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## Asset Prices Under Prospect Theory and Habit Formation

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In this paper, we develop a consumption-based asset pricing model motivated by prospect theory, where habit formation determines the endogenous reference point. This exploits the similarity between habit formation and prospect theory. Both emphasize that the investor does not care about the absolute amount of gain or loss, but rather compares the gain or the loss experienced to a benchmark. The results show that when taking people's loss averse attitude over consumption into consideration, our model is capable of resolving the equity premium puzzle.

*Keywords:* Prospect theory; habit formation; loss aversion; consumption-based asset pricing model.

### 1. Introduction

Due to the inability of expected utility theory to explain the behavior of asset returns, prospect theory has been recognized as a valid alternative to describe an investor's behavior.<sup>1</sup> Prospect theory is first formally proposed

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<sup>1</sup>Other alternatives include: (1) nonexpected utility in Weil (1989) and Epstein and Zin (1990); (2) habit formation in Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999); (3) some kinds of market incompleteness, such as Reitz (1988), asymmetric underlying process in Hung (1994), transaction costs in Aiyagari and Gertler (1991) and Heaton and Lucas (1996), and heterogeneous agents in Mankiw (1986), Mankiw and Zeldes (1991), Weil (1992), Lucas (1994), and Constantinides and Duffie (1996), etc.

in Kahneman and Tversky (1979). They design some psychological tests to see how people make decisions when they face different kinds of gambles. The results show that what affects people's decisions is not their wealth level after the gamble, but the amount of gain or loss from the gamble. In addition, people are found to be more sensitive about the losses than the gains. Moreover, they find that people will become more willing to take risks to avoid losses than to realize gains. These observations illustrate what is often called loss aversion.<sup>2</sup> Several studies, e.g., Benartzi and Thaler (1985) and Barberis, Huang and Santos (2001), have successfully employed prospect theory to explain the behavior of asset returns. However, as pointed out in Campbell (2000), there are some major unsettled issues when prospect theory is incorporated into asset pricing models. One issue is the determination and updating of the reference point in prospect theory. The other is the argument of the choice of objective function in prospect theory. In this paper, these two issues are considered while we incorporate prospect theory into the traditional asset pricing model.

Previous studies all focus on "wealth-based" prospect theory. However, in a standard intertemporal model, investors derive their utilities from consumption rather than wealth. In this paper, we address these issues by developing a consumption-based (rather than wealth-based) asset pricing model with prospect theory. In addition, it is often believed that what people really care about is not the absolute value of their consumption level, but the increase or decrease in their level of consumption. In other words, there exists a reference level of consumption in each consumer's mind. In our model, the reference level is determined and updated along the lines of habit formation. Following the habit formation framework in Abel (1990), the reference level consists of both the individual consumption level and the aggregate consumption level per capita from the last period. The individual consumption level portion represents the habit formation

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<sup>2</sup>In contrast to loss aversion, Gul (1991) proposes a disappointment averse utility function. He extends the standard constant relative risk aversion utility function with one more parameter. The good outcomes in the disappointment aversion utility function are down weighted relative to the bad outcomes. The spirit of disappointment aversion is like loss aversion, but there are two main differences between them. First, in the disappointment averse framework, the investor is always risk averse whenever facing gains or losses, but the loss averse utility function is convex for losses. Second, the disappointment averse utility function adopts the certainty equivalent to be its reference point, but the benchmark of the loss averse utility function is exogenous. The disappointment and loss averse utility functions have been applied to many financial and economic issues.

and the aggregate consumption level portion represents the characteristic of “catching up with the Joneses”. However, the ratio-form habit utility in Abel (1990) is replaced with the subtract-form habit utility in our model.

However, partly because the first derivatives of loss aversion utility functions are not continuous, previous researches apply it only to the wealth-based utility rather than the intertemporal consumption-based asset pricing model, where the derivative of the expected utility over consumption is needed to derive the Euler equation. In our model, we replace the power utility functions of original prospect theory with exponential utility functions. The first-order derivative of our prospect utility function becomes well-defined everywhere. Hence, we can readily apply it into the intertemporal consumption-based asset pricing model.

It is well known that Mehra and Prescott (1985) demonstrate that the traditional consumption-based asset pricing model with a proper risk aversion coefficient cannot generate a large enough equity premium. The observed real return in the US is 0.8% and the equity premium is 6%, but the largest premium that can be obtained with the traditional model is about 0.35%. This is the so-called equity premium puzzle. Since then, there have been many papers that attempt to improve the consumption-based asset pricing model to solve this problem.

For instance, Constantinides (1990) applies habit formation in a production (rather than exchange) and continuous-time economy to solve the problem. In addition, Abel (1990) first introduces habit formation and “catching up with the Joneses” into the consumption-based asset pricing model to try to solve the equity premium puzzle. Adopting the parameter values as in Mehra and Prescott (1985), which used a two-point Markov process for consumption growth with expectation equaling 1.018, variance equaling  $(0.036)^2$ , and correlation of this-period and last-period consumption growths equaling  $-0.14$ , by using “catching up with the Joneses”, and setting the risk aversion coefficient to be 6, Abel was able to generate a value of 463 basis points for equity premium. However, his model derived a seemingly too large riskless rate of return, 2.07%.

As to Campbell and Cochrane (1999), the habit formation is applied to explain not only the equity premium puzzle, but also the procyclical variation of stock prices and the countercyclical variation of stock price volatility. They introduce an external habit specification in their model. By carefully assuming the behavior of the habit as a function of consumption growth and exogenous endowment shocks, they obtain an unrealistic constant real

risk-free rate. Because the expected return of risky asset is a function of the surplus consumption ratio, they are able to obtain an arbitrary expected return for a risky asset given a specific value of the surplus consumption ratio.

Barberis, Huang and Santos (2001) is the first to introduce prospect theory into the consumption-based asset pricing model. However, in their model, they separate the utility function of the representative consumer into two parts. One part is the risk averse consumption utility function, and the other is the utility function of a financial investment. They do not apply prospect theory to the consumption stream, but instead apply a simplified prospect utility function only to the gains or losses from the financial investment. In addition to prospect theory, plus another consumer behavior feature “prior outcomes”, their model is employed to resolve the equity premium puzzle.<sup>3</sup> Like Campbell and Cochrane (1999), they generated an unrealistic constant risk-free interest rate. However, when a simplified linear version of prospect theory is applied to the financial investment of the representative agent, their model is not able to generate a large enough equity premium. The highest possible equity premium was only 1.2%.

Unlike Barberis, Huang and Santos (2001), our model applies the full version of prospect theory not on the return of financial investment, but on the consumption level of the representative agent. In addition, the reference level of prospect theory in our model is not exogenous. It is constructed following habit formation and “catching up with the Joneses”. Because of prospect theory and the endogenous habit-formation reference level, our model can solve the equity premium perfectly.

This paper is organized as follows. In Section 2, we will introduce the asset pricing model and compare it with some previous studies; and we will show how to derive asset returns, too. In Section 3, we report the results and analyze the influence on the equity premium of each parameter in our model. Section 4 concludes the paper.

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<sup>3</sup>Berkelaar and Kouwenberg (2000) also try to use prospect theory to explain the equity premium. They break loss averse attitude into two parts: one part is a probability maximizing strategy, and the other is a growth strategy. They use the martingale method to solve the dynamic asset allocation problem. In their asset allocation model, they found that the representative agent with loss aversion cannot adequately explain the equity premium puzzle.

## 2. The Model

### 2.1. Our model

We assume there exists a representative consumer in the economy, and he is with a time-additive utility function:

$$U_t = \sum_{j=0}^{\infty} \delta^j u(c_{t+j}, v_{t+j}), \quad (2.1)$$

where  $\delta$  is the discount factor, and in order to take prospect theory into account, the utility function  $u(c_t, v_t)$  is as follows:

$$u(c_t, v_t) = \begin{cases} 1 - e^{-\beta(c_t - v_t)}, & \text{if } c_t - v_t \geq 0, \\ -\lambda[1 - e^{\frac{\beta}{\lambda}(c_t - v_t)}], & \text{if } c_t - v_t < 0, \end{cases} \quad (2.2)$$

where  $\beta$  is the risk aversion coefficient. The consumption level at time  $t$  is  $c_t$ , and  $v_t$  is the consumption benchmark level at time  $t$ . When  $c_t - v_t \geq 0$ , it means the consumer's consumption level is better than the benchmark level, so we say he is in a "good state". In this case, his utility is assumed to follow the exponential risk averse utility function  $1 - e^{-\beta(c_t - v_t)}$ . Otherwise, when  $c_t - v_t < 0$ , the consumer's consumption level is worse off than at the benchmark level. Under this scenario, we say he is in a "bad state", and with the utility  $-\lambda[1 - e^{\frac{\beta}{\lambda}(c_t - v_t)}]$ , where  $\lambda$  is the loss aversion coefficient and is set to be larger than 1. This means that when the representative consumer is in a bad state, he will become more sensitive to the relative consumption level than when he is in a good state. To deal with the determination and updating of the reference point,  $v_t$ , in prospect theory, the idea of habit formation is adopted. To include the features of habit formation and "catching up with the Joneses" into our model, the following form of the benchmark of consumption level  $v_t$  is adopted:

$$v_t = wc_{t-1} + (1 - w)\overline{C}_{t-1}, \quad 0 \leq w \leq 1. \quad (2.3)$$

Note that  $v_t$  consists of the last period's individual consumption level  $c_{t-1}$  and the last period's aggregate consumption level per capita  $\overline{C}_{t-1}$ . The function of  $v_t$  in the utility function includes the concepts of both habit formation and "catching up with the Joneses." When  $w = 1$ ,  $v_t$  is influenced only by the last period's individual consumption level, the utility function displays

habit formation. Habit formation states that the utility level of the representative consumer depends only on the difference between today's individual consumption and the consumption level in the last period. If  $w = 0$ ,  $v_t$  only depends on the aggregate consumption level per capita in the last period. In this case, the utility function displays the phenomenon of "catching up with the Joneses", which means that the utility level of the representative consumer is a function of the difference between today's individual consumption and the aggregate consumption level per capita in the last period.

In previous works, if consumption were to unfortunately fall below the habit-benchmark, the investor's marginal utility would not always remain finite and positive. In Campbell and Cochrane (1999), they adopt a highly persistent nonlinear historic-consumption benchmark level to let it be always below consumption. On the other hand, in Abel (1990, 1999), he changes the subtract utility form to a ratio one,  $(c_t/v_t)^{1-\beta}/(1-\beta)$ , to avoid this problem. However, in this paper, by force of prospect theory, we are able to handle the situation  $c_t - v_t < 0$  comfortably.

## 2.2. Comparisons with previous models

### 2.2.1. Original prospect theory

Prospect utility function is proposed by Kahneman and Tversky (1979) as follows:

$$u(x) = \begin{cases} (x - v)^{1-\beta}, & \text{if } x - v \geq 0, \\ -\lambda(v - x)^{1-\beta}, & \text{if } x - v < 0, \end{cases} \quad \text{where } \lambda > 1,$$

where  $x$  represents the wealth level, and  $v$  can be thought of as the wealth level of the previous period or as an imaginary benchmark level, so  $(x - v)$  represents the amount of gains or losses. In addition to  $x$  and  $v$ , there are two other parameters in the utility function: the parameter of  $\beta$  is the risk averse coefficient of the utility function and the parameter of  $\lambda$  is the loss averse coefficient. According to the experiments of Kahneman and Tversky (1979), they found that  $\beta$  is 0.12 and  $\lambda$  is 2.25. What  $\lambda$  means is that when a loss occurs, the utility level will decrease more rapidly in order to reflect loss aversion. Figure 1 displays the meaning of prospect theory. It is seen that when facing the same amount of gain and loss,  $\Delta x$ , the decreased utility caused by the loss  $\overline{BC}$  is larger than the increased utility caused by the same amount of gain  $\overline{AB}$ .

Is it possible to apply Kahneman and Tversky's original prospect utility as the representative agent's utility over consumption directly? It is not so

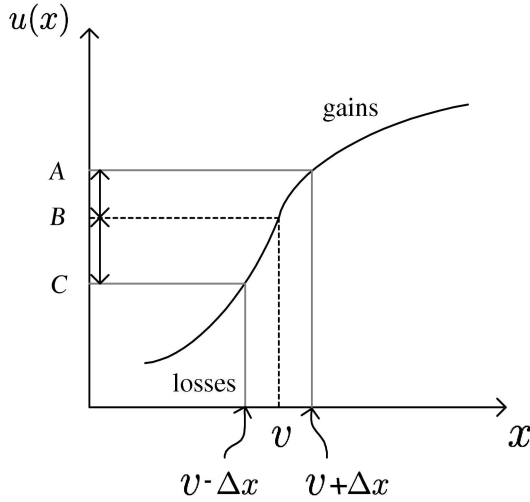


Fig. 1. Diagram of the prospect utility function. If one is with prospect utility, he is risk averse when facing gains ( $x \geq v$ ) and becomes risk loving when facing losses ( $x < v$ ). In addition, when facing the same amount of gain and loss,  $\Delta x$ , against the benchmark  $v$ , the decrease in utility through the loss  $\overline{BC}$  is larger than  $\overline{AB}$ , the increase in utility through the gain.

easy because there is a critical problem when applying this kind of utility function — its first-order derivative at  $x = v$  is not well-defined.<sup>4</sup> In order to overcome this problem, we propose a new version of prospect utility function. We replace the power utility functions inside prospect utility function with exponential utility functions, which is just like Eq. (2.2). As the same setting in the value function of the original prospect theory,  $\beta$  is the risk averse coefficient and  $\lambda$  is the loss averse coefficient. The spirit of the function we propose is the same as the original prospect theory.

Relative to the original prospect theory, it should be noted in Eq. (2.2) of our model that when  $c_t - v_t < 0$ , we change the risk aversion coefficient from  $\beta$  to  $\beta/\lambda$ . Having made this small change, the first-order derivative of prospect utility function in our model will be continuous, which is as

<sup>4</sup>In order to preclude this problem, some previous researches modify their asset pricing models, where the representative agent chooses the optimal consumption and investment ratios, instead of choosing the consumption and investment levels directly. Others apply a simplified linear version of prospect theory and try to separate the consumption or investment levels out of the prospect utility. These features let them avoid the problem that prospect utility function is not differentiable at the reference point.

follows:

$$u'(c_t, v_t) = \begin{cases} \beta e^{-\beta(c_t - v_t)}, & \text{if } c_t - v_t \geq 0, \\ \beta e^{\frac{\beta}{\lambda}(c_t - v_t)}, & \text{if } c_t - v_t < 0. \end{cases}$$

Because  $\lim_{c_t \rightarrow v_t^-} u'(c_t, v_t) = \lim_{c_t \rightarrow v_t^+} u'(c_t, v_t) = \beta$ , we can say that at  $c_t = v_t$ , our prospect utility function is differentiable.<sup>5</sup>

We present all the above analyses in Fig. 2. Although we make some changes to the original prospect utility function, the outline of the improved one looks similar to and maintains the spirit of the original one. After all, the most important thing is to make the function differentiable. Given this improvement, this prospect utility function can be applied to not only the consumption-based asset pricing model but also to many other economic issues.

### 2.2.2. The model in Barberis, Huang and Santos (2001)

Although Barberis, Huang and Santos (2001) is the first to introduce prospect theory into the consumption-based asset pricing model, they do not actually apply prospect utility as the representative agent's utility over his consumption streams. In their model, they separate the utility function of the representative consumer into two parts. One part is a risk averse consumption utility function, and the other is a utility function over financial investment. Their model is briefly shown as follows:

$$\max_{c_t, S_t} E \left[ \sum_{t=0}^{\infty} \left( \delta^t \frac{c_t^{1-\beta}}{1-\beta} + b_t \delta^{t+1} v(X_{t+1}) \right) \right],$$

s.t.  $W_{t+1} = (W_t - c_t)R_f + X_{t+1}$ ,

where

$$X_{t+1} = S_t R_{t+1}^s - S_t R_f = S_t (R_{t+1}^s - R_f),$$

and

$$v(X_{t+1}) = \begin{cases} X_{t+1}, & \text{if } X_{t+1} \geq 0, \\ \lambda(X_{t+1}), & \text{if } X_{t+1} < 0. \end{cases}$$

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<sup>5</sup>The first-order derivative of original prospect utility function is

$$u'(x) = \begin{cases} (1-\beta)(x-v)^{-\beta}, & \text{if } x-v \geq 0, \\ \lambda(1-\beta)(v-x)^{-\beta}, & \text{if } x-v < 0. \end{cases}$$

It is clear that when  $x = v$ , the first-order derivative does not exist.



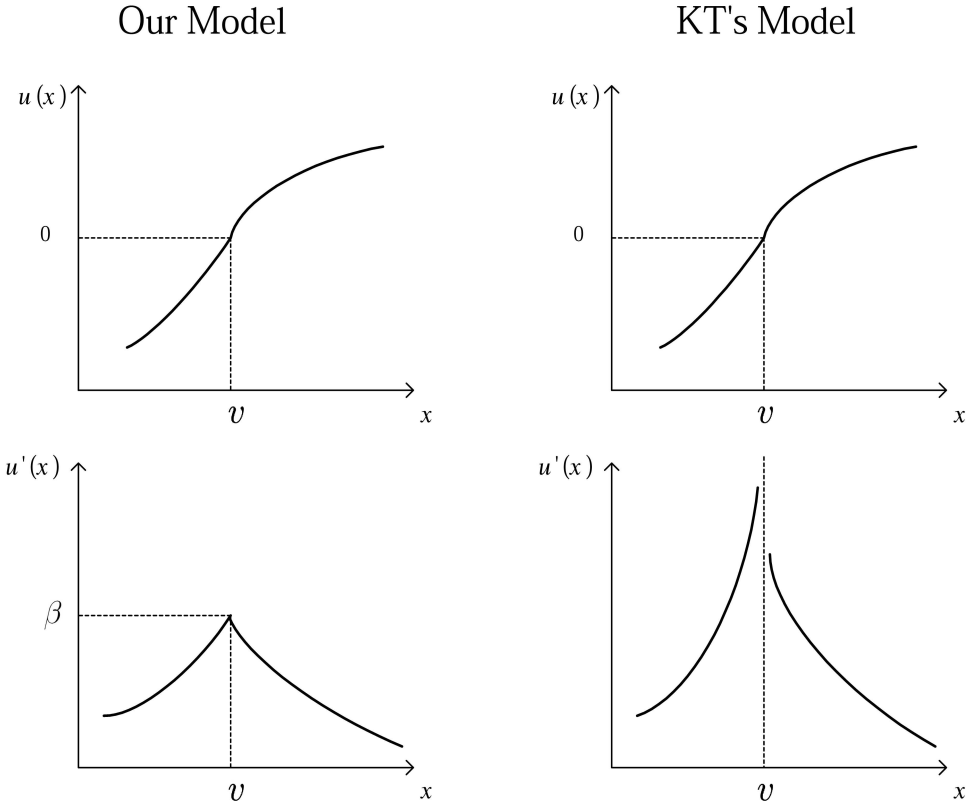


Fig. 2. Difference between the original and our prospect theory. Although we make some changes to the original prospect utility function, the outline of our model looks similar to and maintains the spirit of the original one. At the same time, the function of the first-order derivative of the original form is not continuous (the reference point  $v$  is a singularity point), but in our utility form, the first-order derivative is well-defined everywhere.

Inside the equations,  $W_t$  is the wealth level at time  $t$ . The parameter  $c_t$  represents the consumption of the representative agent and  $S_t$  is the representative agent's amount of stock in possession at time  $t$ . In addition,  $X_{t+1}$  is the excess return of the stock,  $\delta$  is the time discount factor, and  $b_t$  is an exogenous scaling factor. The utility over consumption  $c_t$  is a standard power utility function, and the utility over financial investment is a simplified linear prospect utility function found by setting the risk aversion coefficient in prospect theory to be zero. These settings are employed to overcome the nondifferentiable problem at the reference point.

However, there is a vital problem in this model. Whenever an investor has gains or losses from financial investment,  $X_{t+1}$ , these will bring him utility via  $v(X_{t+1})$ . Minus his consumption at time  $t$ , the residual gains or

losses will accumulate into the next-period wealth,  $W_{t+1}$ . The  $W_{t+1}$  will affect the next-period consumption, from which he can derive utility again. As a result, their model actually counts the effect of the financial return twice. One is via the simplified linear loss aversion utility function, and the other is via the power utility function over the next-period consumption.

### 2.3. Asset returns

In this section, we will derive the asset returns via the Euler equation in our consumption-based asset pricing model. The spirit of the Euler equation states that consumers will sacrifice today's consumption level in exchange for increasing their holdings of some assets. Holding the asset will bring them returns that can be transformed into consumption goods in the next period. The Euler equation is stated as follows:

$$E_t \left[ -\frac{\partial U_t}{\partial c_t} + \delta R_{t+1}^i \left( \frac{\partial U_{t+1}}{\partial c_{t+1}} \right) \right] = 0,$$

where  $R_{t+1}^i$  is the rate of the return of the asset  $i$  between today and the next period. Separating the above equation, we obtain:

$$E_t \left[ \frac{\partial U_t}{\partial c_t} \right] = E_t \left[ \delta R_{t+1}^i \left( \frac{\partial U_{t+1}}{\partial c_{t+1}} \right) \right]. \quad (2.4)$$

Following Eqs. (2.1) and (2.3),  $\partial U_t / \partial c_t$  is represented as follows:

$$\frac{\partial U_t}{\partial c_t} = u_{c_t}(c_t, v_t) + \delta u_{v_{t+1}}(c_{t+1}, v_{t+1}) \frac{\partial v_{t+1}}{\partial c_t}. \quad (2.5)$$

Similarly,

$$\frac{\partial U_{t+1}}{\partial c_{t+1}} = u_{c_{t+1}}(c_{t+1}, v_{t+1}) + \delta u_{v_{t+2}}(c_{t+2}, v_{t+2}) \frac{\partial v_{t+2}}{\partial c_{t+1}}. \quad (2.6)$$

Notice that  $\partial U_t / \partial c_t$  depends not only on the derivative of  $u(c_t, v_t)$  with respect to  $c_t$ , but also on the derivative of  $u(c_{t+1}, v_{t+1})$  with respect to  $c_t$ . By the same reasoning, we can see that in order to obtain  $\partial U_{t+1} / \partial c_{t+1}$ , we should take the derivatives of both  $u(c_{t+1}, v_{t+1})$  and  $u(c_{t+2}, v_{t+2})$  with respect to  $c_{t+1}$  into account.

When calculating  $\partial U_t / \partial c_t$ , because  $u(c_{t+1}, v_{t+1})$  is needed to be taken into consideration,  $\partial U_t / \partial c_t$  does not belong to the information set of time  $t$ . So it is still necessary to maintain the expectation at  $t$  of  $\partial U_t / \partial c_t$ . That is

different from those in previous articles. We divide both sides of Eq. (2.4) by  $E_t[\frac{\partial U_t}{\partial c_t}]$ , then

$$E_t \left[ \delta R_{t+1}^i \left( \frac{\frac{\partial U_{t+1}}{\partial c_{t+1}}}{E_t \left[ \frac{\partial U_t}{\partial c_t} \right]} \right) \right] = 1. \quad (2.7)$$

According to the general definition,  $M_t^{t+1} = \frac{\partial U_{t+1}/\partial c_{t+1}}{E_t[\partial U_t/\partial c_t]}$  is the intertemporal marginal rate of substitution, and Eq. (2.7) can be rewritten as follows:

$$E_t [\delta R_{t+1}^i M_t^{t+1}] = 1. \quad (2.8)$$

If the asset is a risk-free security, then Eq. (2.8) is rewritten as:

$$R_{t+1}^B E_t [\delta M_t^{t+1}] = 1. \quad (2.9)$$

When the asset is a risky security, the situation is different. In this paper, the processes of consumption growth and dividend growth are separately assumed. According to the definition of  $R_{t+1}^s = (P_{t+1}^s + D_{t+1})/P_t^s$ , where  $P_t^s$  means the price of the risky security at time  $t$  and  $D_t$  means the dividend at time  $t$ , Eq. (2.8) can be rewritten as:<sup>6</sup>

$$E_t \left[ \delta \frac{D_{t+1}}{D_t} \left( 1 + \frac{P_{t+1}^s}{D_{t+1}} \right) M_t^{t+1} \right] = \frac{P_t^s}{D_t}. \quad (2.10)$$

In order to derive  $R_{t+1}^B$  and  $R_{t+1}^s$ , we must find  $M_t^{t+1}$  first. From Eqs. (2.5) and (2.6), we can see that in order to obtain  $M_t^{t+1}$ , we need to take  $u(c_t, v_t)$ ,  $u(c_{t+1}, v_{t+1})$ , and  $u(c_{t+2}, v_{t+2})$  into consideration. Since prospect theory is applied, the utility function of each period may not be the same. If  $c_t - v_t \geq 0$ , the utility function is  $1 - e^{-\beta(c_t - v_t)}$ . If  $c_t - v_t < 0$ , because of the characteristic of loss aversion, the utility of the representative agent becomes  $-\lambda[1 - e^{\frac{\beta}{\lambda}(c_t - v_t)}]$ . In order to calculate the intertemporal marginal rate of substitution  $M_t^{t+1}$ , in addition to comparing the values of  $c_t$  and  $v_t$ , the comparisons of the values of  $c_{t+1}$  and  $v_{t+1}$  and the values of  $c_{t+2}$  and  $v_{t+2}$  are also necessary. Because the agent is in either a good state or a bad state at each date, it is needed to decompose the marginal rate of substitution,  $M_t^{t+1}$ , into  $2^3 = 8$  different scenarios.

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<sup>6</sup>If a risky security is treated as a stream of consumption claims, the Euler equation is as follows.

$$E_t \left[ \delta \frac{c_{t+1}}{c_t} \left( 1 + \frac{P_{t+1}^s}{c_{t+1}} \right) M_t^{t+1} \right] = \frac{P_t^s}{c_t},$$

where the return  $R_{t+1}^s = (P_{t+1}^s + c_{t+1})/P_t^s$ .

In order to simplify the representation, we define the following indicator variables:

$$\begin{aligned} l &= I_{\{c_t - v_t \geq 0\}}, & \phi_t &= 1 \cdot l + \left(-\frac{1}{\lambda}\right) \cdot (1 - l), \\ m &= I_{\{c_{t+1} - v_{t+1} \geq 0\}}, & \phi_{t+1} &= 1 \cdot m + \left(-\frac{1}{\lambda}\right) \cdot (1 - m), \\ n &= I_{\{c_{t+2} - v_{t+2} \geq 0\}}, & \phi_{t+2} &= 1 \cdot n + \left(-\frac{1}{\lambda}\right) \cdot (1 - n). \end{aligned}$$

Following the definition of  $M_t^{t+1}$  and some derivative calculations, the marginal rate of substitution  $M_t^{t+1}$  is:

$$M_t^{t+1}(l, m, n) = \frac{e^{-\beta\phi_{t+1}(c_{t+1}-v_{t+1})} - \delta w e^{-\beta\phi_{t+2}(c_{t+2}-v_{t+2})}}{e^{-\beta\phi_t(c_t-v_t)} - \delta w E_t[e^{-\beta\phi_{t+1}(c_{t+1}-v_{t+1})}]}. \quad (2.11)$$

Following above equation and Eqs. (2.9) and (2.10), the pricing equations of the returns of assets can be rewritten as follows:<sup>7</sup>

$$R_{t+1}^B E_t[\delta M_t^{t+1}(l, m, n)] = 1, \quad (2.12)$$

$$E_t \left[ \delta \frac{D_{t+1}}{D_t} \left( 1 + \frac{P_{t+1}^s}{D_{t+1}} \right) M_t^{t+1}(l, m, n) \right] = \frac{P_t^s}{D_t}. \quad (2.13)$$

Like the original consumption-based asset pricing model, we assume the growth rate of both individual consumption and aggregate consumption per capita follow the same distribution:

$$\ln \frac{c_{t+1}}{c_t} = \ln \frac{\overline{C}_{t+1}}{\overline{C}_t} = g_t^{t+1} = \mu_{g_t^{t+1}} + z_g, \quad z_g \sim \mathcal{N}(0, \sigma_{g_t^{t+1}}). \quad (2.14)$$

In Mehra and Prescott (1985), they use not only  $\mu_{g_t^{t+1}}$  and  $z_g$  to describe the consumption process, but also the correlation between  $g_t^{t+1}$  and  $g_{t-1}^t$ . Following their approach, we also take  $\text{corr}(g_t^{t+1}, g_{t-1}^t) = \rho_{gg}$  into consideration.

In our model, dividend growth is not assumed to follow the same distribution as consumption growth, so another lognormal distribution for dividend growth process is assumed. In addition, it is assumed that there is an

<sup>7</sup>If treating the risky security as a pure consumption stream, the corresponding equation is

$$E_t \left[ \delta \frac{c_{t+1}}{c_t} \left( 1 + \frac{P_{t+1}^s}{c_{t+1}} \right) M_t^{t+1}(l, m, n) \right] = \frac{P_t^s}{c_t}.$$

imperfect correlation between consumption and dividend growth processes. We assume

$$\ln \frac{D_{t+1}}{D_t} = y_t^{t+1} = \mu_{g_t^{t+1}} + z_y, z_y \sim \mathcal{N}(0, \sigma_{y_t^{t+1}}), \quad \text{and} \quad \text{corr}(z_y, z_g) = \rho_{yg}. \quad (2.15)$$

In the above equation, the means of the consumption growth and dividend growth are the same, but the standard deviations of these two processes are different. This is because the consumption and dividend streams will ultimately share the same trend in the long run, but the volatilities of these two streams are not the same. Furthermore, because of the assumption of imperfect correlation, the correlation between these two processes is also considered.

#### 2.4. Simulation algorithm

In this paper, just like many previous researches, a simulation method is adopted to derive the returns of assets. In real life, at each point in time  $t$ , the representative agent chooses the consumption level  $c_t$  based on the information of the reference point  $v_t^s$  to maximize his expected utility,  $E_t[U_t]$ . During the maximizing phase, the returns of the riskless and risky assets between today and the next period are determined. The equity premium is derived by calculating the difference between the arithmetic average of the returns of the riskless and risky assets over time.

Our simulation algorithm is formulated to mimic the above situation. First, assuming the reference point  $v_t$  and the chosen consumption level  $c_t$  are given, the conditional rates of return  $R_{t+1}^B(v_t, c_t)$  and  $R_{t+1}^S(v_t, c_t)$  are derived. Second, because  $v_t$  and  $c_t$  changes over time, the arithmetic average over time actually means the arithmetic average over different combinations of the values of  $v_t$  and  $c_t$ . So we obtain the unconditional rates of return  $R_{t+1}^B$  and  $R_{t+1}^S$  through integration over  $v_t$  and  $c_t$ . Finally, the difference between these two values is the equity premium. The details about how to derive the conditional and unconditional expected rates of return are as follows.

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<sup>8</sup>Of course the information of  $P_t^s/D_t$  is known, too. However, because when calculating the rate of return of the risky asset in our algorithm, what should be derived is the ratio  $(P_{t+1}^s + D_{t+1})/P_t^s = (D_{t+1}/D_t)(P_{t+1}^s/D_{t+1} + 1)/(P_t^s/D_t)$ , the effect of  $P_t^s/D_t$  is normalized in our model. Therefore, the expected returns are not conditional on this information.

#### 2.4.1. Conditional expected rates of return

Based on the given values of the reference point  $v_t$  and the consumption level  $c_t$ , the steps to derive the conditional rates of return  $R_{t+1}^B(v_t, c_t)$  and  $R_{t+1}^s(v_t, c_t)$  are:

First, following the assumptions in Eqs. (2.14) and (2.15), 5,000 sets of random samples of  $(g_t^{t+1}, g_{t+1}^{t+2}, y_t^{t+1})$  is generated.<sup>9</sup> Based on the information of  $v_t$  and given the value of  $c_t$ , then 5,000 sets of  $(c_{t+1}, c_{t+2}, v_{t+1}, v_{t+2})$  can be derived.

Second, for each set, Eqs. (2.12) and (2.13) are applied to derive the conditional  $R_{t+1}^B(v_t, c_t)$  and  $R_{t+1}^s(v_t, c_t)$ . However, the value of  $M_t^{t+1}(l, m, m)$  for each set should be decided first. Because there is a term  $E_t[e^{-\beta\phi_{t+1}(c_{t+1}-v_{t+1})}]$  inside  $M_t^{t+1}(l, m, m)$ , this expectation is needed to be calculated before deriving the value of  $M_t^{t+1}(l, m, m)$  for each set. In this model, the arithmetic average of  $e^{-\beta\phi_{t+1}(c_{t+1}-v_{t+1})}$  is derived as the approximation of this expectation. Once we have the value of  $E_t[e^{-\beta\phi_{t+1}(c_{t+1}-v_{t+1})}]$ , and for each set of  $(c_{t+1}, c_{t+2}, v_{t+1}, v_{t+2})$ , we judge whether  $c_{t+j} - v_{t+j}$  ( $j = 0, 1, 2$ ) is larger than zero, then we can settle on the right form of  $M_t^{t+1}(l, m, m)$  for each set. Given the information of  $v_t$  and the chosen value of  $c_t$ , for each set of values  $c_{t+1}$ ,  $c_{t+2}$ ,  $v_{t+1}$ ,  $v_{t+2}$ , and  $y_t^{t+1}$ ,<sup>10</sup> the corresponding values of conditional  $R_{t+1}^B(v_t, c_t)$  and  $R_{t+1}^s(v_t, c_t)$  can be derived according to Eqs. (2.12) and (2.13). Calculate the arithmetic average of rates of return over the 5,000 sample sets, and we will have the values of the conditional expected rates of return,  $R_{t+1}^B(v_t, c_t)$  and  $R_{t+1}^s(v_t, c_t)$ .

#### 2.4.2. Unconditional expected rates of return

In this paper, we calculate the unconditional expected rates of return by numerical integration over possible values of  $v_t$  and  $c_t$ . If we assume the

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<sup>9</sup>These 5,000 sets of random samples are used to describe the possible states of nature of the world in our model. They are sampled once, and kept to calculate all the results under different combinations of the values of the parameters. If different random samples are used to calculate the results of different values of the parameters, it is not able to tell the differences are caused from different values of the parameters or from different random samples.

<sup>10</sup>In the pure consumption case, the value of  $y_t^{t+1}$  is not needed. In addition to  $c_{t+1}$ ,  $c_{t+2}$ ,  $v_{t+1}$ , and  $v_{t+2}$ , we also need the value of  $g_t^{t+1}$  to derive the corresponding values of  $R_{t+1}^B(v_t, c_t)$  and  $R_{t+1}^s(v_t, c_t)$ .

probability density function of  $v_t$  and  $c_t$  to be  $f(v_t, c_t)$ , the unconditional  $R_{t+1}^B$  and  $R_{t+1}^S$  are as follows.

$$R_{t+1}^B = \int_{v_t} \int_{c_t} R_{t+1}^B(v_t, c_t) f(v_t, c_t) dc_t dv_t,$$

$$R_{t+1}^S = \int_{v_t} \int_{c_t} R_{t+1}^S(v_t, c_t) f(v_t, c_t) dc_t dv_t.$$

In our model,  $v_t \sim N(\mu_c, \sigma_c)$  is assumed, where  $\mu_c$  and  $\sigma_c$  are the mean and the standard deviation of the consumption level calculated from the historical quarterly data of per capita consumption in the US. However, knowing  $v_t$  is equivalent to knowing last period's consumption data,  $c_{t-1}$  and  $\overline{C}_{t-1}$ , therefore there must be some correlation between  $v_t$  and today's consumption level,  $c_t$ . In order to not introduce more parameters to describe the correlation between  $c_t$  and  $v_t$ , an assumption is adopted so that  $\ln(c_t/v_t) = g_{t-1}^t$ , where  $g_{t-1}^t$  is the consumption growth rate from  $t-1$  to  $t$  and its distribution is from Eq. (2.14). With this setting, the above equations can be rewritten as follows:

$$R_{t+1}^B = \int_{v_t} \int_{g_{t-1}^t} R_{t+1}^B(v_t, e^{g_{t-1}^t} v_t) h(v_t, g_{t-1}^t) dg_{t-1}^t dv_t,$$

$$R_{t+1}^S = \int_{v_t} \int_{g_{t-1}^t} R_{t+1}^S(v_t, e^{g_{t-1}^t} v_t) h(v_t, g_{t-1}^t) dg_{t-1}^t dv_t.$$

Because of the independence characteristic between  $v_t$  and  $g_{t-1}^t$ , a bivariate normal independent probability distribution function  $h(v_t, g_{t-1}^t)$  is applied for the integration. Via numerical integration, we are able to obtain the unconditional expected rates of return,  $R_{t+1}^B$  and  $R_{t+1}^S$ .

### 3. Numerical Results

In this section, the values of parameters in our model are discussed first. Then we report the results based on our model. Finally, we will show the statistic analysis of each parameter in our model.

#### 3.1. The settings of parameters

Table 1 lists the parameters and their assumed values in our model. We will explain the meaning of each parameter value in detail. First, following the data set of Campbell and Cochrane (1999), we assume both the expected mean of logarithmic consumption growth,  $g_t^{t+1}$ , and dividend growth,  $y_t^{t+1}$ , to be 0.0189. As for the standard deviations of these two random variables, we assume the standard deviation of  $g_t^{t+1}$  to be 0.015, and the standard

Table 1. Parameters and their assigned values.

There are many parameters in our model and the following table lists the value of each parameter. In order to prove that our model is more capable than previous models of solving the equity premium puzzle through combining prospect theory and habit formation into consumption-based asset pricing model, the values of parameters in our model are mostly collected from previous researches.

Parameters	Value
Mean of consumption growth, $\mu_{g_t^{t+1}}$	0.0189
Standard deviation of consumption growth, $\sigma_{g_t^{t+1}}$	0.015
Correlation between $g_t^{t+1}$ and $g_{t-1}^t$ , $\rho_{gg}$	-0.14
Mean dividend growth, $\mu_{y_t^{t+1}}$	0.0189
Standard deviation of dividend growth, $\sigma_{y_t^{t+1}}$	0.112
Correlation between $y_t^{t+1}$ and $g_t^{t+1}$ , $\rho_{yg}$	0.2
Loss aversion coefficient, $\lambda$	2.25
Risk aversion coefficient, $\beta$	1.5
Discount factor, $\delta$	0.99
$w$ (benchmark level $v_t = wc_{t-1} + (1-w)\overline{C_{t-1}}$ )	0.5
Mean of annualized consumption level, $\mu_c$ (US\$1,000)	4.57336
Standard deviation of annualized consumption level, $\sigma_c$ (US\$1,000)	1.02046

deviation of  $y_t^{t+1}$  to be 0.112. As for the correlation between  $y_t^{t+1}$  and  $g_t^{t+1}$ ,  $\rho_{yg}$ , one may expect the correlation to approach 1 because dividends and consumption processes ultimately share the same long-term trend. However, in fact, the growth rates of stock market dividends and consumption are only weakly correlated in US data. Following Campbell and Cochrane (1999), the value of the parameter is assumed to be 0.2. In addition,  $\rho_{gg}$ , which is the correlation between  $g_t^{t+1}$  and  $g_{t-1}^t$ , is taken into consideration, too. In Abel (1990), following Mehra and Prescott (1985), the value of the parameter is assumed to be -0.14.

Third, the primary feature of our model is to employ prospect theory to describe the behavior of the representative consumer. Hence, there is one more parameter  $\lambda$  in our model. The parameter  $\lambda$  is called the loss averse coefficient because we use it to describe the loss averse behavior of the consumer. When the value of  $\lambda$  becomes larger, the consumer is more loss averse and therefore more afraid of incurring the loss. In our model, according to the suggested value in the original prospect theory, we set  $\lambda$  to be 2.25. In addition to the loss averse coefficient, we also have the risk averse coefficient  $\beta$  as in the traditional models. Unlike the unreasonably large value of the risk aversion coefficient used in previous studies to match the observed large risk premium, in our model, we assume  $\beta$  to be a more realistic value,



1.5, thus representing a consumer possessing mild degree of risk aversion in the good state, and with mild degree of risk loving in the bad state.

The parameter  $w$  represents the features of “catching up with the Joneses” and habit formation. In our model, we have a benchmark consumption level  $v_t = wc_{t-1} + (1 - w)\overline{C_{t-1}}$ . When  $w = 0$ ,  $v_t = \overline{C_{t-1}}$ , this means that the utility function of the consumer displays the characteristic of “catching up with the Joneses”. Another extreme case is when  $w = 1$ ,  $v_t = c_{t-1}$ . This means the utility function of the consumer has the characteristic of habit formation. In fact, we do not have a clear idea about how the representative agent feels, so we assume the reference point of the consumption level of the parameter  $w$  to be 0.5. This means that the utility of the representative consumer has both features of “catching up with the Joneses” and habit formation.

The benchmark  $v_t$  plays an important role in our model. Not only because its construction contains habit formation and “catching up with the Joneses”, but because when we set  $v_t = 0$ , our model degenerates to a classical exponential utility model. We will compare our results with the ones derived from the degenerate model.

Finally, we assume the discount factor  $\delta$  to be a reasonable value, 0.99. In addition, in order to derive the unconditional expected rates of return, we need to have the initial distribution of consumption level per capita. In the paper, the annualized consumption level per capita is assumed to follow a normal distribution whose mean and standard deviation are calculated from the quarterly data of consumption level per capita from 1959:4 to 1996:1 in the US. These values are listed in the end of Table 1.

### 3.2. Results and comparative statics

In the following paragraphs, it is shown that introducing prospect theory into the consumption-based asset pricing model is the key reason why we are able to resolve the equity premium puzzle. In addition to showing the values of  $R_{t+1}^B$  and  $R_{t+1}^s$  derived by our model, we also report comparative analyses of some parameters in this section. We want to see how changes in the values of parameters affect the equity premium.

Based on the values of the parameters in the last section, we can derive an equity premium of 6.43% and a risk-free real interest rate of about 0.98% in our model. This result appears much closer to the historical data. Furthermore, Table 2 lists the results of our model and those of the classical exponential form utility model in both dividend-claim and consumption-claim

Table 2. Summary of results in different models and different cases.

The table lists the results of our prospect utility model and those of the classical exponential form utility model in both dividend-claim and consumption-claim cases.

	Exponential Utility + Consumption Claim (E + C)	Exponential Utility + Dividend Claim (E + D)	Prospect Utility + Consumption Claim (P + C)	Prospect Utility + Dividend Claim (P + D)
$R_{t+1}^s$ (%)	14.80	15.99	6.31	7.41
$R_{t+1}^B$ (%)	13.73	13.73	0.98	0.98
$R_{t+1}^s - R_{t+1}^B$ (%)	1.07	2.26	5.33	6.43

cases. When we apply classical exponential utility into the consumption-based asset pricing model, and only consider the consumption process, the equity premium would be 1.07%; however if we take both consumption and dividend processes into consideration, we obtain a 2.26% equity premium. Relatively speaking, in our prospect theory consumption-based asset pricing model, if we only consider consumption process, the equity premium would be 5.33%, and if taking both consumption and dividend processes into consideration, we obtain a 6.43% equity premium.

When we compare the results, some important issues should be noted. First, with the same setting of parameters, the classical exponential utility model cannot generate a large enough value of equity premium. This result again proves the existence of the equity premium puzzle. Furthermore, the scale of the rates of return of both the riskless and risky assets seems too large in the exponential-form case. If the rates of return of both the riskless and risky assets are within the right scale, the derived equity premium may be smaller than those in the table. Second, separating the consumption and dividend processes does help alleviate the equity premium puzzle. However, with this alone, we still cannot solve the puzzle. Third, it is found that introducing the dividend process brings only about one more percent of equity premium; however, if we assume the representative agent is with prospect utility, we can obtain about four more percent of equity premium. According to these results, we conclude that by force of prospect theory, our model is more capable of solving the equity premium puzzle.

Because the motivations of this paper is to mainly apply prospect theory to consumer behavior, the main focus of the following subsection is to show equity premium as a function of the risk averse and loss averse coefficients in our prospect utility. As to the role of habit formation in our model, it is

only applied to complement the deficiency of prospect theory and is used to determine the reference point of prospect theory in our consumption-based asset pricing model. Even though habit formation is not the key factor to what we want to elaborate in this paper, the results of the correlation between equity premium and  $w$  is still shown in the following context.

### 3.2.1. *The effect of risk averse coefficient on equity premium*

Figure 3 and Table 3 present equity premium as a function of the risk aversion coefficient,  $\beta$ . No matter whether treating the risky asset as a stream

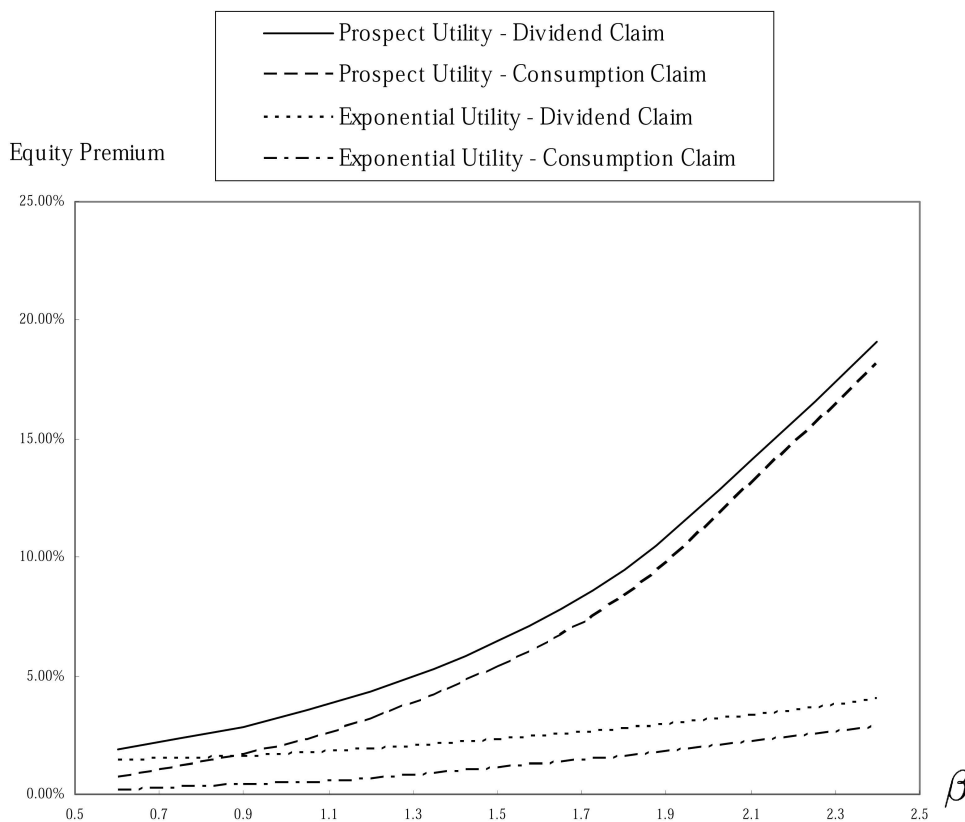


Fig. 3. The effect of risk averse coefficient on equity premium. It is easy to see that the equity premium becomes larger when the risk aversion coefficient becomes larger in both the consumption-claim or dividend-claim cases and in both prospect theory utility and exponential utility models. Moreover, one can see that in the classical exponential utility model, the effect of  $\beta$  on equity premium is almost linear. In our model, however, it looks like prospect utility will enlarge the effect of  $\beta$  on the equity premium. Because  $\beta$  dramatically affects the equity premium in our model, we do not need a very large  $\beta$  value to obtain a large enough equity premium.

Table 3. The effect of risk averse coefficient on equity premium.

Given different values of the risk averse coefficient  $\beta$ , this table lists the equity premiums derived from our prospect utility model (P) and the classical exponential form utility model (E) in both dividend-claim (D) and consumption-claim (C) cases. The values in parentheses are the riskless rates of return.

$\beta$	0.6	0.9	1.2	1.5	1.8	2.1	2.4
E+C (%)	0.15 (6.23)	0.36 (8.77)	0.66 (11.28)	1.07 (13.73)	1.57 (16.14)	2.17 (18.51)	2.87 (20.83)
E+D (%)	1.39 (6.23)	1.58 (8.77)	1.87 (11.28)	2.26 (13.73)	2.74 (16.14)	3.33 (18.51)	4.01 (20.83)
P+C (%)	0.71 (1.00)	1.69 (1.00)	3.18 (0.99)	5.33 (0.98)	8.39 (0.97)	13.13 (0.95)	18.15 (0.93)
P+D (%)	1.92 (1.00)	2.87 (1.00)	4.33 (0.99)	6.43 (0.98)	9.47 (0.97)	14.15 (0.95)	19.08 (0.93)

of consumption claims or dividend claims, it is easy to see that the equity premium becomes larger when the risk aversion coefficient becomes larger in both prospect theory utility model and exponential utility model. Moreover, one can see that in the degenerate model, the effect of  $\beta$  on equity premium is almost linear. In our model, however, it looks like prospect utility will enlarge the effect of  $\beta$  on the equity premium. The larger the  $\beta$  value, the quicker the equity premium will increase. Because  $\beta$  dramatically affects the equity premium in our model, we do not need a very large  $\beta$  value to obtain a large enough equity premium. In Table 3, when we consider dividend growth in the exponential utility model, one may wonder why when  $\beta$  equals 2.4, the equity premium is about 4.01%, which seems large enough to solve the puzzle. However, the risk-free return is now about 20.83%, which is an unreasonably large value. In fact, based on reasonable values of parameters, whenever the classical exponential utility model generates a large enough equity premium, the accompanying returns of riskless and risky assets will be too high.

In a pure risk averse framework, it is natural that when an investor becomes more risk averse (an increase in  $\beta$ ), the risky asset's equity premium will rise in order to attract the investor to buy the asset. However, in the prospect utility case, the situation is different. Whenever the value of  $\beta$  increases, the utility function not only becomes more concave in the good state, but also becomes more convex in the bad state. But why does our prospect utility perform like a risk averse utility? We believe there are two reasons and they are as follows.

Table 4. The analysis of the random samples.

This table summarizes the occurring probability of each scenario based on the five thousand sets of random samples in our model.

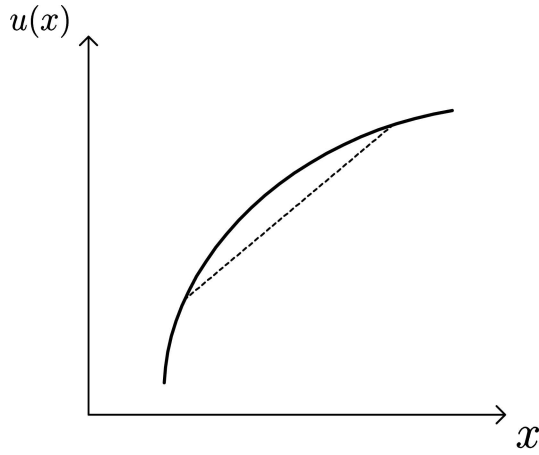
$c_t - v_t$	$c_{t+1} - v_{t+1}$	$c_{t+2} - v_{t+2}$	Prob.
+	+	+	71.67%
+	+	-	8.58%
+	-	+	8.77%
+	-	-	0.56%
-	+	+	8.34%
-	+	-	1.00%
-	-	+	1.02%
-	-	-	0.07%

First, from Table 4, it can be found that although there are eight scenarios in our model, the probability for the scenarios to be  $c_t - v_t \geq 0$ ,  $c_{t+1} - v_{t+1} \geq 0$ , and  $c_{t+2} - v_{t+2} \geq 0$  is about 71.67%. These numbers show that because of the persistent and mild growing trend of the consumption level, the representative agent in our model is almost in the good state. That means that the effect of concavity plays a dominating role in the result, so an increase in  $\beta$  in our prospect utility case will enlarge the equity premium.

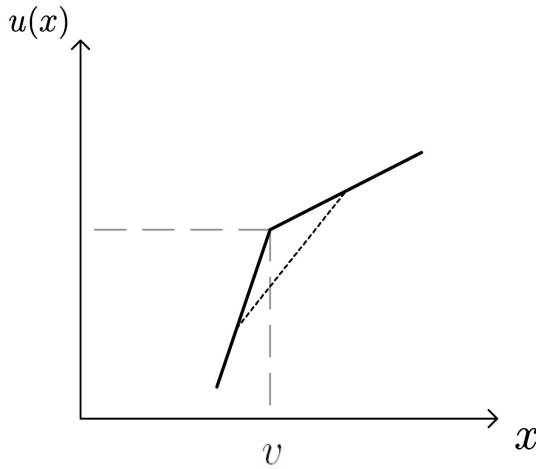
In addition, one may think that when the agent is mild risk loving in the bad state, he would be more willing to accept a small or negative risk premium. As a result, the equilibrium risk premium decreases. In a one-period model, because the decision is made only once, it is agreed that the risk loving attitude results in a smaller risk premium. However, in a multi-period situation, whenever the agent is in the bad state, he becomes risk loving temporarily and will start accepting risky assets. The acceptance of risky asset is thought to bring him enough return to cover the loss, which increases the probability of being in the good state in the future. Once he enters the good state, he will behave more risk averse to ensure he will end up with gains. Because of this, the risk premium increases at times and decreases at other times. Therefore, we cannot conclude that the average risk premium would necessarily decrease only based upon the risk loving attitude in the bad state.

Second, from Fig. 4, globally looking, loss aversion is somehow another kind of risk aversion. Thus with the help of loss aversion, our prospect utility can generate a larger equity premium than a pure risk averse utility.

Our main intention in this paper is the analysis of the effect of loss averse coefficient, and the detailed results are shown in the next context.



Risk averse utility



Risk neutral + Loss averse utility

Fig. 4. Loss aversion versus risk aversion. The upper figure is a classical risk averse utility function, and the lower one is a utility function where loss aversion is taken into consideration under the condition when the representative agent is risk neutral in both good and bad states. It is easy to see that loss aversion somehow means global risk aversion.

### 3.2.2. The effect of loss averse coefficient on equity premium

The primary feature of prospect theory is to introduce the loss averse attitude of human beings. Because there is no  $\lambda$  inside the degenerate model, we report the results for prospect utility model only. The solid line in Fig. 5 presents the equity premium as a function of the level of loss aversion

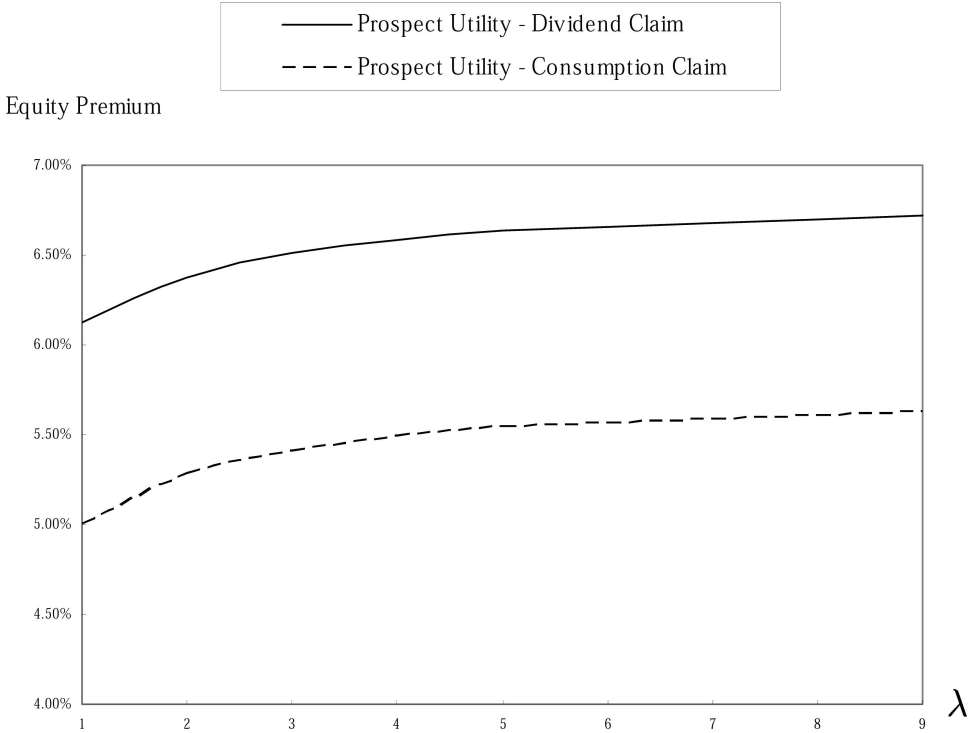


Fig. 5. The effect of loss averse coefficient on equity premium. The solid line presents the equity premium as a function of the loss aversion coefficient in dividend-claim case, and the dash line portrays the results in consumption-claim case. In both cases, when  $\lambda$  becomes larger, the equity premium becomes higher. The phenomenon reflects that if one is more loss averse, he needs more gains to cover the pains of losses that have occurred, so the equity premium rises. In addition, when  $\lambda$  is large enough, it becomes harder to increase the equity premium by enlarging  $\lambda$ . We think the results mean that human beings do have loss averse attitude, but the degree of the loss aversion coefficient is limited.

in the dividend-claim case, and the dash line portrays the results of our consumption-claim case. In both cases, when  $\lambda$  becomes larger, the equity premium becomes higher. The result is consistent with our expectations — if one is more loss averse, he needs more gains to cover the pains of losses that have occurred, so the equity premium rises. This is also why we want to apply loss aversion in prospect theory to resolve the equity premium puzzle.

In addition, marginal increasing level of equity premium decreases with respect to the loss aversion coefficient. From Table 5, when the value of  $\lambda$  is 1 (which means when it is in the bad state, the decreasing utility level of the representative agent is not amplified) and becomes 2, the equity premium rises significantly. However, when  $\lambda$  is large to an extent, it becomes harder to increase equity premium by enlarging  $\lambda$ . In our opinion, the above results

Table 5. The effect of loss averse coefficient on equity premium.

Given different values of the loss averse coefficient  $\lambda$ , this table lists the equity premium derived from our prospect utility model, and the value in parentheses is the real rate of return of the riskless asset.

$\lambda$	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
P+C (%)	5.00 (1.03)	5.28 (0.98)	5.41 (0.97)	5.49 (0.96)	5.54 (0.96)	5.56 (0.96)	5.58 (0.95)	5.60 (0.95)	5.62 (0.95)
P+D (%)	6.12 (1.03)	6.38 (0.98)	6.51 (0.97)	6.58 (0.96)	6.63 (0.96)	6.66 (0.96)	6.68 (0.95)	6.69 (0.95)	6.72 (0.95)

reflect the fact that human beings do have loss averse attitude, but the degree of the level of loss aversion is limited.

Figure 6 and Table 6 present the equity premium as a function of the loss averse coefficient when given different values of  $\beta$ . Because the results in consumption-claim and dividend-claim cases are alike, we present the results of the dividend-claim case only. We find that when the risk aversion coefficient becomes larger, the effect of the loss aversion coefficient becomes more significant. In the case when  $\beta$  is 0.9, the equity premium rises from 2.78% to 2.95% when the value of  $\lambda$  is from 1 to 9. Relatively speaking, when  $\beta$  is 2.1, the equity premium rises from 13.50% to 15.87% when the value of  $\lambda$  is from 1 to 9.

Despite the definitions of risk aversion and loss aversion being independent, above results show that interactions exist between the ways risk aversion and loss aversion affect the equity premium. Please note that in order to let the first-order derivative of prospect utility be continuous, when  $c_t - v_t < 0$ , we change the risk aversion coefficient from  $\beta$  to  $\beta/\lambda$ . Interactions may arise from the modification.<sup>11</sup> Note that when  $c_t \geq v_t$ , the absolute risk aversion coefficient (ARA),  $-u''/u'$ , equals the constant  $\beta$ . When

<sup>11</sup>Thanks to the referee for reminding us about this argument. In order to prove this point with our model, the following more general utility framework has been tried:

$$u(c_t, v_t) = \begin{cases} 1 - e^{-\beta(c_t - v_t)}, & \text{if } c_t - v_t \geq 0, \\ -\lambda_1 \left[ 1 - e^{\frac{\beta}{\lambda_2}(c_t - v_t)} \right], & \text{if } c_t - v_t < 0. \end{cases}$$

We focus on the equity premium derived in the case when  $\lambda_2 = 1$ . Once we find that no matter what the value of  $\beta$  is, the pattern of the equity premium versus  $\lambda_1$  remains the same, we believe the modification ( $\beta \rightarrow \beta/\lambda$  in Equation (2.2)) brings the side effect (the interaction). Otherwise, if the value of  $\beta$  still affects the pattern of the equity premium versus  $\lambda_1$ , the interaction may not necessarily come from the modification. However, maybe because the first-order derivative is continuous only when  $\lambda_1 = \lambda_2$ , the results of this general framework are not stable whenever  $\lambda_1 \neq \lambda_2$ .



Equity Premium

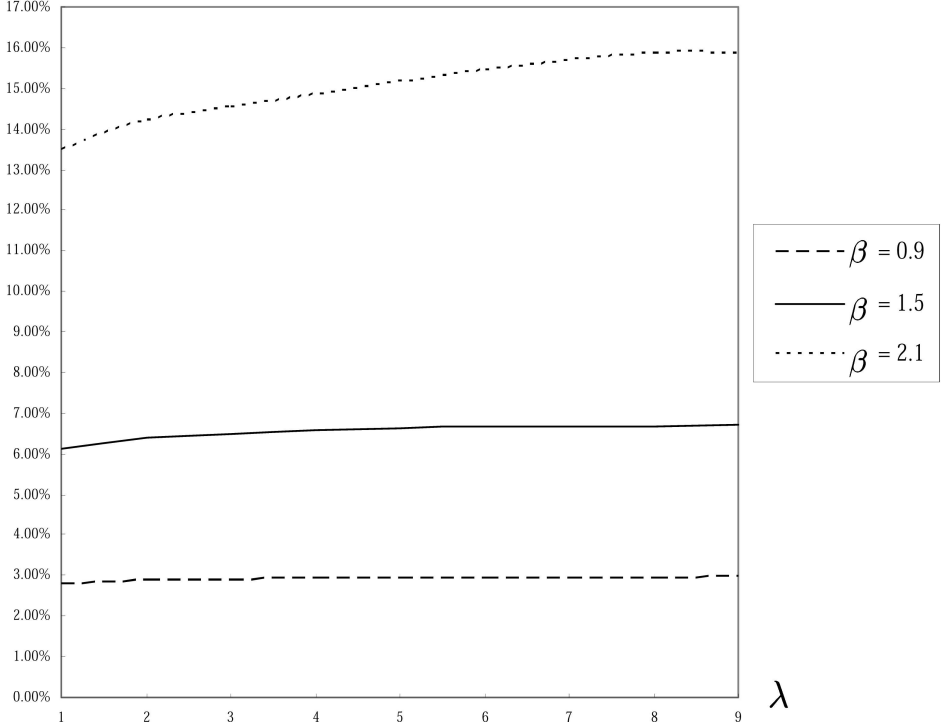


Fig. 6. The effects of risk and loss averse coefficients on equity premium. When given different values of  $\beta$ , we present equity premium as a function of the loss aversion coefficient. Inside the figure, no matter what the value of  $\beta$  is, when  $\lambda$  becomes larger, the equity premium becomes higher. Moreover, we find that when the risk aversion coefficient becomes larger, the effect of the loss aversion coefficient becomes more significant. The interactions between risk and loss averse coefficients may arise from our modification of the original prospect utility function.

$c_t < v_t$ , the ARA is  $-\beta/\lambda$ . It is noted that  $\partial \text{ARA} / \partial \lambda = 0$  when  $c_t \geq v_t$ , and  $\partial \text{ARA} / \partial \lambda = \beta/\lambda^2$  when  $c_t < v_t$ . Therefore, in the bad state, the effect of loss aversion on ARA increases when  $\beta$  increases. As a result, globally looking, when the value of  $\beta$  becomes larger, the effect of  $\lambda$  weakly increases.

3.2.3. The effect of  $w$  on equity premium

From Eq. (2.3), we can see that when the value of  $w$  approaches 1, the reference point in our model is near an internal habit, which is based on the agent’s last-period consumption. On the contrary, when the value of  $w$  approaches 0, the reference point in our model is near an external habit,

Table 6. The effects of risk and loss averse coefficients on equity premium.

Given different values of the loss averse coefficient and risk averse coefficient, this table lists the equity premium derived from our prospect utility model, and the value in parentheses is the real rate of return of the riskless asset.

$\lambda$	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\beta = 0.9$	2.78	2.86	2.90	2.91	2.93	2.94	2.94	2.95	2.95
(%)	(1.02)	(1.00)	(0.99)	(0.99)	(0.99)	(0.98)	(0.98)	(0.98)	(0.98)
$\beta = 1.5$	6.12	6.38	6.51	6.58	6.63	6.66	6.68	6.69	6.72
(%)	(1.03)	(0.98)	(0.97)	(0.96)	(0.96)	(0.96)	(0.95)	(0.95)	(0.95)
$\beta = 2.1$	13.50	14.21	14.51	14.82	15.14	15.45	15.65	15.85	15.87
(%)	(1.02)	(0.96)	(0.94)	(0.93)	(0.92)	(0.92)	(0.91)	(0.91)	(0.91)

Table 7. The effect of  $w$  on equity premium.

Given different values of  $w$ , because there is no reference point in the exponential form utility model, this table only lists the equity premiums derived from our prospect utility model (P) in both dividend-claim (D) and consumption-claim (C) cases. The value in parentheses is the real rate of return of the riskless asset.

$w$	0.3	0.4	0.5	0.6	0.7
P + C (%)	2.06	3.15	5.33	10.45	27.78
	(1.07)	(1.03)	(0.98)	(0.90)	(0.77)
P + D (%)	3.22	4.29	6.43	11.82	27.35
	(1.07)	(1.03)	(0.98)	(0.90)	(0.77)

which is based on last-period's consumption per capita. In Table 7 and Fig. 7, the results for  $w$  ranging from 0.3 to 0.7 is presented in both dividend-claim and consumption-claim cases.

However, because analyzing  $w$  was not our original intention when developing this model, the results are not satisfactory. The results in Table 7 and Fig. 7 show a trend — when the value of  $w$  approaches 1, equity premium becomes larger. That means that, from the aspect of solving the equity premium puzzle, internal habit seems to be a better description of consumer behavior than external habit. Nevertheless, with the above analysis, we are not clear whether people should have more internal habit or more external habit.

#### 4. Conclusion

In this paper, we develop a consumption-based asset pricing model motivated by prospect theory. The similarity between habit formation and

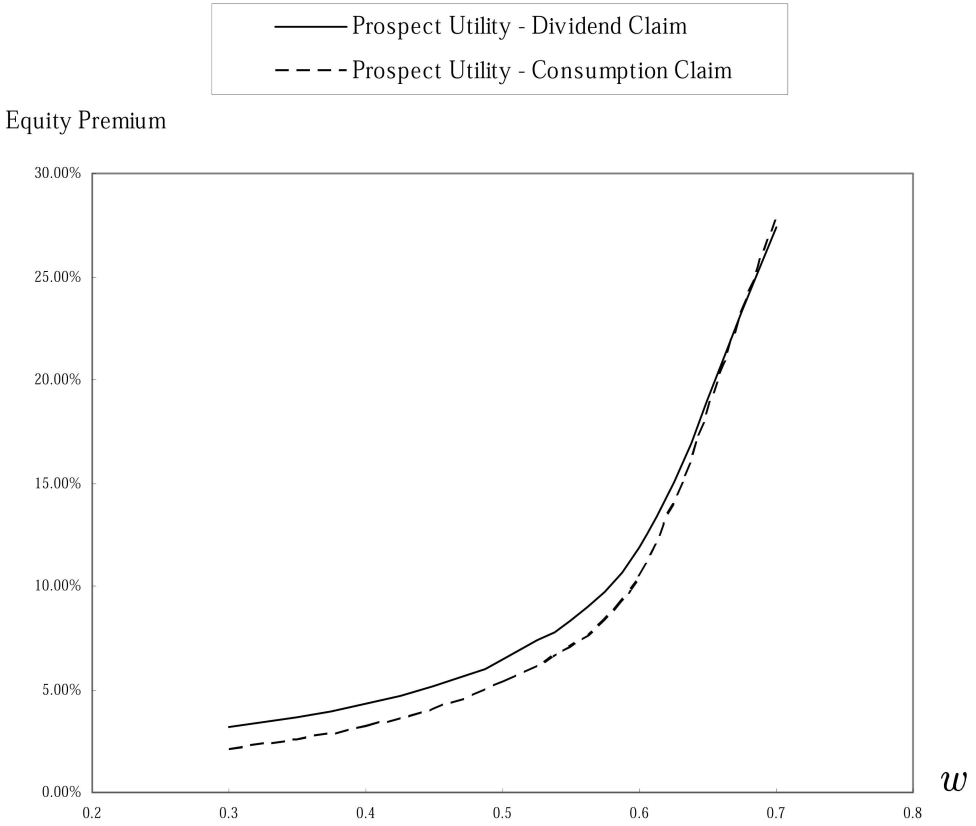


Fig. 7. The effect of  $w$  on equity premium. This figure compares the effects of internal and external habits on equity premium. It shows a trend that when the value of  $w$  approaches 1 (more internal habit the representative agent has), equity premium becomes larger. That means that, from the aspect of solving for the equity premium puzzle, internal habit seems to be a better description of consumer behavior than external habit.

prospect theory is exploited to complement a deficiency of the original prospect theory, where habit formation and “catching up with the Joneses” are incorporated to determine the reference point in prospect utility endogenously. Moreover, in our model, the power utility functions of the original prospect theory are replaced with exponential utility functions. After this change, our prospect utility function is still similar to the original one, but the first-order derivative of our prospect utility function becomes well-defined everywhere. That solves another drawback of the original prospect theory. Therefore, we can readily apply prospect theory to many issues in economics and finance.

There are three major contributions in this paper. First, this is the first model that tries to apply prospect theory as the utility function over

consumption under the traditional consumption-based asset pricing framework. Second, the concept of habit formation and “catching up with the Joneses” is endogenously built into the reference point of prospect theory. Third, it is proved that introducing prospect theory to describe consumer behavior is the key way to solve the equity premium puzzle.

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