	**0.171**	-0.010	**0.917**	**0.196**	0.005	**0.916**	**0.196**
	(0.726)	**(4.005)**	**(2.129)**	-(0.166)	**(35.998)**	**(2.043)**	(0.075)	**(36.146)**	**(2.041)**
PFE	0.066	**0.910**	**0.141**	0.046	**0.759**	**0.157**	0.054	**0.750**	**0.159**
	(1.100)	**(3.877)**	**(2.312)**	(0.888)	**(19.159)**	**(2.124)**	(1.062)	**(21.070)**	**(2.154)**
PG	0.059	**0.643**	**0.110**	0.004	**0.544**	**0.130**	0.011	**0.541**	**0.130**
	(0.956)	**(2.587)**	**(2.258)**	(0.084)	**(11.770)**	**(2.369)**	(0.242)	**(11.634)**	**(2.385)**
T	0.092	**0.841**	0.120	0.017	**0.697**	**0.145**	0.005	**0.708**	**0.149**
	(1.165)	**(3.972)**	(1.881)	(0.280)	**(19.245)**	**(2.113)**	(0.089)	**(20.425)**	**(2.145)**
UTX	0.075	**1.070**	**0.111**	0.032	**1.012**	**0.136**	0.030	**1.012**	**0.136**
	(1.037)	**(6.985)**	**(2.123)**	(0.830)	**(36.348)**	**(2.359)**	(0.774)	**(37.611)**	**(2.392)**
VZ	0.099	**0.763**	**0.131**	0.089	**0.646**	**0.157**	0.060	**0.646**	**0.159**
	(1.311)	**(2.903)**	**(2.207)**	(1.490)	**(19.314)**	**(2.144)**	(1.025)	**(19.224)**	**(2.146)**
WMT	0.057	**0.570**	**0.135**	**0.098**	**0.454**	**0.149**	**0.113**	**0.455**	**0.150**
	(0.915)	**(2.068)**	**(2.405)**	**(2.060)**	**(13.396)**	**(2.279)**	**(2.364)**	**(14.199)**	**(2.297)**
XOM	0.067	**0.926**	0.107	0.014	**0.859**	0.133	0.019	**0.852**	0.133
	(0.694)	**(5.377)**	(1.702)	(0.191)	**(31.434)**	(1.729)	(0.248)	**(31.373)**	(1.731)

For each stock, this table reports the means and their corresponding t-statistics of the daily option-implied, realized, and historical α, β, and σ_e for the whole period of 2008-2013. Since the realized and historical estimates are generated with overlapping samples, the Newey-West method is utilized to adjust the standard errors of the realized and historical α and β. For other estimates, their standard errors are approximated by the standard deviations of their sampling distributions. Boldface figures indicate 5% significant results.

We first observe that the means of option-implied estimates are close to those of realized estimates. Across the 21 stocks, the averages of the differences between the means of the option-implied and realized α, β, and σ_e are 0.025, 0.100, and -0.024, respectively. Second, although their means are at similar levels, the standard deviations of the option-implied estimates (not reported in Table 3) are in general smaller than those of the realized and historical estimates, particularly for the option-implied α. Third, by analyzing t-statistics, one can infer common phenomena among option-implied, realized, and historical estimates: the estimated β's are significant with large t-values for all stocks, with the significant estimated σ_e's for fewer cases, whereas the estimated α's rarely differ significantly from 0.[13] Significant β and insignificant α based on our option implicit market model verify that the CAPM holds in 21-trading-day periods even when using the forward-looking option-implied estimates of α, β, and σ_e. These results are consistent with those based on the market model and can be anticipated: if a significantly nonzero alpha were to exist for any of the 21 stocks, which are perhaps those stocks that catch the most attention of traders worldwide, it would no doubt be exploited by traders to make a profit and thus bring the nonzero alpha back to a near-zero level. Finally, since there is only a 21-trading-day lag between the realized and historical estimates, it is not surprising that the means and corresponding t-statistics of the realized and

13 Equipped with the regression-based realized and historical estimates on each trading day, similar phenomena for any examined stock can be obtained by counting the frequency of the dates that yield significant α and β: β is significant for almost all dates, but it is rare to observe a significant α. In our data set, the significance frequency of α is usually less than 5% of the total examined trading days, regardless of different stocks or examined periods.

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historical estimates are close.[14]

Moreover, Figure 1 cross-sectionally compares the means of option-implied and realized estimates based on the results in Table 3, by depicting the scatter diagrams for the means of the realized α, β, and σ_e versus the means of the option-implied α, β, and σ_e of the 21 stocks from 2008 to 2013. In each diagram, we also report the least-squares linear regression line among the scatter points of different stocks. First, we observe in Panels (b) and (c) that β and σ_e generated by our option-implicit market model are on average close to the realized β and σ_e in the subsequent one month for different stocks, because the slope coefficient and R-squared value for the beta (sigma_e) analysis in Panel (b) (Panel (c)) are 1.018 and 0.9713 (1.0691 and 0.9213), respectively, both of which are close to unity. Second, in contrast, the means of option-implied α do not match the means of one-month-ahead realized α well across different stocks. The slope coefficient and R-squared value for the alpha analysis in Panel (a) are 0.6122 and 0.0149, both of which deviate from 1. We attribute these mediocre results to the fact that most of the means of the realized α and all of the means of our option-implied α are insignificant for different stocks in Table 3. Consequently, the low R-squared value for the regression line in Panel (a) of Figure 1 is not surprising.

14 Recall that for each trading day t, the daily excess returns on the subsequent (previous) 21 days are employed to estimate the realized (historical) α, β, and σ_e based on the market model. Hence the historical α, β, and σ_e on trading day $t+21$ are exactly the same as the realized α, β, and σ_e on trading day t. When calculating the average differences between the realized and historical α, β, and σ_e over any period, only those cases on the first and last 21 trading days of the examined period contribute discrepancies; thus the average differences between the historical and realized α, β, and σ_e are extremely small. However, even with these small average differences, we will show that the historical market model is actually inferior to our option implicit market model in predicting the realized α, β, and σ_e.

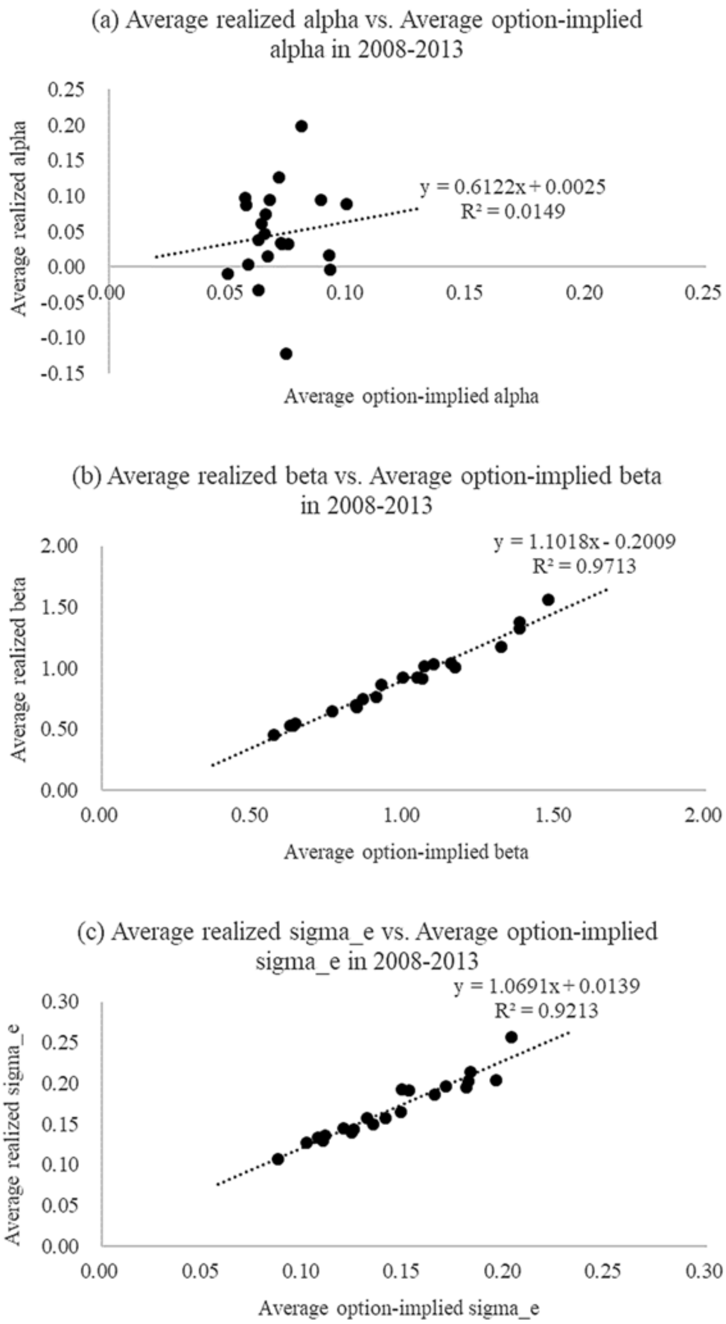

Figure 1. Scatter Diagrams for Averages of Realized Estimates versus Averages of Option-Implied Estimates in 2008-2013. The three diagrams are depicted based on the results in Table 3.

However, comparing the means of the option-implied (or historical) estimates with those of the realized estimates is not appropriate when examining the forecasting ability of the option-implied estimates, which is of most concern to us here. Therefore, in the next subsection, we measure the prediction performance of our model by calculating the MAE (mean of absolute errors) of the daily difference between our option-implied estimates with the realized estimates for the whole period in addition to several subperiods.

5.3.2 Tests of Prediction Ability

Tables 4 to 6 present the MAE analyses when using our option-implied estimates (or the historical estimates) to predict the realized α, β, and σ_e. A smaller MAE indicates a superior model to predict the dynamic variation in α, β, or σ_e on the following 21 trading days. We also conduct a t-test with Newey-West adjustment to statistically compare the MAEs of the option-implied and historical estimates for different periods of time. A negative (positive) t-statistic indicates a smaller MAE and thus superior prediction power of the option-implied (historical) estimate, and the boldface figures indicate the 5%-significance t-statistic results. Note that the 21 stocks are sorted in Tables 4 to 6 according to their numbers of qualified option quotes, as we will show later that the prediction powers of the option-implied estimates are more satisfactory for stocks with more qualified option quotes.

In Table 4, we observe that our option-implied α are superior to the historical α in predicting the realized α because all of the MAEs of our option-implied α (versus the realized α) are smaller than the corresponding historical α across all examined stocks in any examined period. In fact, the time series of the option-implied α is more stable than that of the historical α in all cases due to the smaller standard deviations of absolute errors between the option-implied and realized α. For the respective periods of 2008-2010, 2011-2013, or 2008-2013, all t-statistics are negatively significant in Table 4,

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which shows that the option-implied α dominates the historical α in predicting the future α in both crisis and normal periods.

For Table 5, several phenomena are worth discussing. First, the option-implied β shows a slightly better prediction ability than the historical β from 2008 to 2013. The MAEs of the option-implied β (versus the realized β) are smaller than those of the historical β in 11 of the 21 stocks. Among the 11 (10) stocks for which the option-implied (historical) β generates smaller MAEs, 7 (5) are 5% significant.

Second, the prediction ability of our option-implied β dominates that of the historical β in the normal market condition from 2011 to 2013. Specifically, our option-implied β performs better for 15 of the 21 stocks in terms of smaller MAEs, and 9 of these 15 stocks generate significantly negative t-statistic results from 2011 to 2013.

Third, the option-implied β does not necessarily perform better than the historical β in predicting realized β in the crisis period from 2008 to 2010. For our model, although 12 of the 21 stocks generate smaller MAEs for our option-implied β, only 2 of these are significant. However, in comparison to the satisfactory prediction ability of our model in the normal market condition from 2011 to 2013, we would rather attribute the mediocre performance of our model to the inability of (professional) option traders to predict the future in an unprecedented crisis than to the existence of structural model problems. In other words, our results do not support the hypothesis that option traders possessed more information during the crisis period in 2008-2010. Another point worth emphasizing is that since the option-implied estimates exhibit superior forecasting ability in the normal period as compared to the crisis period, results that are more favorable to us could be obtained by including more subsequent normal years in the empirical sample. To ensure fair comparisons, we thus constrain ourselves to examining only the three-year crisis period and the subsequent three-year normal period.

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Table 4. Mean of Absolute Error (MAE) Analyses for Predicting Realized α

Mean (s.d.) of AE vs realized alpha	JPM		IBM		BA		CAT		AXP		DIS		MCD		MRK		HD		PG		MMM	
	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha
2008	1.55	2.48	0.66	0.78	0.80	1.28	0.86	1.06	0.76	1.14	0.51	0.84	0.44	0.56	0.83	1.30	0.89	1.45	0.51	0.71	0.49	0.83
	(1.52)	(2.03)	(0.54)	(0.58)	(0.51)	(0.84)	(0.84)	(1.22)	(0.61)	(0.77)	(0.39)	(0.63)	(0.28)	(0.48)	(0.64)	(0.82)	(0.65)	(1.03)	(0.35)	(0.52)	(0.51)	(0.78)
2009	0.67	1.00	0.36	0.58	0.86	1.29	0.92	1.47	0.81	1.48	0.56	0.84	0.42	0.54	0.66	1.07	0.59	0.83	0.56	0.75	0.36	0.58
	(0.52)	(1.01)	(0.41)	(0.47)	(0.60)	(1.00)	(0.80)	(1.22)	(0.88)	(1.50)	(0.44)	(0.60)	(0.31)	(0.48)	(0.55)	(0.78)	(0.50)	(0.68)	(0.38)	(0.49)	(0.32)	(0.44)
2010	0.44	0.65	0.28	0.42	0.52	0.76	0.48	0.67	0.58	0.80	0.37	0.55	0.31	0.37	0.63	0.86	0.63	0.77	0.28	0.49	0.35	0.57
	(0.37)	(0.48)	(0.25)	(0.34)	(0.38)	(0.56)	(0.30)	(0.45)	(0.38)	(0.55)	(0.25)	(0.41)	(0.25)	(0.31)	(0.39)	(0.50)	(0.45)	(0.53)	(0.19)	(0.31)	(0.26)	(0.36)
2008-2010	0.88	1.37	0.43	0.59	0.73	1.11	0.75	1.07	0.71	1.13	0.48	0.74	0.39	0.49	0.71	1.08	0.70	1.01	0.45	0.65	0.40	0.66
	(1.06)	(1.55)	(0.45)	(0.50)	(0.53)	(0.85)	(0.72)	(1.08)	(0.65)	(1.04)	(0.37)	(0.57)	(0.28)	(0.44)	(0.54)	(0.73)	(0.55)	(0.82)	(0.34)	(0.47)	(0.38)	(0.57)
t-stat. for diff. btwn. AEs (2008-2010)	**-5.30**		**-5.14**		**-7.48**		**-4.51**		**-6.28**		**-6.65**		**-3.02**		**-7.93**		**-5.26**		**-6.25**		**-8.72**	
2011	0.51	0.62	0.41	0.72	0.37	0.55	0.67	0.90	0.42	0.62	0.43	0.52	0.35	0.41	0.41	0.57	0.39	0.48	0.33	0.52	0.31	0.39
	(0.46)	(0.55)	(0.34)	(0.45)	(0.27)	(0.38)	(0.42)	(0.70)	(0.29)	(0.44)	(0.30)	(0.38)	(0.25)	(0.31)	(0.37)	(0.48)	(0.31)	(0.41)	(0.21)	(0.34)	(0.26)	(0.31)
2012	0.57	0.84	0.32	0.49	0.43	0.52	0.69	0.85	0.31	0.40	0.31	0.46	0.28	0.44	0.43	0.61	0.44	0.54	0.32	0.48	0.17	0.26
	(0.57)	(0.76)	(0.23)	(0.43)	(0.28)	(0.38)	(0.46)	(0.64)	(0.23)	(0.31)	(0.31)	(0.37)	(0.23)	(0.34)	(0.29)	(0.45)	(0.23)	(0.43)	(0.24)	(0.40)	(0.11)	(0.18)
2013	0.39	0.57	0.40	0.61	0.53	0.60	0.46	0.56	0.28	0.41	0.36	0.51	0.30	0.42	0.42	0.67	0.27	0.43	0.33	0.51	0.20	0.29
	(0.28)	(0.41)	(0.32)	(0.41)	(0.34)	(0.42)	(0.37)	(0.40)	(0.22)	(0.28)	(0.26)	(0.35)	(0.20)	(0.31)	(0.33)	(0.52)	(0.21)	(0.32)	(0.26)	(0.40)	(0.16)	(0.21)
2011-2013	0.49	0.68	0.38	0.61	0.45	0.56	0.61	0.77	0.34	0.48	0.37	0.49	0.31	0.42	0.42	0.62	0.37	0.48	0.33	0.50	0.23	0.31
	(0.46)	(0.60)	(0.30)	(0.44)	(0.30)	(0.40)	(0.43)	(0.61)	(0.26)	(0.37)	(0.29)	(0.37)	(0.23)	(0.32)	(0.33)	(0.49)	(0.26)	(0.39)	(0.24)	(0.38)	(0.19)	(0.25)
t-stat. for diff. btwn. AEs (2011-2013)	**-4.46**		**-7.73**		**-3.41**		**-3.66**		**-5.69**		**-5.27**		**-4.79**		**-5.57**		**-4.59**		**-7.21**		**-5.03**	
2008-2013	0.68	1.02	0.40	0.60	0.58	0.83	0.68	0.92	0.52	0.79	0.42	0.61	0.35	0.46	0.56	0.85	0.53	0.74	0.39	0.58	0.31	0.48
	(0.84)	(1.22)	(0.38)	(0.47)	(0.45)	(0.72)	(0.60)	(0.89)	(0.52)	(0.83)	(0.34)	(0.49)	(0.26)	(0.39)	(0.47)	(0.66)	(0.46)	(0.69)	(0.30)	(0.43)	(0.31)	(0.47)
t-stat. for diff. btwn. AEs (2008-2013)	**-6.23**		**-8.60**		**-7.42**		**-5.41**		**-7.29**		**-7.88**		**-4.97**		**-9.07**		**-6.20**		**-8.81**		**-9.10**	

Mean (s.d.) of AE vs realized alpha	VZ		UTX		MSFT		WMT		XOM		T		GE		JNJ		INTC		PFE	
	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha	Option-implied alpha	Historical alpha
2008	0.59	0.81	0.44	0.70	0.69	1.15	0.53	0.65	0.78	1.02	0.68	0.96	0.90	1.20	0.39	0.51	0.59	1.02	0.64	0.74
	(0.50)	(0.64)	(0.33)	(0.57)	(0.55)	(0.89)	(0.38)	(0.48)	(0.75)	(0.76)	(0.51)	(0.72)	(0.72)	(0.94)	(0.30)	(0.38)	(0.53)	(0.80)	(0.40)	(0.65)
2009	0.52	0.69	0.45	0.63	0.59	0.97	0.44	0.74	0.54	0.63	0.43	0.61	0.96	1.18	0.35	0.54	0.55	0.79	0.53	0.96
	(0.35)	(0.50)	(0.33)	(0.50)	(0.52)	(0.66)	(0.36)	(0.63)	(0.36)	(0.40)	(0.32)	(0.50)	(0.85)	(0.89)	(0.26)	(0.39)	(0.45)	(0.61)	(0.50)	(0.76)
2010	0.43	0.58	0.22	0.38	0.41	0.53	0.22	0.30	0.31	0.43	0.32	0.36	0.48	0.81	0.26	0.38	0.44	0.67	0.54	0.70
	(0.30)	(0.38)	(0.18)	(0.29)	(0.29)	(0.39)	(0.16)	(0.24)	(0.20)	(0.27)	(0.22)	(0.26)	(0.36)	(0.45)	(0.20)	(0.29)	(0.35)	(0.43)	(0.39)	(0.45)
2008-2010	0.52	0.69	0.37	0.57	0.56	0.89	0.40	0.57	0.55	0.70	0.49	0.66	0.80	1.08	0.33	0.47	0.53	0.82	0.55	0.82
	(0.40)	(0.53)	(0.31)	(0.49)	(0.48)	(0.73)	(0.34)	(0.52)	(0.54)	(0.58)	(0.41)	(0.59)	(0.71)	(0.82)	(0.26)	(0.36)	(0.45)	(0.65)	(0.44)	(0.64)
t-stat. for diff. btwn. AEs (2008-2010)	**-4.83**		**-7.30**		**-7.34**		**-5.57**		**-3.67**		**-4.23**		**-4.66**		**-5.72**		**-6.61**		**-6.45**	
2011	0.31	0.50	0.33	0.53	0.48	0.57	0.34	0.52	0.31	0.49	0.29	0.48	0.49	0.57	0.35	0.48	0.58	0.93	0.41	0.59
	(0.19)	(0.34)	(0.30)	(0.43)	(0.32)	(0.37)	(0.28)	(0.41)	(0.22)	(0.32)	(0.24)	(0.31)	(0.35)	(0.51)	(0.27)	(0.32)	(0.47)	(0.76)	(0.29)	(0.43)
2012	0.46	0.60	0.36	0.52	0.42	0.51	0.47	0.59	0.22	0.31	0.40	0.53	0.30	0.53	0.23	0.35	0.51	0.71	0.29	0.50
	(0.29)	(0.42)	(0.22)	(0.38)	(0.29)	(0.37)	(0.38)	(0.45)	(0.18)	(0.25)	(0.28)	(0.35)	(0.21)	(0.43)	(0.20)	(0.30)	(0.53)	(0.54)	(0.21)	(0.34)
2013	0.51	0.63	0.25	0.37	0.54	0.77	0.33	0.46	0.28	0.33	0.35	0.45	0.35	0.51	0.32	0.43	0.41	0.62	0.33	0.57
	(0.33)	(0.51)	(0.21)	(0.30)	(0.48)	(0.61)	(0.22)	(0.30)	(0.22)	(0.31)	(0.24)	(0.38)	(0.24)	(0.36)	(0.21)	(0.33)	(0.35)	(0.46)	(0.27)	(0.45)
2011-2013	0.43	0.58	0.31	0.47	0.48	0.62	0.38	0.52	0.27	0.38	0.34	0.49	0.38	0.54	0.30	0.42	0.50	0.76	0.34	0.55
	(0.29)	(0.43)	(0.25)	(0.38)	(0.38)	(0.48)	(0.31)	(0.39)	(0.21)	(0.31)	(0.26)	(0.35)	(0.28)	(0.43)	(0.23)	(0.32)	(0.46)	(0.61)	(0.26)	(0.41)
t-stat. for diff. btwn. AEs (2011-2013)	**-5.11**		**-6.57**		**-4.21**		**-5.80**		**-5.35**		**-6.28**		**-5.10**		**-5.41**		**-7.26**		**-7.36**	
2008-2013	0.47	0.63	0.34	0.52	0.52	0.75	0.39	0.55	0.41	0.54	0.42	0.57	0.58	0.80	0.31	0.44	0.51	0.79	0.44	0.67
	(0.35)	(0.49)	(0.28)	(0.44)	(0.43)	(0.63)	(0.32)	(0.46)	(0.43)	(0.49)	(0.35)	(0.50)	(0.58)	(0.71)	(0.24)	(0.34)	(0.46)	(0.63)	(0.37)	(0.54)
t-stat. for diff. btwn. AEs (2008-2013)	**-6.64**		**-9.35**		**-7.82**		**-7.60**		**-5.39**		**-6.43**		**-6.10**		**-7.24**		**-9.16**		**-9.24**	

For each examined equity stock, this table reports the means and standard deviations of absolute errors (AEs) of the daily option-implied and historical α versus the daily realized α in different periods. For the periods of 2008-2010, 2011-2013, and 2008-2013, a t-test with Newey-West adjustment is performed on the differences of the AEs of option-implied and historical α. A negative (positive) t-statistic means that the option-implied (historical) α has a smaller MAE in the examined period. Boldface t-statistics indicate 5% significance.

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Table 5. Mean of Absolute Error (MAE) Analyses for Predicting Realized β

Mean (s.d.) of AE vs realized beta	JPM		IBM		BA		CAT		AXP		DIS		MCD		MRK		HD		PG		MMM	
	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta
2008	0.55	0.64	0.23	0.25	0.34	0.37	0.31	0.29	0.22	0.23	0.23	0.25	0.24	0.29	0.44	0.37	0.34	0.28	0.28	0.22	0.22	0.15
	(0.41)	(0.51)	(0.17)	(0.18)	(0.22)	(0.25)	(0.23)	(0.27)	(0.19)	(0.20)	(0.20)	(0.17)	(0.19)	(0.20)	(0.38)	(0.26)	(0.27)	(0.20)	(0.28)	(0.18)	(0.17)	(0.12)
2009	0.50	0.49	0.22	0.23	0.25	0.25	0.28	0.29	0.40	0.57	0.24	0.30	0.28	0.28	0.36	0.39	0.23	0.30	0.17	0.15	0.20	0.22
	(0.46)	(0.39)	(0.22)	(0.16)	(0.19)	(0.18)	(0.23)	(0.23)	(0.38)	(0.41)	(0.21)	(0.20)	(0.32)	(0.26)	(0.34)	(0.31)	(0.18)	(0.20)	(0.12)	(0.13)	(0.15)	(0.18)
2010	0.29	0.40	0.20	0.21	0.27	0.31	0.28	0.30	0.30	0.37	0.26	0.25	0.19	0.17	0.32	0.25	0.29	0.37	0.20	0.19	0.19	0.18
	(0.28)	(0.32)	(0.27)	(0.24)	(0.25)	(0.31)	(0.26)	(0.24)	(0.26)	(0.31)	(0.23)	(0.22)	(0.17)	(0.15)	(0.29)	(0.23)	(0.24)	(0.31)	(0.15)	(0.13)	(0.20)	(0.17)
2008-2010	0.44	0.51	0.22	0.23	0.29	0.31	0.29	0.29	0.31	0.39	0.25	0.27	0.24	0.25	0.37	0.34	0.28	0.32	0.22	0.19	0.20	0.19
	(0.40)	(0.42)	(0.22)	(0.20)	(0.22)	(0.26)	(0.24)	(0.25)	(0.29)	(0.35)	(0.22)	(0.20)	(0.24)	(0.22)	(0.34)	(0.27)	(0.24)	(0.25)	(0.20)	(0.15)	(0.17)	(0.16)
t-stat. for diff. btwn. AEs (2008-2010)	**-2.08**		-0.71		-1.33		-0.31		**-3.35**		-1.15		-0.52		1.52		-1.54		**2.15**		1.48	
2011	0.23	0.31	0.24	0.28	0.16	0.18	0.29	0.34	0.22	0.24	0.22	0.24	0.17	0.14	0.20	0.21	0.30	0.32	0.32	0.24	0.17	0.19
	(0.18)	(0.29)	(0.19)	(0.25)	(0.12)	(0.17)	(0.25)	(0.29)	(0.17)	(0.20)	(0.17)	(0.18)	(0.14)	(0.11)	(0.17)	(0.18)	(0.23)	(0.29)	(0.38)	(0.19)	(0.16)	(0.13)
2012	0.41	0.44	0.24	0.25	0.28	0.33	0.37	0.46	0.22	0.24	0.28	0.25	0.24	0.33	0.31	0.33	0.26	0.26	0.26	0.20	0.19	0.24
	(0.29)	(0.31)	(0.22)	(0.18)	(0.23)	(0.26)	(0.31)	(0.36)	(0.16)	(0.16)	(0.22)	(0.22)	(0.21)	(0.21)	(0.20)	(0.20)	(0.17)	(0.21)	(0.22)	(0.18)	(0.19)	(0.18)
2013	0.20	0.25	0.38	0.44	0.38	0.42	0.36	0.45	0.23	0.29	0.30	0.29	0.27	0.34	0.32	0.31	0.31	0.41	0.22	0.24	0.25	0.23
	(0.15)	(0.17)	(0.29)	(0.31)	(0.30)	(0.32)	(0.28)	(0.25)	(0.16)	(0.19)	(0.30)	(0.18)	(0.21)	(0.25)	(0.37)	(0.38)	(0.24)	(0.27)	(0.17)	(0.18)	(0.20)	(0.18)
2011-2013	0.28	0.34	0.29	0.32	0.27	0.31	0.34	0.42	0.22	0.26	0.27	0.26	0.23	0.27	0.28	0.28	0.29	0.33	0.27	0.22	0.20	0.22
	(0.23)	(0.27)	(0.25)	(0.27)	(0.25)	(0.28)	(0.28)	(0.31)	(0.16)	(0.18)	(0.24)	(0.20)	(0.20)	(0.22)	(0.27)	(0.28)	(0.22)	(0.27)	(0.27)	(0.19)	(0.19)	(0.17)
t-stat. for diff. btwn. AEs (2011-2013)	**-2.89**		-1.94		**-2.31**		**-3.77**		**-2.86**		0.43		**-2.52**		-0.32		**-2.03**		**2.34**		-1.23	
2008-2013	0.36	0.42	0.25	0.28	0.28	0.31	0.31	0.36	0.26	0.32	0.26	0.26	0.23	0.26	0.33	0.31	0.29	0.32	0.24	0.21	0.20	0.20
	(0.34)	(0.37)	(0.24)	(0.24)	(0.23)	(0.27)	(0.26)	(0.28)	(0.24)	(0.28)	(0.23)	(0.20)	(0.22)	(0.22)	(0.31)	(0.28)	(0.23)	(0.26)	(0.24)	(0.17)	(0.18)	(0.17)
t-stat. for diff. btwn. AEs (2008-2013)	**-3.11**		-1.86		**-2.45**		**-2.99**		**-4.05**		-0.39		**-1.98**		1.03		**-2.38**		**3.02**		0.01	

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Mean (s.d.) of AE vs realized beta	VZ		UTX		MSFT		WMT		XOM		T		GE		JNJ		INTC		PFE	
	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta	Option-implied beta	Historical beta
2008	0.25	0.24	0.21	0.21	0.36	0.26	0.29	0.28	0.33	0.35	0.23	0.24	0.35	0.38	0.28	0.19	0.35	0.34	0.24	0.19
	(0.18)	(0.19)	(0.14)	(0.15)	(0.30)	(0.19)	(0.23)	(0.24)	(0.30)	(0.25)	(0.17)	(0.15)	(0.29)	(0.33)	(0.28)	(0.13)	(0.26)	(0.27)	(0.15)	(0.19)
2009	0.32	0.26	0.17	0.18	0.29	0.30	0.28	0.18	0.19	0.18	0.22	0.20	0.31	0.33	0.25	0.18	0.28	0.27	0.27	0.31
	(0.31)	(0.21)	(0.13)	(0.13)	(0.23)	(0.20)	(0.21)	(0.13)	(0.16)	(0.15)	(0.17)	(0.15)	(0.23)	(0.26)	(0.22)	(0.14)	(0.19)	(0.19)	(0.19)	(0.19)
2010	0.24	0.21	0.21	0.26	0.29	0.27	0.25	0.16	0.20	0.22	0.24	0.20	0.28	0.26	0.20	0.16	0.42	0.31	0.32	0.30
	(0.21)	(0.15)	(0.22)	(0.24)	(0.24)	(0.24)	(0.27)	(0.12)	(0.16)	(0.14)	(0.14)	(0.17)	(0.23)	(0.18)	(0.14)	(0.11)	(0.33)	(0.28)	(0.27)	(0.21)
2008-2010	0.27	0.23	0.20	0.22	0.31	0.28	0.27	0.21	0.24	0.25	0.23	0.21	0.32	0.33	0.24	0.18	0.35	0.31	0.28	0.29
	(0.24)	(0.19)	(0.17)	(0.18)	(0.26)	(0.21)	(0.24)	(0.18)	(0.23)	(0.20)	(0.16)	(0.16)	(0.25)	(0.27)	(0.22)	(0.13)	(0.27)	(0.25)	(0.23)	(0.20)
t-stat. for diff. btwn. AEs (2008-2010)	**2.11**		-1.61		**2.04**		**3.61**		-0.74		1.31		-0.59		**4.11**		**2.18**		-0.30	
2011	0.19	0.18	0.22	0.25	0.26	0.24	0.21	0.13	0.23	0.28	0.22	0.15	0.35	0.21	0.16	0.18	0.34	0.39	0.29	0.31
	(0.15)	(0.13)	(0.20)	(0.26)	(0.22)	(0.27)	(0.17)	(0.12)	(0.23)	(0.29)	(0.15)	(0.11)	(0.28)	(0.20)	(0.15)	(0.16)	(0.31)	(0.38)	(0.22)	(0.23)
2012	0.24	0.26	0.21	0.22	0.27	0.31	0.24	0.26	0.18	0.23	0.29	0.27	0.24	0.21	0.30	0.25	0.31	0.35	0.27	0.19
	(0.20)	(0.18)	(0.16)	(0.16)	(0.21)	(0.28)	(0.25)	(0.19)	(0.13)	(0.16)	(0.20)	(0.17)	(0.17)	(0.15)	(0.25)	(0.19)	(0.19)	(0.20)	(0.17)	(0.13)
2013	0.35	0.44	0.22	0.25	0.36	0.40	0.22	0.19	0.29	0.28	0.22	0.23	0.33	0.29	0.21	0.28	0.37	0.49	0.34	0.35
	(0.23)	(0.31)	(0.17)	(0.18)	(0.31)	(0.38)	(0.19)	(0.17)	(0.21)	(0.21)	(0.17)	(0.17)	(0.29)	(0.21)	(0.17)	(0.20)	(0.38)	(0.44)	(0.29)	(0.22)
2011-2013	0.26	0.29	0.22	0.24	0.30	0.32	0.22	0.19	0.23	0.27	0.24	0.22	0.31	0.24	0.22	0.23	0.34	0.41	0.30	0.28
	(0.21)	(0.25)	(0.18)	(0.20)	(0.25)	(0.32)	(0.21)	(0.17)	(0.20)	(0.23)	(0.18)	(0.16)	(0.26)	(0.19)	(0.20)	(0.19)	(0.30)	(0.36)	(0.23)	(0.21)
t-stat. for diff. btwn. AEs (2011-2013)	**-2.30**		-1.27		-0.78		**2.20**		**-2.42**		1.79		**3.81**		-0.76		**-3.23**		0.94	
2008-2013	0.27	0.27	0.21	0.23	0.31	0.30	0.25	0.20	0.24	0.26	0.24	0.22	0.31	0.28	0.23	0.21	0.34	0.36	0.29	0.29
	(0.23)	(0.22)	(0.18)	(0.19)	(0.26)	(0.27)	(0.22)	(0.18)	(0.21)	(0.22)	(0.17)	(0.16)	(0.25)	(0.24)	(0.21)	(0.17)	(0.29)	(0.31)	(0.23)	(0.21)
t-stat. for diff. btwn. AEs (2008-2013)	0.05		-1.92		0.56		**4.00**		**-2.08**		**2.12**		**2.06**		**2.18**		-0.86		0.43	

Percentage Analysis of Prediction Power Comparison

	Option-Implied beta Outperforming Significantly	Option-Implied beta Outperforming Insignificantly	Historical beta Outperforming Insignificantly	Historical beta Outperforming Significantly
2008-2010	10% (=2/21)	48% (=10/21)	14% (=3/21)	29% (=6/21)
2011-2013	43% (=9/21)	29% (=6/21)	14% (=3/21)	14% (=3/21)
2008-2013	33% (=7/21)	19% (=4/21)	24% (=5/21)	24% (=5/21)

For each examined equity stock, this table reports the means and standard deviations of the absolute errors (AEs) of the daily option-implied and historical β versus the daily realized β in different periods. For the periods of 2008-2010, 2011-2013, and 2008-2013, a t-test with Newey-West adjustment is performed on the differences of the AEs of option-implied and historical β. A negative (positive) t-statistic means that the option-implied (historical) β has a smaller MAE in the examined period. Boldface t-statistics indicate 5% significance. The distributions of significantly (insignificantly) outperforming percentages of our option-implied and historical β are also reported.

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Last, we find that the option-implied β shows superior prediction power for stocks with more qualified option quotes. The upper (bottom) half in Table 5 presents the sample of the examined stocks with more (fewer) qualified option quotes. The option-implied β generates smaller MAEs than the historical β (or equivalently negative t-statistic values) in 8, 9, and 8 of the 11 stocks in 2008-2010, 2011-2013, and 2008-2013, respectively, for the upper half of Table 5. Moreover, there are more significantly negative (positive) t-statistic values in the upper (bottom) half of Table 5. These observations suggest that with more qualified option quotes, one obtains more informative option-implied estimates through the option implicit market model.

Table 6 shows the MAE analyses for the option-implied and historical σ_e versus the realized σ_e. The MAEs of both estimates are of similar magnitude, but the option-implied σ_e still provides an advantage in predicting the realized σ_e. We find in Table 6 that the option-implied σ_e generates smaller MAEs for 16, 13, and 15 of the 21 stocks in the period of 2008-2010, 2011-2013, and 2008-2013, respectively. The results of the t-test show that among the 16, 13, and 15 smaller MAEs for the option-implied σ_e, 6, 4, and 8 are significant in 2008-2010, 2011-2013, and 2008-2013, respectively. In addition, the MAEs of both estimates are generally smaller in the normal condition period from 2011 to 2013 than in the crisis period from 2008 to 2010, which suggests that it is difficult to estimate firm-specific risk in the crisis period, regardless of whether the historical return information or the option-implied information is used. Moreover, the MAEs of both estimates are exceptionally large for AXP and JPM in 2008 and 2009. Note that AXP (American Express Company) and JPM (JPMorgan Chase) are financial companies which experienced extreme difficulties in the subprime crisis and the subsequent credit crisis from 2008 to 2009. This shows that under a crisis, although firm-specific risk may become more severe, neither the proposed option-based model nor the market model

Table 6. Mean of Absolute Error (MAE) Analyses for Predicting Realized σ_e

Mean (s.d.) of AE vs realized sigma_e	JPM		IBM		BA		CAT		AXP		DIS		MCD		MRK		HD		PG		MMM	
	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e
2008	0.23	0.23	0.05	0.06	0.07	0.08	0.09	0.09	0.09	0.09	0.06	0.07	0.06	0.06	0.14	0.17	0.08	0.08	0.05	0.04	0.08	0.07
	(0.19)	(0.18)	(0.05)	(0.05)	(0.05)	(0.05)	(0.08)	(0.06)	(0.13)	(0.09)	(0.05)	(0.05)	(0.05)	(0.05)	(0.13)	(0.14)	(0.07)	(0.07)	(0.04)	(0.03)	(0.09)	(0.07)
2009	0.08	0.10	0.06	0.07	0.06	0.09	0.08	0.10	0.08	0.10	0.07	0.09	0.04	0.05	0.09	0.11	0.06	0.07	0.04	0.05	0.07	0.06
	(0.09)	(0.11)	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.06)	(0.09)	(0.11)	(0.06)	(0.08)	(0.03)	(0.03)	(0.10)	(0.10)	(0.06)	(0.06)	(0.03)	(0.04)	(0.06)	(0.05)
2010	0.04	0.05	0.03	0.04	0.05	0.05	0.04	0.04	0.04	0.04	0.05	0.05	0.03	0.03	0.06	0.06	0.03	0.04	0.03	0.03	0.05	0.04
	(0.03)	(0.03)	(0.02)	(0.02)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.02)	(0.02)	(0.05)	(0.05)	(0.02)	(0.03)	(0.02)	(0.03)	(0.04)	(0.04)
2008-2010	0.12	0.13	0.05	0.06	0.06	0.07	0.07	0.08	0.07	0.08	0.06	0.07	0.04	0.05	0.09	0.11	0.06	0.06	0.04	0.04	0.06	0.06
	(0.15)	(0.14)	(0.04)	(0.04)	(0.05)	(0.05)	(0.06)	(0.06)	(0.09)	(0.09)	(0.05)	(0.06)	(0.04)	(0.04)	(0.10)	(0.11)	(0.06)	(0.06)	(0.03)	(0.03)	(0.07)	(0.06)
t-stat. for diff. btwn. AEs (2008-2010)	-1.07		**-3.63**		**-4.12**		**-2.27**		-1.94		**-2.02**		-1.86		**-2.90**		-1.36		-0.37		1.59	
2011	0.04	0.05	0.05	0.06	0.04	0.05	0.05	0.04	0.05	0.04	0.04	0.06	0.03	0.03	0.05	0.05	0.05	0.06	0.03	0.03	0.04	0.05
	(0.04)	(0.05)	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.02)	(0.02)	(0.04)	(0.05)	(0.05)	(0.05)	(0.03)	(0.02)	(0.04)	(0.04)
2012	0.07	0.07	0.04	0.07	0.04	0.04	0.05	0.04	0.05	0.04	0.04	0.05	0.02	0.03	0.04	0.04	0.04	0.03	0.04	0.06	0.03	0.02
	(0.07)	(0.07)	(0.03)	(0.05)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)	(0.05)	(0.02)	(0.02)	(0.03)	(0.04)	(0.03)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
2013	0.03	0.03	0.05	0.10	0.05	0.06	0.03	0.04	0.03	0.04	0.03	0.03	0.02	0.02	0.05	0.05	0.04	0.05	0.05	0.05	0.03	0.03
	(0.02)	(0.02)	(0.06)	(0.08)	(0.05)	(0.05)	(0.03)	(0.04)	(0.03)	(0.04)	(0.03)	(0.02)	(0.02)	(0.02)	(0.04)	(0.04)	(0.03)	(0.03)	(0.05)	(0.04)	(0.04)	(0.04)
2011-2013	0.05	0.05	0.05	0.08	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.05	0.03	0.03	0.05	0.05	0.04	0.05	0.04	0.05	0.04	0.03
	(0.05)	(0.06)	(0.04)	(0.06)	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.04)	(0.02)	(0.02)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)
t-stat. for diff. btwn. AEs (2011-2013)	-0.56		**-6.60**		-1.81		-0.66		-1.71		**-2.63**		-0.89		0.53		-1.23		-1.58		1.06	
2008-2013	0.08	0.09	0.05	0.07	0.05	0.06	0.05	0.06	0.05	0.06	0.05	0.06	0.04	0.04	0.07	0.08	0.05	0.06	0.04	0.04	0.05	0.05
	(0.12)	(0.12)	(0.04)	(0.06)	(0.04)	(0.05)	(0.05)	(0.05)	(0.07)	(0.07)	(0.05)	(0.05)	(0.03)	(0.03)	(0.08)	(0.09)	(0.05)	(0.05)	(0.04)	(0.03)	(0.06)	(0.05)
t-stat. for diff. btwn. AEs (2008-2013)	-1.15		**-6.91**		**-4.07**		**-2.11**		**-2.32**		**-2.96**		-1.92		**-2.24**		-1.72		-1.29		1.79	

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Mean (s.d.) of AE vs realized sigma_e	VZ		UTX		MSFT		WMT		XOM		T		GE		JNJ		INTC		PFE	
	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e	Option-implied sigma_e	Historical sigma_e
2008	0.10	0.10	0.05	0.06	0.10	0.13	0.05	0.05	0.08	0.08	0.08	0.06	0.17	0.16	0.07	0.06	0.09	0.09	0.06	0.07
	(0.10)	(0.11)	(0.04)	(0.05)	(0.08)	(0.05)	(0.04)	(0.04)	(0.08)	(0.08)	(0.07)	(0.05)	(0.16)	(0.14)	(0.06)	(0.04)	(0.07)	(0.08)	(0.05)	(0.06)
2009	0.04	0.04	0.04	0.04	0.08	0.12	0.04	0.05	0.04	0.04	0.04	0.05	0.13	0.14	0.04	0.04	0.05	0.04	0.07	0.07
	(0.05)	(0.03)	(0.03)	(0.03)	(0.06)	(0.08)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)	(0.04)	(0.12)	(0.12)	(0.03)	(0.03)	(0.03)	(0.03)	(0.09)	(0.07)
2010	0.04	0.04	0.02	0.03	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.06	0.05	0.03	0.03	0.03	0.03	0.04	0.05
	(0.03)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.05)	(0.04)	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.05)
2008-2010	0.06	0.06	0.04	0.04	0.08	0.10	0.04	0.04	0.05	0.05	0.05	0.05	0.12	0.12	0.04	0.04	0.05	0.05	0.06	0.06
	(0.08)	(0.07)	(0.04)	(0.04)	(0.06)	(0.07)	(0.03)	(0.04)	(0.06)	(0.05)	(0.05)	(0.04)	(0.13)	(0.12)	(0.05)	(0.04)	(0.05)	(0.06)	(0.07)	(0.06)
t-stat. for diff. btwn. AEs (2008-2010)	-0.10		-1.79		**-4.03**		-1.75		-0.14		0.72		0.34		1.04		0.05		-0.54	
2011	0.03	0.03	0.06	0.05	0.03	0.04	0.04	0.05	0.04	0.04	0.05	0.04	0.06	0.05	0.04	0.04	0.05	0.05	0.03	0.03
	(0.03)	(0.02)	(0.05)	(0.05)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)	(0.03)	(0.05)	(0.04)	(0.02)	(0.02)	(0.04)	(0.04)	(0.02)	(0.03)
2012	0.03	0.02	0.03	0.02	0.04	0.05	0.05	0.06	0.03	0.02	0.04	0.03	0.04	0.04	0.03	0.03	0.04	0.04	0.02	0.03
	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)	(0.04)	(0.05)	(0.05)	(0.02)	(0.01)	(0.04)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
2013	0.04	0.04	0.03	0.03	0.08	0.11	0.02	0.03	0.03	0.03	0.04	0.04	0.05	0.06	0.04	0.03	0.05	0.07	0.04	0.04
	(0.03)	(0.03)	(0.02)	(0.02)	(0.08)	(0.09)	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.03)	(0.05)	(0.04)	(0.02)	(0.02)	(0.04)	(0.05)	(0.05)	(0.04)
2011-2013	0.03	0.03	0.04	0.03	0.05	0.06	0.04	0.04	0.03	0.03	0.04	0.04	0.05	0.05	0.03	0.03	0.05	0.05	0.03	0.03
	(0.03)	(0.02)	(0.04)	(0.03)	(0.06)	(0.07)	(0.04)	(0.04)	(0.03)	(0.02)	(0.04)	(0.03)	(0.05)	(0.04)	(0.02)	(0.02)	(0.04)	(0.04)	(0.03)	(0.03)
t-stat. for diff. btwn. AEs (2011-2013)	-0.03		**2.48**		**-2.62**		-1.55		1.50		0.85		0.56		1.41		**-2.85**		0.39	
2008-2013	0.05	0.05	0.04	0.04	0.06	0.08	0.04	0.04	0.04	0.04	0.05	0.04	0.09	0.08	0.04	0.04	0.05	0.05	0.05	0.05
	(0.06)	(0.05)	(0.04)	(0.03)	(0.06)	(0.07)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.10)	(0.10)	(0.04)	(0.03)	(0.04)	(0.05)	(0.05)	(0.05)
t-stat. for diff. btwn. AEs 2008-2013)	-0.10		0.77		**-4.53**		**-2.23**		0.85		1.03		0.52		1.55		-1.67		-0.30	

Percentage Analysis of Prediction Power Comparison				
	Option-Implied sigma_e Outperforming Significantly	Option-Implied sigma_e Outperforming Insignificantly	Historical sigma_e Outperforming Insignificantly	Historical Sigme_e Outperforming Significantly
2008-2010	29% (=6/21)	48% (=10/21)	24% (=5/21)	0% (=0/21)
2011-2013	19% (=4/21)	43% (=9/21)	33% (=7/21)	5% (=1/21)
2008-2013	38% (=8/21)	33% (=7/21)	29% (=6/21)	0% (=0/21)

For each stock, this table reports the means and standard deviations of the absolute errors (AEs) of the daily option-implied and historical σ_e versus the daily realized σ_e in different periods. For the periods of 2008-2010, 2011-2013, and 2008-2013, a t-test with Newey-West adjustment is performed on the differences of the AEs of option-implied and historical σ_e. A negative (positive) t-statistic means that the option-implied (historical) σ_e has a smaller MAE in the examined period. Boldface t-statistics indicate 5% significance. The distributions of significantly (insignificantly) outperforming percentages of our option-implied and historical σ_e are also reported.

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predicts this effect. Last, for the upper half of Table 6, which presents the results for stocks with more qualified option quotes, the superiority of the option-implied σ_e is more pronounced. Of the 11 stocks in the upper half, 10 (5), 9 (2), and 9 (6) are with (significantly) smaller MAEs generated by the option-implied estimate of σ_e in 2008-2010, 2011-2013, and 2008-2013, respectively. In contrast, the option-implied σ_e performs (significantly) better only in 6, 4, and 5 (1, 2, and 2) cases for the 10 stocks in the bottom half of Table 6 in 2008-2010, 2011-2013, and 2008-2013, respectively. This finding again implies the preferable prediction power of the option-implied estimates given sufficient observations of qualified option quotes.

6. Robustness Test

For the empirical illustration in Section 5, since actual α, β, and σ_e cannot be observed, we employ the market model (single-index model), the most common approach in both academic and practice, to estimate the realized α, β, and σ_e in a future horizon and use these to approximate the actual values. However, this comparison could be susceptible to unexpected impacts from errors when approximating the actual α, β, and σ_e with the realized α, β, and σ_e for a future period of time. Therefore, in this section we conduct a more direct comparison by predicting the future one-month (21-trading-day) stock returns (conditional on the contemporaneous index returns) based on the option-implied and historical α and β as a robustness test. The tests in this section not only avoid estimating the realized α, β, and σ_e, but also are more appropriate when evaluating the major purpose of our option-implicit market model: predicting expected stock returns.

For each stock, we obtain the prediction errors of the option-implied and historical estimates on a trading day t via

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$$\text{Prediction error}_{imp}(t) = Y_T(t) - R_f(t,T) - \frac{\alpha_{imp}(t)}{12}$$
$$-\beta_{imp}(t)\left[M_T(t) - R_f(t,T)\right], \quad (19)$$

$$\text{Prediction error}_{hist}(t) = Y_T(t) - R_f(t,T) - \frac{\alpha_{hist}(t)}{12}$$
$$-\beta_{hist}(t)\left[M_T(t) - R_f(t,T)\right], \quad (20)$$

where $T - t = \frac{21}{252} = \frac{1}{12}$, $Y_T(t)$ and $M_T(t)$ denote the future 21-trading-day (from t to T) equity and index gross returns, respectively, $R_f(t,T)$ is the gross risk-free rate from t to T, and $\alpha_{imp}(t)$ and $\beta_{imp}(t)$ ($\alpha_{his}(t)$ and $\beta_{his}(t)$) are the option-implied (historical) alpha and beta obtained on the examined trading day t. We then calculate the means of $\left|\text{Prediction error}_{imp}(t)\right|$ and $\left|\text{Prediction error}_{hist}(t)\right|$ (MAEs) and the t-statistic for the difference of the two MAEs based on the Newey-West standard error for 2008-2010, 2010-2013, and 2008-2013, respectively. A negatively significant t-statistic indicates that our option-implied α and β outperform the historical counterparts in predicting the future one-month stock return conditional on the contemporaneous index return. The MAEs based on the option-implied and historical estimates, their differences, and the corresponding t-statistic of all examined stocks in 2008-2010, 2010-2013, and 2008-2013 are shown in Table 7.

Panel (a) of Table 7 shows return-prediction MAEs with the presence of the option-implied α and the historical α if it is significant,[15] i.e., if $\alpha_{hist}(t)$ is not significant based on the market model, there is no $\frac{\alpha_{hist}(t)}{12}$ in Equation (20). In contrast, Panel (b) of Table 7 shows the results without the option-implied and historical α, i.e., there is no $\frac{\alpha_{imp}(t)}{12}$ ($\frac{\alpha_{hist}(t)}{12}$) term in Equation (19) ((20)). We

15 For option-implied α, since we cannot identify its significance based on only the option quotes on a single trading day, we here employ the full version of Equation (19) to calculate the prediction error for the future 21-trading-day return.

Table 7. Mean of Absolute Prediction Errors for 21-Day Ahead Returns of Different Examined Stocks

Panel (a)									
Mean of absolute errors (MAEs) for predicting 21-day ahead returns (%)	Option-implied results (with alpha)			Historical results (with sig. alpha)			Diff. (%) and its t-statistic		
	2008-2010	2011-2013	2008-2013	2008-2010	2011-2013	2008-2013	2008-2010	2011-2013	2008-2013
AXP	6.42	2.95	4.62	7.02	3.30	5.08	-0.59 **-(1.96)**	-0.35 **-(2.24)**	-0.47 **-(2.69)**
BA	6.03	3.72	4.86	6.53	4.09	5.30	-0.50 **-(2.12)**	-0.37 **-(2.44)**	-0.44 **-(3.00)**
CAT	6.12	4.78	5.45	7.51	5.80	6.66	-1.39 **-(3.56)**	-1.02 **-(3.66)**	-1.21 **-(4.78)**
DIS	3.87	2.99	3.42	4.26	3.45	3.84	-0.39 **-(2.57)**	-0.45 **-(3.43)**	-0.42 **-(4.04)**
GE	6.45	3.30	4.86	7.20	3.58	5.37	-0.76 **-(2.45)**	-0.28 -(1.63)	-0.52 **-(2.80)**
HD	5.83	3.12	4.45	6.41	3.61	4.99	-0.58 **-(2.45)**	-0.49 **-(4.79)**	-0.53 **-(4.02)**
IBM	3.53	3.18	3.36	4.07	3.97	4.02	-0.54 **-(2.98)**	-0.78 **-(4.22)**	-0.66 **-(4.88)**
INTC	4.70	4.17	4.44	5.54	4.89	5.22	-0.84 **-(3.58)**	-0.72 **-(3.00)**	-0.78 **-(4.46)**
JNJ	3.01	2.57	2.78	3.03	2.64	2.82	-0.02 -(0.13)	-0.06 -(0.45)	-0.04 -(0.40)
JPM	7.21	4.22	5.69	8.60	4.72	6.63	-1.39 **-(3.11)**	-0.50 **-(2.61)**	-0.94 **-(3.74)**
MCD	3.68	2.79	3.24	4.27	3.11	3.70	-0.60 **-(3.94)**	-0.32 **-(2.27)**	-0.46 **-(4.21)**
MMM	3.65	1.87	2.75	3.71	1.93	2.80	-0.06 -(0.41)	-0.06 -(0.81)	-0.06 -(0.74)
MRK	6.17	3.66	4.92	6.90	4.22	5.57	-0.73 **-(2.22)**	-0.56 **-(3.00)**	-0.64 **-(3.27)**

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MSFT	4.44	4.07	4.26	4.87	4.37	4.62	-0.42 **-(2.04)**	-0.30 **-(2.30)**	-0.36 **-(2.82)**
PFE	4.78	3.05	3.85	4.77	3.75	4.22	0.01 (0.07)	-0.70 **-(4.54)**	-0.37 **-(2.87)**
PG	3.86	2.76	3.32	4.30	3.17	3.74	-0.44 **-(2.96)**	-0.41 **-(2.81)**	-0.42 **-(3.94)**
T	4.02	2.99	3.51	4.01	3.16	3.58	0.01 (0.07)	-0.17 -(1.46)	-0.08 -(0.69)
UTX	3.00	2.57	2.78	3.18	2.95	3.06	-0.18 -(1.38)	-0.38 **-(2.91)**	-0.28 **-(2.91)**
VZ	4.38	3.59	3.97	4.96	3.84	4.39	-0.58 **-(3.32)**	-0.26 -(1.49)	-0.41 **-(3.24)**
WMT	3.54	3.13	3.33	3.63	3.57	3.60	-0.09 -(0.62)	-0.45 **-(3.68)**	-0.27 **-(2.68)**
XOM	4.38	2.37	3.36	5.07	2.80	3.91	-0.68 **-(2.92)**	-0.43 **-(3.74)**	-0.55 **-(4.13)**
Average of MAEs of all equities	4.72	3.23	3.96	5.23	3.66	4.44			
Average of mean errors (for predicting 21-day ahead returns) of all equities (%)	-0.23	-0.28	-0.25	0.24	0.12	0.18			

* Option-implied estimates outperforming significantly: 77.78% (=49/63)
* Option-implied estimates outperforming insignificantly: 19.05% (=12/63)
* Historical estimates outperforming insignificantly: 3.17% (=2/63)
* Historical estimates outperforming significantly: 0% (=0/63)

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Panel (b)

Mean of absolute errors (MAEs) for predicting 21-day ahead returns (%)	Option-implied results (without alpha)			Historical results (without alpha)			Diff. (%) and its t-statistic		
	2008-2010	2011-2013	2008-2013	2008-2010	2011-2013	2008-2013	2008-2010	2011-2013	2008-2013
AXP	6.18	2.93	4.49	6.47	2.91	4.62	-0.29 **-(1.96)**	0.02 (0.41)	-0.13 -(1.61)
BA	6.04	3.63	4.83	6.01	3.67	4.83	0.03 (0.33)	-0.04 -(0.53)	0.00 -(0.04)
CAT	6.29	4.64	5.47	6.47	4.68	5.58	-0.18 -(1.41)	-0.05 -(0.52)	-0.11 -(1.38)
DIS	3.81	2.94	3.37	4.00	3.06	3.52	-0.19 **-(2.32)**	-0.12 -(1.60)	-0.15 **-(2.68)**
GE	6.29	3.17	4.71	6.27	3.12	4.68	0.02 (0.13)	0.05 (0.70)	0.04 (0.40)
HD	5.82	3.27	4.53	5.57	3.48	4.51	0.25 **(2.07)**	-0.21 **-(2.86)**	0.02 (0.22)
IBM	3.49	3.17	3.33	3.76	3.29	3.53	-0.27 **-(2.56)**	-0.13 -(1.95)	-0.20 **-(3.06)**
INTC	4.68	4.14	4.41	4.74	4.21	4.48	-0.06 -(0.40)	-0.07 -(0.77)	-0.06 -(0.68)
JNJ	2.91	2.51	2.70	2.75	2.44	2.59	0.16 (1.51)	0.08 (1.35)	0.12 (1.89)
JPM	7.15	4.08	5.59	8.02	4.09	6.03	-0.87 **-(3.21)**	-0.02 -(0.21)	-0.44 **-(2.92)**
MCD	3.84	2.74	3.30	3.93	2.67	3.31	-0.08 -(1.11)	0.07 (1.18)	-0.01 -(0.18)
MMM	3.69	1.87	2.77	3.47	1.92	2.68	0.22 **(2.10)**	-0.05 -(0.95)	0.08 (1.40)
MRK	6.08	3.64	4.86	5.84	3.55	4.70	0.24 (1.25)	0.09 (1.12)	0.16 (1.51)
MSFT	4.30	4.00	4.15	4.32	4.09	4.21	-0.01 -(0.11)	-0.10 -(0.99)	-0.05 -(0.71)

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PFE	4.66	3.13	3.83	4.66	3.08	3.81	0.00 -(0.01)	0.05 (0.66)	0.02 (0.30)
PG	3.72	2.73	3.23	3.82	2.74	3.29	-0.10 -(1.01)	-0.01 -(0.08)	-0.05 -(0.84)
T	3.94	2.93	3.44	3.73	2.80	3.27	0.21 **(2.07)**	0.13 (1.59)	0.17 **(2.49)**
UTX	3.00	2.60	2.80	3.01	2.67	2.84	-0.01 -(0.10)	-0.07 -(1.21)	-0.04 -(0.65)
VZ	4.50	3.49	3.99	4.49	3.50	3.98	0.02 (0.18)	-0.01 -(0.14)	0.00 (0.06)
WMT	3.67	3.07	3.37	3.30	3.14	3.22	0.37 **(2.92)**	-0.08 -(1.42)	0.15 **(1.99)**
XOM	4.27	2.39	3.31	4.46	2.48	3.45	-0.19 -(1.61)	-0.09 **-(2.16)**	-0.14 **-(2.17)**
Average of MAEs of all equities	4.68	3.19	3.93	4.72	3.22	3.96			
Average of mean errors (for predicting 21-day ahead returns) of all equities (%)	0.47	0.21	0.34	0.34	0.30	0.32			

* Option-implied estimates outperforming significantly: 15.87% (=10/63)

* Option-implied estimates outperforming insignificantly: 44.44% (=28/63)

* Historical estimates outperforming insignificantly: 30.16% (=19/63)

* Historical estimates outperforming significantly: 9.52% (=6/63)

This table reports the analyses of the mean of absolute prediction errors for 21-day ahead returns of different stocks in different periods. The prediction errors based on the option-implied and historical estimates are defined in Equations (19) and (20). The MAEs in Panel (a) are obtained by including option-implied α's and historical significant α's. The MAEs in Panel (b) are obtained without option-implied and historical α's. Our option-implied estimates generate more accurate predictions for the subsequent one-month (21-trading-day) stock returns (conditional on the contemporaneous index returns) than the historical estimates, particularly under the more common setting in which significant historical α's are included in Panel (a).

examine the prediction errors without α because the $\alpha_{imp}(t)$ obtained by our model is consistently close to zero, and the significance frequency of the $\alpha_{hist}(t)$ is usually less than 5% of the total trading days examined, regardless of different stocks or examined periods.

Overall speaking, by comparing the averages of return-prediction MAEs and the averages of mean errors across all stocks in each examined period, shown at the bottom of the two panels, we observe that without α (Panel (b)), the averages of MAEs are smaller, but the averages of mean errors generally deviate more from zero. In Panel (a), our option-implied α and β generates smaller return-prediction MAEs in 61 (49 significantly) cases out of the total 63 cases (the combinations of the 21 stocks and 3 examined periods of 2008-2010, 2010-2013, and 2008-2013). In contrast, the historical estimates perform better only in two cases, neither of which is significant. Finally, in Panel (b), our option-implied estimates outperform the historical estimates in 38 (10 significantly) cases, representing 60.32% (15.87%) of the total 63 cases, and the historical estimates generate smaller return-prediction MAEs in 25 (6 significantly) cases, representing 39.68% (9.52%) of the total 63 cases.

Several conclusions can be obtained from Table 7. First, our option-implied estimates generate more accurate predictions for the future one-month (21-trading-day) stock returns (conditional on the contemporaneous index returns) than the historical estimates, regardless of whether α is included. Second, under the more common setting of including significant historical α in Panel (a), the superiority of our option-implied α and β for predicting future stock returns is clearly more pronounced. Third, without option-implied and historical α, i.e., in Panel (b), since only information about β is utilized, the degree of superiority of our option-implied β over the historical β is similar to that in Table 5, where the option-implied β is closer to the realized β in 38 (18 significantly) cases, representing 60.32% (28.57%) of the total 63 cases, and the historical β is closer to the realized β in 25 (14 significantly) cases, representing 39.68% (22.22%) of the total 63 cases. Although the significant cases in Panel (b) of Table 7 and in Table 5 do not match exactly, they are still somewhat consistent. For example, for the first seven stocks in Table 5 (with more qualified option quotes), including JPM, IBM, BA, CAT, AXP, DIS, and MCD, the option-implied β in general perform better in Panel (b) of Table 7. On the other hand, for the last five stocks (except INTC) in Table 5 (with less qualified option quotes), the historical β are superior to the option-implied β not only in Table 5 but also in Panel (b) of Table 7.

7. Conclusion

Option prices are commonly believed to provide additional information on the volatility of the underlying stock return for future periods. Although option prices are informative about future volatility, there is little research on using option prices to estimate future levels of alpha, beta, and firm-specific risk, particularly the latter two, which are theoretically related to the future volatilities of the index and individual stock returns.

This paper develops an equity option pricing model involving the volatility of the market return and the level of alpha, beta, and firm-specific risk of the underlying stock, by incorporating the market model within the framework of the multivariate RNVR. This market-model option pricing formula enables the estimation of alpha, beta, and firm-specific risk of individual stocks based solely on the prices of equity and index options. As a result, although our model employs only information on option prices, it plays an analogous role to the market model in testing the CAPM or generating forward-looking estimates of alpha, beta, and firm-specific risk of individual stocks.

We conduct empirical experiments to evaluate the proposed option implicit market model with the prices of the SPX index options and equity options for 21 stocks continuously listed as DJIA components from 2008 to 2013. Our option-implied estimates demonstrate that the CAPM holds for the examined 21 stocks. As for the prediction accuracy comparison for the realized alpha, beta, and firm-specific in the subsequent one calendar month, the significantly better-performing percentages of our option-implied estimates outnumber (sometimes by multiple times) those of the historical estimates except for estimating beta in the financial crisis of 2008-2010. Moreover, in any examined time period, at least 52% of our option-implied estimates outperform the historical estimates, including both significant and insignificant cases. We thus conclude that our option-implied estimates show superior (slightly better) prediction power over the regression-based historical estimates in the forecasting of future alpha, beta, and firm-specific risk in normal (crisis) periods. Finally, our option-implied alpha and beta also show significant superiority over historical alpha and beta in predicting subsequent 21-trading-day (one-month) stock returns.

Our model could also, for example, be extended for combination into the RNVR framework with more general asset pricing models such as Fama-French's (1992) three-factor or even Fama-French's (2015) five-factor model, by formulating appropriate transformed normal distributions for systematic risk factors other than the market return. Moreover, a complete empirical analysis could be conducted for different periods and different markets to further understand the general behavior of option-implied, forward-looking estimations of alpha, beta, and firm-specific risk.

Appendix 1. Derivation of the Market Model Option Pricing Formulas

***Proof of Proposition 1*:** Note that one can evaluate the present values for the market portfolio return and the firm-specific risk component, which are respectively denoted as P_m and P_e, based on Eq. (10) by simply replacing the payoff function $C(M_T, e_T)$ with $[M_T\ e_T]^{\mathbf{T}}$:

$$\begin{bmatrix} P_m \\ P_e \end{bmatrix} = R_f^{-1} \int_{-\infty}^{\infty} \int_0^{\infty} \begin{bmatrix} M_T \\ e_T \end{bmatrix} \frac{1}{\sqrt{2\pi}\sigma_m\sqrt{T}M_T} \exp\left(-\frac{1}{2\sigma_m^2 T}\left[\ln M_T - \left(\mu_m T + \gamma\rho_{wm}\sigma_w\sigma_m\sqrt{T}\right)\right]^2\right) \cdot$$

$$\frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}\left[e_T - \left(0 + \gamma\rho_{we}\sigma_w\sigma_e\sqrt{T}\right)\right]^2\right) dM_T de_T.$$

Since the right-hand-side integral actually calculates $E\begin{bmatrix} M_T \\ e_T \end{bmatrix}$, we have $\begin{bmatrix} P_m \\ P_e \end{bmatrix} = R_f^{-1}\begin{bmatrix} \exp\left(\mu_m T + \gamma\rho_{wm}\sigma_w\sigma_m\sqrt{T} + \frac{1}{2}\sigma_m^2 T\right) \\ 0 + \gamma\rho_{we}\sigma_w\sigma_e\sqrt{T} \end{bmatrix}$, which completes the proof.

***Proof of Theorem 2*:** Equation (12) is shown as follows.

$$V = R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_0 \max\left(\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right] R_q^{-1} - k, 0\right) \cdot$$

$$\frac{1}{\sqrt{2\pi}\sigma_m\sqrt{T}} \exp\left(-\frac{1}{2\sigma_m^2 T}(Z_m - \mu_m^*)^2\right) \frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}} \exp\left(-\frac{1}{2\sigma_e^2 T}(Z_e - \mu_e^*)^2\right) dZ_m dZ_e$$

$$= R_f^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_0 \max\left(\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right] R_q^{-1} - k, 0\right) \phi^*(Z_m)$$

$$\phi^*(Z_e) dZ_m dZ_e, \tag{A.1}$$

where $\mu_m^* = \ln R_f - \frac{1}{2}\sigma_m^2 T$, $\mu_e^* = \alpha T$, $\phi^*(Z_m) = \frac{1}{\sqrt{2\pi}\sigma_m\sqrt{T}} \exp\left(\frac{-1}{2\sigma_m^2 T}(Z_m - \mu_m^*)^2\right)$, and $\phi^*(Z_e) = \frac{1}{\sqrt{2\pi}\sigma_e\sqrt{T}} \exp\left(\frac{-1}{2\sigma_e^2 T}(Z_e - \mu_e^*)^2\right)$. The market-model option pricing formula can be derived in the following three cases:

Case 1. When $\beta > 0$, we can infer that βe^{Z_m} is positive for any value of Z_m. Therefore, the payoff function of the call option in Equation (A.1) is in the

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money if $\left[(1-\beta)R_f + Z_e\right]R_q^{-1} - k > 0$ and thus $Z_e > R_q k - (1-\beta)R_f$. In contrast, if $\left[(1-\beta)R_f + Z_e\right]R_q^{-1} - k \leq 0$, a proper constraint for Z_m should be imposed such that the call option is in the money at maturity. Consequently, the following two situations are considered:

i. $Z_e > R_q k - (1-\beta)R_f$ and $-\infty < Z_m < \infty$,

ii. $Z_e \leq R_q k - (1-\beta)R_f$ and $\beta e^{Z_m} - \left[R_q k - (1-\beta)R_f - Z_e\right] > 0$, which implies $Z_m > \ln\left(\frac{R_q k - (1-\beta)R_f - Z_e}{\beta}\right)$.

Then Equation (A.1) can be rewritten as

$$V = R_f^{-1} S_0 \int_{R_q k-(1-\beta)R_f}^{\infty} \int_{-\infty}^{\infty} \left\{\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right]R_q^{-1} - k\right\} \phi^*(Z_m)\, dZ_m$$

$$\phi^*(Z_e)\, dZ_e + R_f^{-1} S_0 \int_{-\infty}^{R_q k-(1-\beta)R_f} \int_{\ln(a-bZ_e)}^{\infty} \left\{\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right]R_q^{-1} - k\right\}$$

$$\phi^*(Z_m)\, dZ_m\, \phi^*(Z_e)\, dZ_e, \tag{A.2}$$

where $a = \frac{R_q k-(1-\beta)R_f}{\beta}$ and $b = \frac{1}{\beta}$.

For the first integral in Equation (A.2),

$$R_f^{-1} S_0 \int_{R_q k-(1-\beta)R_f}^{\infty} \int_{-\infty}^{\infty} \left\{\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right]R_q^{-1} - k\right\} \phi^*(Z_m)\, dZ_m\, \phi^*(Z_e)\, dZ_e$$

$$= R_f^{-1} S_0 \int_{R_q k-(1-\beta)R_f}^{\infty} \left[\left(R_f + Z_e\right)R_q^{-1} - k\right] \phi^*(Z_e) dZ_e{}^{\dagger}$$

$$= R_f^{-1} S_0 \left[\int_{R_q k-(1-\beta)R_f}^{\infty} \left(R_f R_q^{-1} - k\right)\phi^*(Z_e)\, dZ_e + \int_{R_q k-(1-\beta)R_f}^{\infty} R_q^{-1} Z_e \phi^*(Z_e) dZ_e\right]$$

$$= R_f^{-1} S_0 \left[\left(R_f R_q^{-1} - k\right) N\left(\frac{\alpha T + (1-\beta)R_f - R_q k}{\sigma_e\sqrt{T}}\right) + R_q^{-1}\sigma_e\sqrt{T} n\left(\frac{R_q k-(1-\beta)R_f - \alpha T}{\sigma_e\sqrt{T}}\right)\right]$$

$$= S_0 e^{-rT}\left[\left(e^{(r-q)T} - k + e^{-qT}\alpha T\right)N(M_1) + e^{-qT}\sigma_e\sqrt{T} n(-M_1)\right], \tag{A.3}$$

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† Note that $\int_c^{\infty}\left\{\left[(1-\beta)R_f + Z_e\right]R_q^{-1} - k\right\}\phi^*(Z_m)\, dZ_m + \int_c^{\infty} \beta R_q^{-1} e^{Z_m}\, \phi^*(Z_m)\, dZ_m = \left\{\left[(1-\beta)R_f + Z_e\right]R_q^{-1} - k\right\} N\left(\frac{-c + \ln R_f - \frac{\sigma_m^2 T}{2}}{\sigma_m\sqrt{T}}\right) + \beta R_q^{-1} R_f N\left(\frac{-c + \ln R_f + \frac{\sigma_m^2 T}{2}}{\sigma_m\sqrt{T}}\right)$. When c approaches negative infinity, it converges to $\left\{\left[(1-\beta)R_f + Z_e\right]R_q^{-1} - k\right\}\cdot 1 + \beta R_q^{-1} R_f \cdot 1 = \left(R_f + Z_e\right)R_q^{-1} - k$.

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where $M_1 = \frac{\alpha T + (1-\beta)e^{rT} - e^{qT}k}{\sigma_e \sqrt{T}}$; the last equation is derived based on the definitions of $R_f \equiv e^{rT}$ and $R_q \equiv e^{qT}$. Regarding the second integral in Equation (A2),

$$R_f^{-1} S_0 \int_{-\infty}^{R_q k-(1-\beta)R_f} \int_{\ln(a-bZ_e)}^{\infty} \left\{\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right] R_q^{-1} - k\right\}$$

$$\phi^*(Z_m)\, dZ_m\, \phi^*(Z_e)\, dZ_e$$

$$= R_f^{-1} S_0 \int_{-\infty}^{R_q k-(1-\beta)R_f} \left\{\int_{\ln(a-bZ_e)}^{\infty} \left\{\left[(1-\beta)R_f + Z_e\right] R_q^{-1} - k\right\} \phi^*(Z_m)\, dZ_m\right.$$

$$\left. + \int_{\ln(a-bZ_e)}^{\infty} \beta\, R_q^{-1} e^{Z_m} \phi^*(Z_m)\, dZ_m \right\} \phi^*(Z_e)\, dZ_e$$

$$= R_f^{-1} S_0 \int_{-\infty}^{R_q k-(1-\beta)R_f} \left\{ \left\{\left[(1-\beta)R_f + Z_e\right] R_q^{-1} - k\right\} N\left(\frac{-\ln(a-bZ_e) + \left(\ln R_f - \frac{\sigma_m^2 T}{2}\right)}{\sigma_m \sqrt{T}}\right)\right.$$

$$\left. + \beta R_q^{-1} R_f\, N\left(\frac{-\ln(a-bZ_e) + \left(\ln R_f + \frac{\sigma_m^2 T}{2}\right)}{\sigma_m \sqrt{T}}\right)\right\} \phi^*(Z_e)\, dZ_e$$

$$= S_0 e^{-rT} \int_{-\infty}^{e^{qT}k-(1-\beta)e^{rT}} \left\{\left\{\left[(1-\beta)e^{rT} + Z_e\right] e^{-qT} - k\right\} N(D_2)\right.$$

$$\left. + \beta e^{(r-q)T} N(D_1)\right\} \phi^*(Z_e)\, dZ_e, \qquad \text{(A.4)}$$

where $D_1 = \frac{-\ln(a-bZ_e) + \left(rT + \frac{\sigma_m^2 T}{2}\right)}{\sigma_m \sqrt{T}}$ and $D_2 = \frac{-\ln(a-bZ_e) + \left(rT - \frac{\sigma_m^2 T}{2}\right)}{\sigma_m \sqrt{T}} = D_1 - \sigma_m \sqrt{T}$.

Case 2: When $\beta = 0$, the integral for Z_m is not needed and thus can be dropped. In addition, the call option is in the money at maturity when $\left(R_f + Z_e\right) R_q^{-1} - k > 0$, that is, $Z_e > R_q k - R_f$. Consequently, we obtain the option price V as

$$V = R_f^{-1} S_0 \int_{R_q k - R_f}^{\infty} \left[\left(R_f + Z_e\right) R_q^{-1} - k\right] \phi^*(Z_e)\, dZ_e$$

$$= R_f^{-1} S_0 \left[\int_{R_q k - R_f}^{\infty} \left(R_f R_q^{-1} - k\right) \phi^*(Z_e)\, dZ_e + \int_{R_q k - R_f}^{\infty} R_q^{-1} Z_e \phi^*(Z_e)\, dZ_e\right]$$

$$= R_f^{-1} S_0 \left[\left(R_f R_q^{-1} - k + R_q^{-1}\alpha T\right) N\left(\frac{\alpha T + R_f - R_q k}{\sigma_e \sqrt{T}}\right) + R_q^{-1} \sigma_e \sqrt{T}\, n\left(\frac{R_q k - R_f - \alpha T}{\sigma_e \sqrt{T}}\right)\right]$$

$$= S_0 e^{-rT}\left[\left(e^{(r-q)T} - k + e^{-qT}\alpha T\right) N(M_2) + e^{-qT}\sigma_e \sqrt{T} n(-M_2)\right], \qquad \text{(A.5)}$$

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where $M_2 = \frac{\alpha T + e^{rT} - e^{qT} k}{\sigma_e \sqrt{T}}$.

Case 3: When $\beta < 0$, as long as $Z_e > R_q k - (1-\beta) R_f$ and $\beta e^{Z_m} + [Z_e - R_q k + (1-\beta) R_f] \geq 0$, the call option is in the money at maturity. Thus we obtain the constraints for Z_e and Z_m as $Z_e > R_q k - (1-\beta) R_f$ and $Z_m \leq \ln\left(\frac{R_q k - (1-\beta) R_f - Z_e}{\beta}\right)$.
$= \ln(a - bZ_e)$

Consequently, the option price is expressed as

$$V = R_f^{-1} S_0 \int_{R_q k-(1-\beta)R_f}^{\infty} \int_{-\infty}^{\ln(a-bZ_e)} \left\{\left[(1-\beta)R_f + \beta e^{Z_m} + Z_e\right] R_q^{-1} - k\right\}$$

$$\phi^*(Z_m)\, dZ_m\, \phi^*(Z_e)\, dZ_e$$

$$= R_f^{-1} S_0 \int_{R_q k-(1-\beta)R_f}^{\infty} \left\{\int_{-\infty}^{\ln(a-bZ_e)} \left\{\left[(1-\beta)R_f + Z_e\right] R_q^{-1} - k\right\} \phi^*(Z_m)\, dZ_m\right.$$

$$\left. + \int_{-\infty}^{\ln(a-bZ_e)} \beta R_q^{-1} e^{Z_m}\, \phi^*(Z_m)\, dZ_m\right\} \phi^*(Z_e)\, dZ_e$$

$$= R_f^{-1} S_0 \int_{R_q k-(1-\beta)R_f}^{\infty} \left\{ \left\{\left[(1-\beta)R_f + Z_e\right] R_q^{-1} - k\right\} N\left(\frac{\ln(a-bZ_e) - \left(\ln R_f - \frac{\sigma_m^2 T}{2}\right)}{\sigma_m \sqrt{T}}\right)\right.$$

$$\left. + \beta R_q^{-1} R_f\, N\left(\frac{\ln(a-bZ_e) - \left(\ln R_f + \frac{\sigma_m^2 T}{2}\right)}{\sigma_m \sqrt{T}}\right)\right\} \phi^*(Z_e)\, dZ_e$$

$$= S_0 e^{-rT} \int_{e^{qT} k-(1-\beta)e^{rT}}^{\infty} \left\{\left\{\left[(1-\beta)e^{rT} + Z_e\right] e^{-qT} - k\right\} N(-D_2)\right.$$

$$\left. + \beta e^{(r-q)T} N(-D_1)\right\} \phi^*(Z_e)\, dZ_e. \qquad \text{(A.6)}$$

***Proof of Corollary 1*:** Since $\sigma_e = 0$ and $\alpha = 0$ for the index option, we can derive $e_T = Z_e = \mu_e^* = 0$ with probability one. Because Z_e is fixed at zero, the integral over Z_e can be dropped. Furthermore, given $\beta = 1$ and thus $R_q k - (1-\beta) R_f = R_q k$ being larger than zero, we obtain $-\infty < Z_e = 0 < R_q k - (1-\beta) R_f$ and thus need consider only the second integral in Equation (A.2):

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$$V = R_f^{-1} S_0 \int_{\ln(a-b\cdot 0)=\ln(R_q k)}^{\infty} \left(R_q^{-1} e^{Z_m} - k\right) \phi^*(Z_m)\, dZ_m$$

$$= R_f^{-1} S_0 \int_{\ln(R_q k)}^{\infty} R_q^{-1} e^{Z_m}\, \phi^*(Z_m)\, dZ_m - R_f^{-1} S_0 \int_{\ln(R_q k)}^{\infty} k\, \phi^*(Z_m)\, dZ_m$$

$$= R_f^{-1} S_0 R_q^{-1} R_f N\left(\frac{-\ln(R_q k) + \ln R_f + \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right) - R_f^{-1} S_0 k N\left(\frac{-\ln(R_q k) + \ln R_f - \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right)$$

$$= S_0 R_q^{-1} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + \ln R_f - \ln R_q + \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right) - R_f^{-1} K N\left(\frac{\ln\left(\frac{S_0}{K}\right) + \ln R_f - \ln R_q - \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right).$$

By substituting R_f and R_q for e^{rT} and e^{qT}, we rewrite the above equation to be identical to the Black-Scholes formula for the market index call options:

$$V = S_0 e^{-qT} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r-q)T + \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right) - K e^{-r_f T} N\left(\frac{\ln\left(\frac{S_0}{K}\right) + (r-q)T - \frac{\sigma_m^2 T}{2}}{\sigma_m \sqrt{T}}\right). \qquad \text{(A.7)}$$

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Appendix 2. Calibration Example

The options of the SPX and BA on January 2, 2008 are employed to illustrate the calibration process. First, the risk-free zero curve on this day is presented in Table 8. Second, following the process described in Section 4.1, we derive the term structure of the implied volatilities of the SPX on this day as shown in Table 9. Third, the qualified BA options after the screening process are summarized in Table 10. Moreover, based on the zero curve and the implied volatilities of the SPX reported in Tables 8 and 9, respectively, the spline interpolation is employed to derive the risk-free interest rate and σ_m, which match different times to maturity for each BA equity option. The combination of the grid search for implied β (with the lower and upper bounds $[\beta_l, \beta_u] = [-0.0707, 1.4060]$ on this day) and the nonlinear least squares optimization procedure for the implied α and σ_e is performed to minimize the sum of the percentage errors between the market and theoretical option prices. Given the estimated dividend yield $q = 1.4784\%$ on this day, we obtain option-implied α, β, and σ_e of 0.0821, 0.9593, and 0.2192, respectively, and the square root of mean squared percentage errors is 8.33%. It is worth noting that the option values and their implied volatilities based on our option pricing formula exhibit patterns across different strike prices that are very similar to those actually observed in option markets. Finally, for comparison, the realized (historical) α, β, and σ_e are -0.0504, 0.7036, and 0.3221 (-0.7621, 0.9184, and 0.1781), respectively, on January 2, 2008.

Table 8. Risk-Free Zero Rates on January 2, 2008

Days to maturity	Zero rate
7	4.5592%
14	4.5699%
49	4.5918%
77	4.4937%
105	4.4035%
140	4.2994%
168	4.2156%

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Table 9. Term Structure of $\sigma_m(t,T)$ on January 2, 2008

Days to maturity $(T-t)$	Implied volatilities of the SPX
2	20.6617%
17	18.0426%
45	16.3139%
80	16.4157%
89	20.4649%
108	17.3406%
171	19.9511%
180	19.9621%

Table 10. Calibration Details on January 2, 2008

S_0 (\$)	K (\$)	$T-t$ (yrs)	Market option prices (implied vol.)	r (%)	σ_m (%)	Implied α	Implied β	Implied σ_e	Theoretical option prices (implied vol.)
86.62	70	0.0466	16.8504 (0.5588)	4.5749	18.0426				17.0402 (0.6651)
86.62	75	0.0466	11.9000 (0.4265)	4.5749	18.0426				12.0672 (0.4968)
86.62	80	0.0466	7.1985 (0.3531)	4.5749	18.0426				7.2667 (0.3694)
86.62	85	0.0466	3.1479 (0.2938)	4.5749	18.0426				3.2734 (0.3116)
86.62	90	0.0466	0.8196 (0.2696)	4.5749	18.0426				0.9424 (0.2891)
86.62	75	0.1233	12.4497 (0.3526)	4.5973	16.3139	(0.0821,	0.9593,	0.2192)	12.9535 (0.4327)
86.62	80	0.1233	8.1473 (0.3196)	4.5973	16.3139				8.5121 (0.3596)
86.62	85	0.1233	4.5990 (0.2960)	4.5973	16.3139				4.8426 (0.3169)
86.62	90	0.1233	2.1955 (0.2845)	4.5973	16.3139				2.2891 (0.2925)
86.62	95	0.1233	0.8752 (0.2781)	4.5973	16.3139				0.8674 (0.2771)
86.62	100	0.1233	0.3240 (0.2812)	4.5973	16.3139				0.2564 (0.2662)

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以選擇權價格估計隱性市場模型

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本文旨在以選擇權價格估計隱性市場模型，主要創新之處在於提出一個與偏好無關的選擇權評價公式，透過在多元風險中立評價架構，使用市場模型來描述選擇權標的股票報酬，評價公式中則包含了大盤指數波動率、與個股股票報酬的 alpha、beta、及公司特定風險。因此，我們可以經由校準個股和指數選擇權價格來估計選擇權隱性市場模型；亦即估計前瞻性 alpha、beta、與公司特定風險。實證研究顯示我們的模型可以準確地校準個股選擇權價格，選擇權隱含之估計值亦支持 CAPM 的成立，並且選擇權隱含之估計值在預測未來的 alpha、beta、與公司特定風險上，相較基於歷史資料之回歸估計更為有效。

關鍵詞：前瞻性 alpha；前瞻性 beta；前瞻性公司特定風險；選擇權隱性市場模型；風險中立評價關係。

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