Pricing Convertible Bonds Subject to Default Risk

Mao-Wei Hung and Jr-Yan Wang

Mao-Wei Hung
is with the College of Management at the National Taiwan University in Taipei, Taiwan.
hung@mba.ntu.edu.tw

Jr-Yan Wang
is with the College of Management at the National Taiwan University in Taipei, Taiwan.

This article provides a new method to price convertible bonds that are subject to default risk. As some convertible bonds do not convert, we should price them using a risky discount rate in order to reflect the future default probability. Once the bonds are converted into shares of stock, it is appropriate to use the risk-free interest rate as the discount rate.

We carefully distinguish the risky discount rate from the risk-free interest rate, and take into account the stochastic characteristics of the two discount processes.

The literature recognizes two main approaches to pricing convertible bonds. The first approach assumes specific stochastic processes for the stock price and interest rates, and then uses Ito's lemma to derive a partial differential equation. It then exploits boundary conditions in order to solve the partial differential equation (see, for example, Brennan and Schwartz [1977, 1980]).

The drawback of this approach is that the parameters of the stochastic process are exogenous, so the term structure generated may not match today's term structure perfectly. In fact, most authors on this subject do not even consider the default risk of the convertible bond, and hence do not include the risky discount term structure.

There is, however, one exception; Tsiveriotis and Fernandes [1998] take default risk into account. They evaluate both the cash flow portion and the equity portion of the convertible bond, and use two partial differential equations to represent the behavior of the convertible bond.

The second approach is the traditional tree model, which has been widely adopted. This approach constructs a tree model to represent the behavior of the company's stock price, and assumes a non-stochastic risky discount rate as well as a non-stochastic risk-free interest rate. This method distinguishes risk-free and risky discount yields, but ignores the stochastic characteristics of these two processes.

We focus on how to price convertible bonds that may be defaultable. There are generally two different approaches to model default risk: structural models, and reduced-form models. The structural models of default risk attempt to model the lower boundary on firm value that triggers reorganization of a company. Leland [1994] and Leland and Toft [1996] are some examples of this approach. The reduced-form models focus on specifying the default process and the recovery rate. Examples are Das and Tufano [1995], Duffie and Singleton [1999], and Jarrow and Turnbull [1995].

To combine the stock price process, the stochastic risk-free interest rate process, and the risky discount yield process into one single tree, we follow the reduced-form modeling methodology. We extend Jarrow and Turnbull [1995] to price convertible bonds that may be defaultable.

Jarrow and Turnbull [1995] introduce the default probability to represent that the company may have some probability of defaulting in each period. They use the normal risk-free interest rate tree plus the default probability to represent the risky discount rate. Once default occurs, bondholders can receive only a portion of the face value of the bond (the recovery rate).

We combine this model with the base model for valuing convertible bonds to derive one single tree that can represent not only the stock price process, but also the risk-free interest rate process and the risky discount rate process. Our tree model can easily solve the problems encountered when pricing convertible bonds.

We provide a general numerical example to illustrate the model, and use it to price a real convertible bond traded in the market.

I. PRICING MODEL

First, we review the traditional model for pricing convertible bonds. Second, using the structure and the algorithm of the traditional model, we add the stochastic risk-free interest rate process as well as a stochastic risky discount rate process to derive a new model for pricing convertible bonds.

Traditional Model

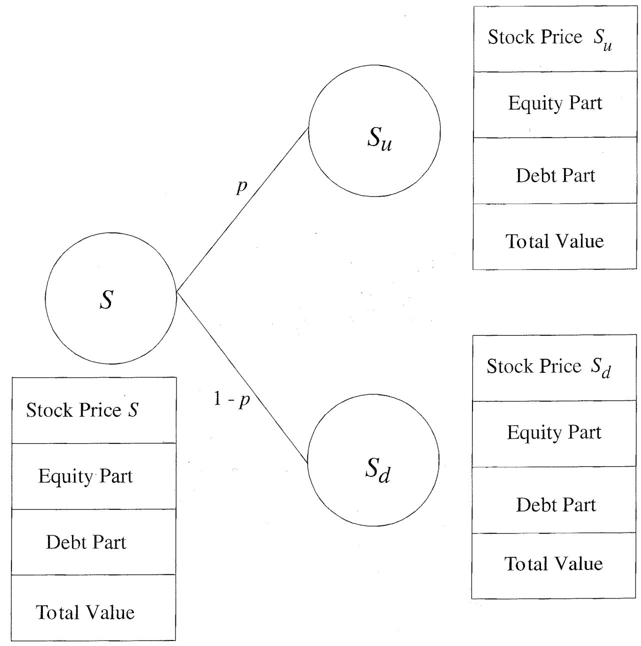
The traditional method to price convertible bonds constructs a stochastic binomial stock price tree and makes assumptions about the behavior of the risk-free yield and the risky yield. For simplicity, the risk-free yield and risky yield are often assumed to be constant.

When rolling back through the tree, two optimal conditions at each node of the tree must be checked. The optimal time for the holder to exercise the conversion option is when the conversion value exceeds its market value. The optimal time for the issuer to call the convertible bond is when the convertible's market value exceeds its call price. When the issuer exercises the call option, this often forces the holders to convert. Thus, the optimal value of the convertible bond at each node can be stated as:

$$\max[\min(\text{Market Value, Call Price}), \text{Conversion Value}] \quad (1)$$

Exhibit 1 illustrates the pricing method. In addition to the stock price, we should include three more values at each node to show how to price the convertible bond, for four elements at each node. The first value is the stock price. The second is the equity component of the convertible bond. This value arises from the contingency that the convertible bond will ultimately become stock. The third value is the debt component of the convertible bond, which arises from the contingency that the convertible bond remains in the form of a bond. The last value is the total value of the convertible bond. This represents the convertible bond's market value at the current node.

EXHIBIT 1
Four Elements at Each Node to Price a Convertible Bond



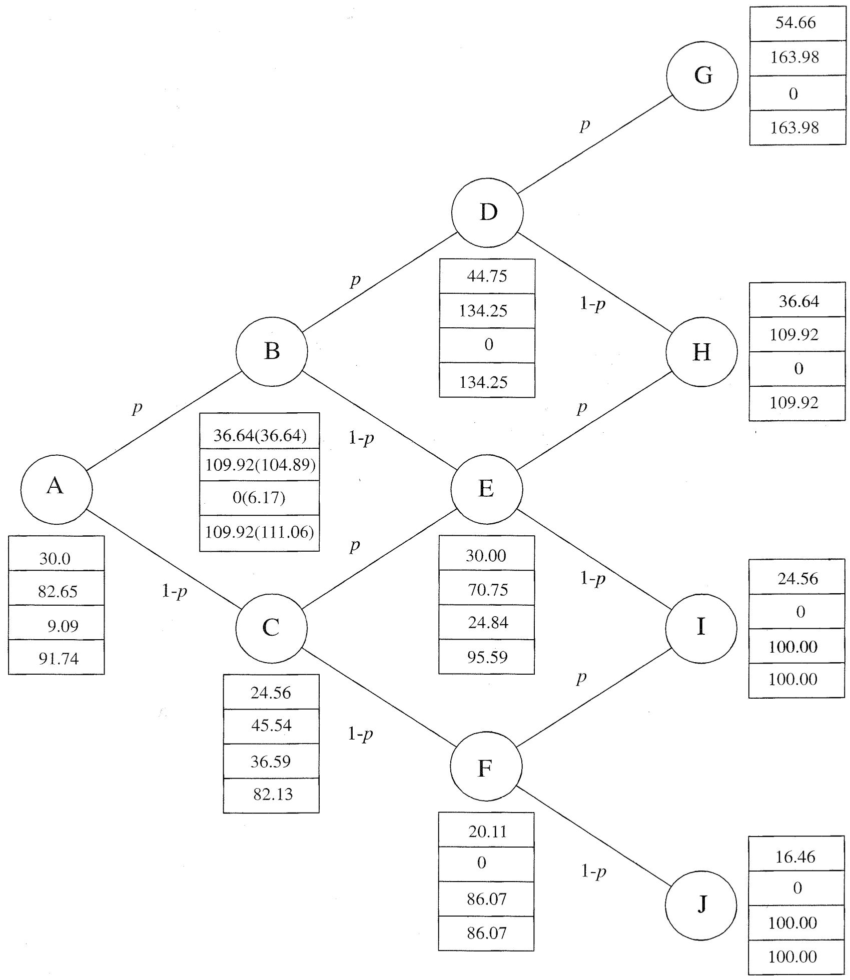
To price a convertible bond in this framework, the first step is to decide the payoff at the terminal nodes of the stock price tree, and then to roll back through the stock price tree. A numerical example shows how to roll back through the tree.

Assume a three-period zero-coupon convertible bond with face value equal to 100. The holder of this convertible bond can exchange it for three shares of the company's stock, and the convertible bond can be called for 105 at any time. We also assume today's stock price is 30.

In the traditional convertible bond pricing model, first construct a stock price tree. The parameters of this tree are set to $u = 1.2214$, $d = 0.8187$, and $p = 0.7114$. We also assume the one-period risk-free yield is 10%, and the one-period risky yield is 15%. The stock price tree is given in Exhibit 2.

Exhibit 2

Example of Traditional Method to Price a Convertible Bond



s(0) = 30, p = 0.7114, u = 1.2214, d = 0.8187, Conversion ratio = 3, Call price = 105, Risk-free yield = 10%, and Risky yield = 15%.

After building the stock price tree, we determine the terminal payoff for the bond. Take the node G in Exhibit 2 as an example. At node G, the stock price is S_{uuu} = 54.66; the total value of the convertible bond is 54.66 × 3 = 163.98. This is because the holder will exercise the conversion option rather than letting the convertible bond stay a bond, and receiving only the face value of 100. Thus, at this node, the equity part of the convertible bond is 163.98, and the debt part of the convertible bond is 0. The total value of node G is 163.98 + 0 = 163.98.

After deciding the payoffs at all terminal nodes, we begin rolling back. At node D, the equity part is (0.7114 × 163.98 + 0.2886 × 109.92)$e^{-0.1}$ = 134.25. The debt part is (0.7114 × 0 + 0.2886 × 0)$e^{-0.15}$ = 0. The total value is 134.25 + 0 = 134.25.

One thing is noteworthy in this framework. Rolling back through the tree, we should choose the correct discount rate. When we calculate the equity part, we use 10% as the discount rate. For the debt part, 15% is an appropriate discount yield.

While rolling back, we should test whether conversion is optimal for the bondholder and whether the issuer should call the bond. At node E, the equity part is (0.7114 × 109.92 + 0.2886 × 0)$e^{-0.1}$ = 70.75. The debt part is (0.7114 × 0 + 0.2886 × 100)$e^{-0.15}$ = 24.84. The total value is 70.75 + 24.84 = 95.59. Because the total value 95.59 is lower than the call price 105, the issuer will not exercise the call option.

At node B, however, the equity part is (0.7114 × 134.25 + 0.2886 × 70.75)$e^{-0.1}$ = 104.89. The debt part is (0.7114 × 0 + 0.2886 × 24.84)$e^{-0.15}$ = 6.17. The total value should be 104.89 + 6.17 = 111.06. Because the total value 111.06 is higher than the call price 105, the convertible bond will be called at this node. Since the conversion value

(36.64 × 3 = 109.92) is also higher than the call price (105), the holder of the convertible will be induced to convert earlier. The actual equity part is 109.92, which is equal to the conversion value 36.64 × 3, and the actual debt part is equal to 0 because of the conversion.

Upon rolling back, we can obtain a value for the convertible bond, 91.74, which is the total value at node A.

Our Model

Our model not only distinguishes the risk-free interest rate from the risky discount rate, but also takes the stochastic behavior of these two discount rates into consideration. To achieve our goal, we adopt Jarrow and Turnbull's model for the risk-free and risky discount rate process model. The salient feature of their model is that they combine the risk-free and risky discount rates in one tree. They use the default probability $\lambda_{\mu t}$ and the recovery rate δ to represent the behavior of a corporate bond.

A corporate bond has a positive probability of default in each period. If the corporation were to default, the holders of the bond would receive not the whole face value, but rather a fraction of the face value. This fraction is usually called the recovery rate δ.

The original risk-free interest rate tree is plotted in Exhibit 3A, and Jarrow and Turnbull's model is in Exhibit 3B, where π is the pseudo-probability for the risk-free interest rate process. For simplicity, we assume π to be 0.5. The recovery rate is given exogenously; we assume that it is δ = 32% if default occurs.

Derivation of $\lambda_{\mu t}$. Given the recovery rate δ and the pseudo-probability π, we can use real data to derive each value for $\lambda_{\mu t}$. The real data include the prices of default-free zero-coupon bonds $p(0, T)$ and the prices of the company's defaultable zero-coupon bonds $v(0, T)$.

A two-period example will show how to solve for $\lambda_{\mu 0}$ and $\lambda_{\mu 1}$. First, we consider only the time interval between 0 and 1. The price relationship between these two periods can be described as follows:

$$\begin{aligned} v(0, 1) &= e^{-r(0)}[\lambda_{\mu 0} \times \delta + (1 - \lambda_{\mu 0})1] \\ &= p(0, 1)[\lambda_{\mu 0} \times \delta + (1 - \lambda_{\mu 0})1] \end{aligned} \tag{2}$$

By using prices $p(0, 1)$, $v(0, 1)$, and the recovery rate δ, we can derive $\lambda_{\mu 0}$, the default probability in the first period. Next, we consider the time interval between 0 and 2. The pricing relationship can be described as follows:

EXHIBIT 3A
Two-Period Risk-Free Interest Rate Tree

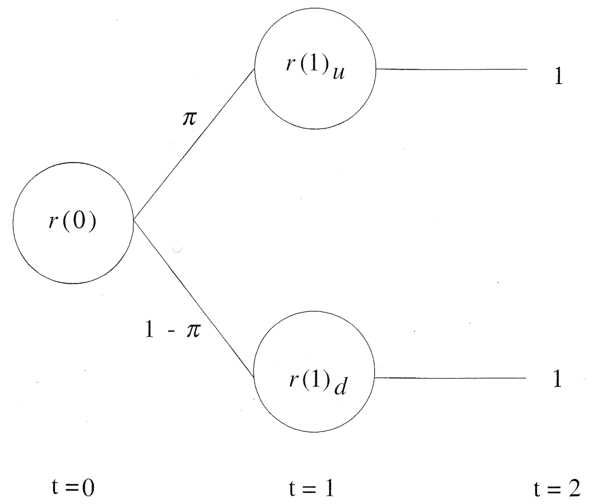
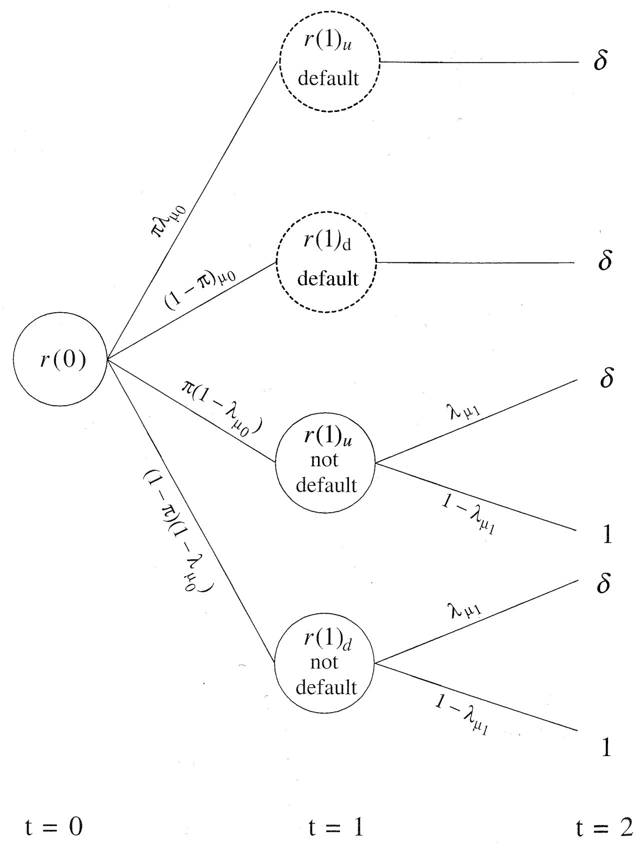
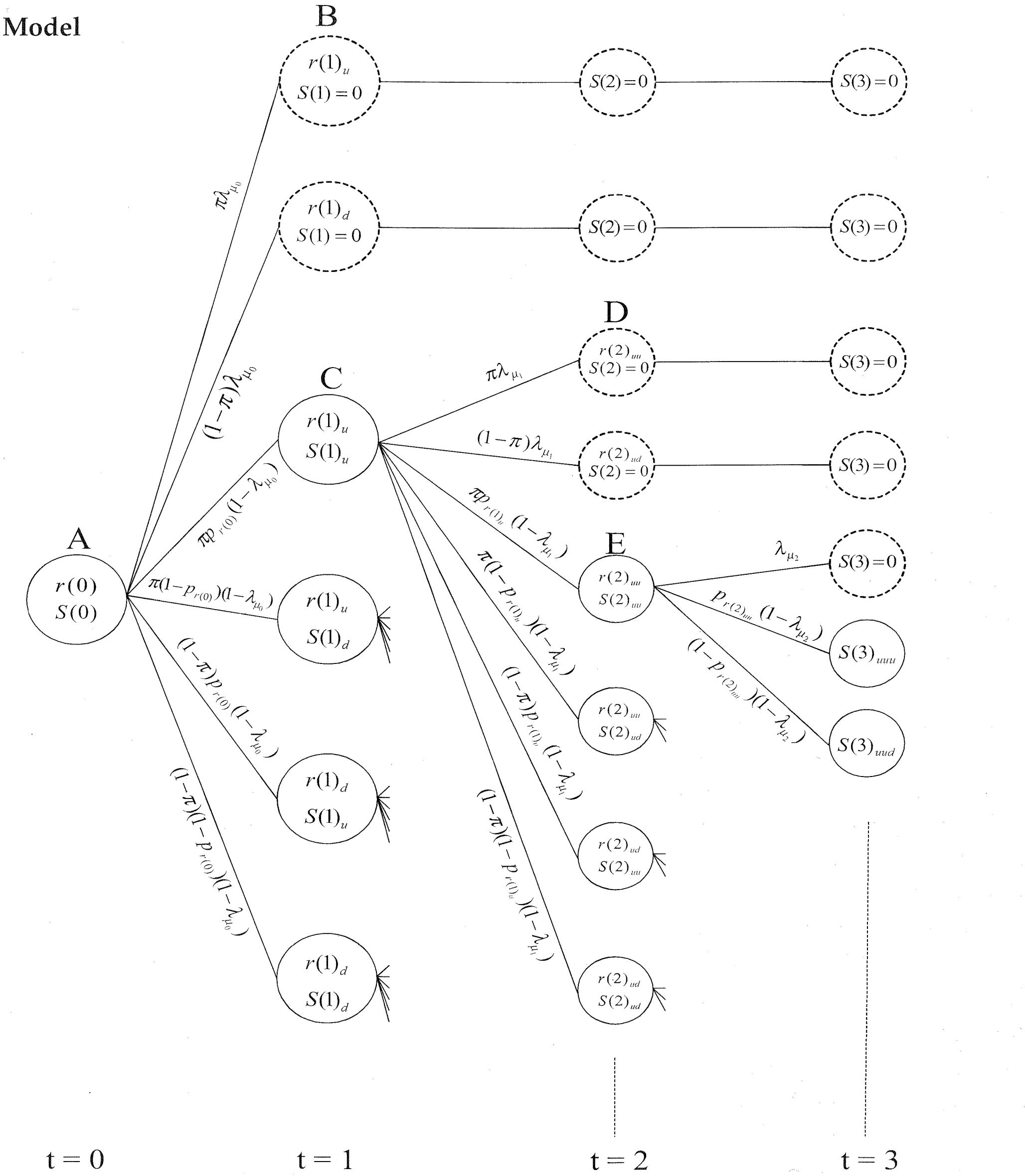


EXHIBIT 3B
Two-Period Risky Interest Rate Tree



E X H I B I T 4
Constructing Tree for Our Model



$$\begin{aligned} v(0, 2) = {} & e^{-r(0)}\{e^{-r(1)_u}\pi \times \lambda_{\mu 0} \times \delta + \\ & e^{-r(1)_d}(1-\pi)\lambda_{\mu 0} \times \delta + \\ & e^{-r(1)_u}\pi(1-\lambda_{\mu 0})[\lambda_{\mu 1}\delta + (1-\lambda_{\mu 1})1] + \\ & e^{-r(1)_d}(1-\pi)(1-\lambda_{\mu 0})[\lambda_{\mu 1}\delta + (1-\lambda_{\mu 1})1]\} \\ = {} & p(0, 2)\{\lambda_{\mu 0} \times \delta + (1-\lambda_{\mu 0}) \times \\ & [\lambda_{\mu 1} \times \delta + (1-\lambda_{\mu 1})1]\} \end{aligned} \tag{3}$$

where

$$p(0, 2) = e^{-r(0)}[e^{-r(1)_u} \times \pi \times 1 + e^{-r(1)_d} \times (1-\pi) \times 1] \tag{4}$$

Because $p(0, 2)$ and $v(0, 2)$ are given, and $\lambda_{\mu 0}$ has been derived, we can obtain the default probability $\lambda_{\mu 1}$ of period 2 easily from these equations.

Stochastic Discount Yield Convertible Bond Tree. According to this procedure, we can calculate $\lambda_{\mu 1}$ at each time period recursively. After that, we can combine these two models to derive our new method for pricing convertible bonds. We consider three kinds of probabilities: p_{rt}, π, and $\lambda_{\mu 1}$, where p_{rt} is the probability that the stock price will go up when the risk-free rate at period t is r_t; π is the probability that the risky yield and the risk-free yield go up; and $\lambda_{\mu 1}$ is the probability that the corporate bond may default at period $t \sim t+1$.

The simplified convertible bond pricing tree is shown in Exhibit 4. We construct a three-period model, for example, where $\lambda_{\mu 0}$, $\lambda_{\mu 1}$, and $\lambda_{\mu 2}$ can be calculated from the known data: $p(0, 1)$, $p(0, 2)$, $p(0, 3)$, $v(0, 1)$, $v(0, 2)$, and $v(0, 3)$.

There are three main cases as one moves through the tree. The first case, including both node A and node C, should have six branches if the bond has not defaulted at this node. Each branch represents a different situation in the next period:

1. Default occurs; r goes up, and S jumps to 0.
2. Default occurs; r goes down, and S jumps to 0.
3. Default doesn't occur; r goes up, and S goes up.
4. Default doesn't occur; r goes up, and S goes down.
5. Default doesn't occur; r goes down, and S goes up.
6. Default doesn't occur; r goes down, and S goes down.

We use r to represent the risk-free discount yield and S to represent the stock price. Each branch's probability is represented by $\lambda_{\mu t}$, π, and p_{rt}. The $\lambda_{\mu t}$ represents the probability of the default occurring; π is the probability when r goes up; and p_{rt} is the probability of S going up when the risk-free rate is r_t. We assume the stockholder will receive nothing whenever bankruptcy occurs, so the stock price in the first two branches jumps to zero.

If default occurs, the tree enters case 2. Nodes B and D are examples of case 2. The bond price will not fluctuate again, and neither will the company's risky discount yield. The bond price will be equal to the product of the recovery rate and the bond's face value.

Case 3 is a special condition. In Exhibit 4, node E is the example, and the special circumstances can be seen at the nodes just before the terminal nodes. Although these nodes, like those in case 1, have not defaulted, we do not need to have six branches. This is because $r(2)$ already represents the discount yield between $t = 2$ and $t = 3$. We should consider only the stochastic characteristics of the stock price and the probability that the company will default.

Thus, the nodes for condition 3 should have only three branches. These three branches represent:

1. Default occurs, and S jumps to 0.
2. Default doesn't occur, and S goes up.
3. Default doesn't occur, and S goes down.

After finishing the construction of the tree, just as in the traditional model of pricing a convertible bond, we should decide the payoff at the terminal nodes first. The data for each node still include the stock price, the equity component of the convertible bond, the debt component of the convertible bond, and the total value of the convertible bond.

Finally, we roll back through the tree just as when we priced the convertible bond above. This time, however, we do not take the risky discount rate into consideration. We will use only the risk-free interest rate as the discount yield. This is because the risky part has already been represented in each period's default probability $\lambda_{\mu t}$ and recovery rate δ. Since we use the default probability $\lambda_{\mu t}$ and the recovery rate δ to represent the risky part, we can combine the stock price process, the risk-free interest rate process, and the risky discount rate process to form one tree. Once having rolled back through the tree, the total value of the node $t = 0$ is the convertible bond value, and this value of the convertible bond takes default risk into consideration.

The tree appears very complicated, so we use a computer program to help implement this model. A simplified and more readable version of the algorithm for the model, written in C language, is in an appendix available from the authors.

II. NUMERICAL EXAMPLE

We use a numerical example to compare our model to the traditional convertible bond pricing model we have discussed. The parameters of the convertible bond are all the same, but the risk-free yield and the risky yield will not be assumed to be constant. While the usual interest rate tree is adopted to represent the risk-free yield, the default probability $\lambda_{\mu t}$ and the recovery rate δ in the same tree are used to represent the difference between the risky yield and the risk-free yield.

EXHIBIT 5

Data on Default-Free and Defaultable Zero-Coupon Bonds

Maturity T	Prices of Default-Free Zero-Coupon Bonds $p(0, T)$	Prices of Company's Defaultable Zero-Coupon Bonds $v(0, T)$
1	90.4837	86.0708
2	81.8731	74.0818
3	74.0818	63.7628

Derivation of Default Probabilities $\lambda_{\mu t}$

The data we need are the prices of default-free zero-coupon bonds and the price of a defaultable zero-coupon bond of the company in the market. We assume the data values given in Exhibit 5 in order to fit the constant risk-free discount yield 10% and the risky discount yield 15%. Therefore $p(0, 1) = 100e^{-0.1} = 90.4837$, $v(0, 1) = 100e^{-0.15} = 86.0708$, $p(0, 2) = 100e^{-0.1 \times 2} = 81.8731$, and $v(0, 2) = 100e^{-0.15 \times 2} = 74.0818$.

Using Exhibit 5, and assuming the standard deviation σ of a default-free zero-coupon bond to be 10%, we can obtain the original risk-free interest rate tree. The result is shown in Exhibit 6 with $\pi = 0.5$.

Next, we can use the prices of the company's defaultable zero-coupon bonds, the recovery rate $\delta = 32\%$, and the recursive method described above to calculate each period's default probability $\lambda_{\mu t}$. The result is shown in Exhibit 7.

Given the default probabilities, the three-period risky interest rate tree is shown in Exhibit 8.

EXHIBIT 6
Three-Period Risk-Free Interest Rate Tree

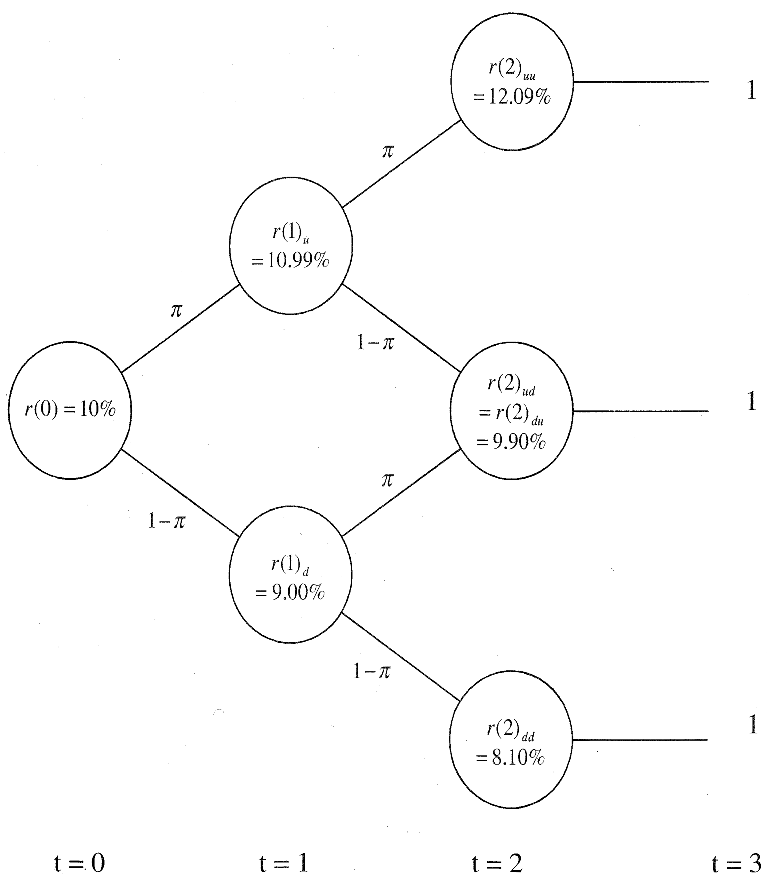
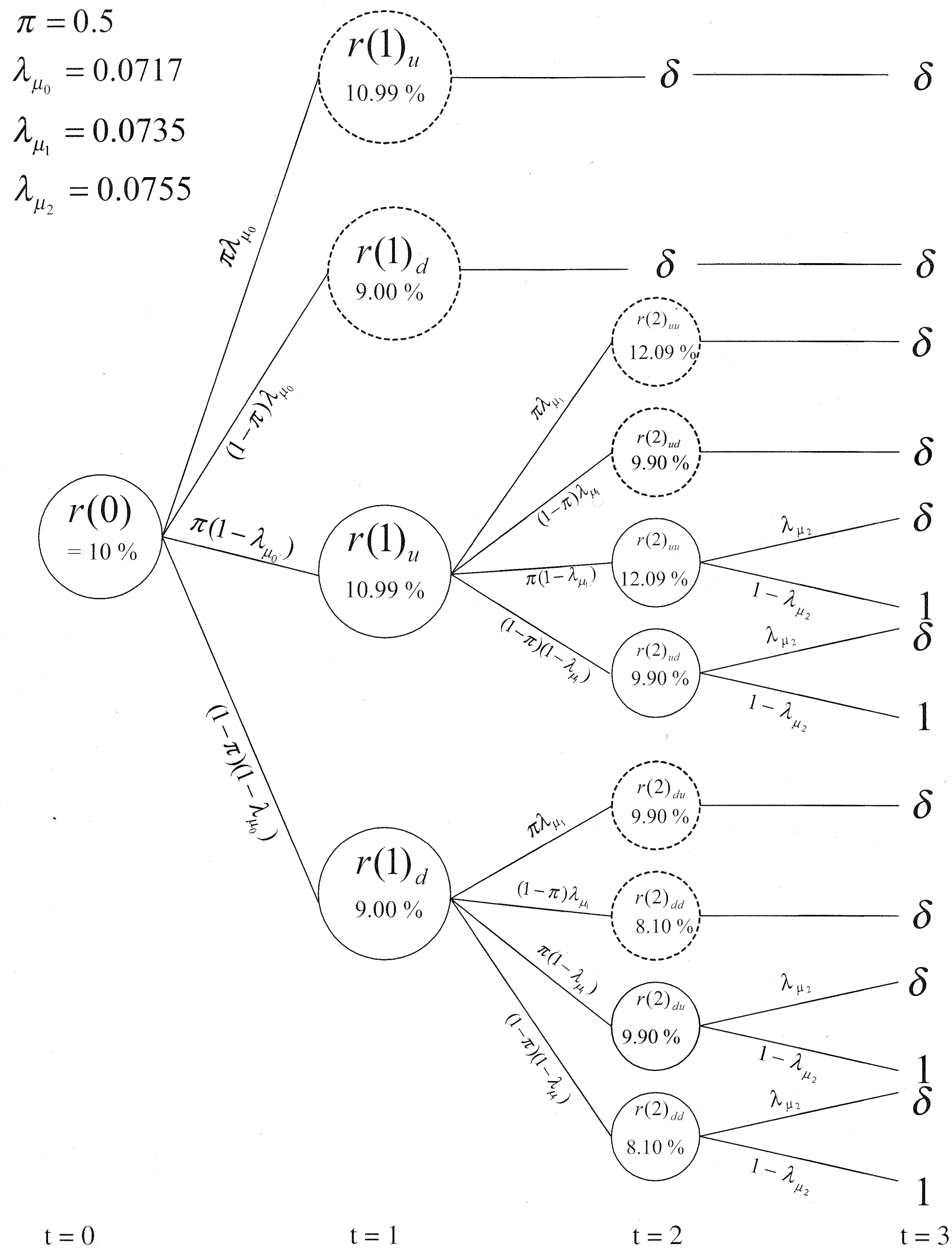


EXHIBIT 7
Default Probabilities in Each Period

Time period	0-1	1-2	2-3
Default Probability λ_{μ_t}	0.0717	0.0735	0.0755

EXHIBIT 8

Three-Period Risky Interest Rate Tree



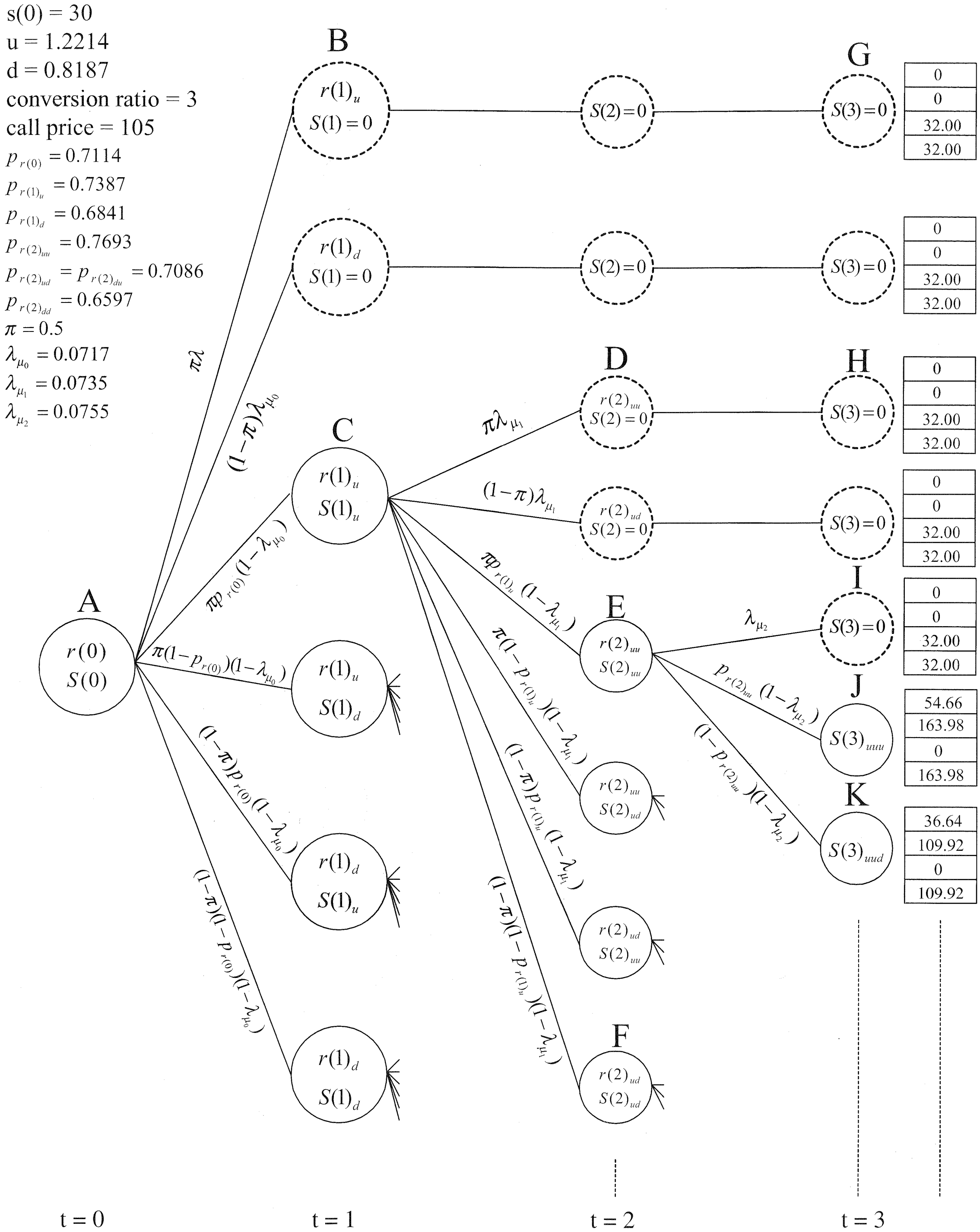
Pricing Convertible Bonds Subject to Default Risk

After we obtain the default probability for each period, we use the tree in Exhibit 4 to calculate the price of the convertible bond, as in the usual convertible bond pricing model. We decide the payoff of the terminal nodes first, and then roll back through the tree. This time we do not use two different discount rates. This is because the risky yield has already been replaced by each period's default probability $\lambda_{\mu t}$ and recovery rate δ.

Exhibit 9 gives us the layout of the whole stochastic discount yield convertible bond tree. We show the payoff of some of the terminal nodes. Take nodes I and K as examples. At node I, because of bankruptcy, the stock price of the company jumps to zero. The rational holder of the convertible bond will not exercise the right to get three shares of zero-value stock. What the holder can receive is only the recovery rate multiplied by the face value. So the equity part of node I is zero, and the debt part is 32.00.

Exhibit 9

Determining Payoff of Each Terminal Node



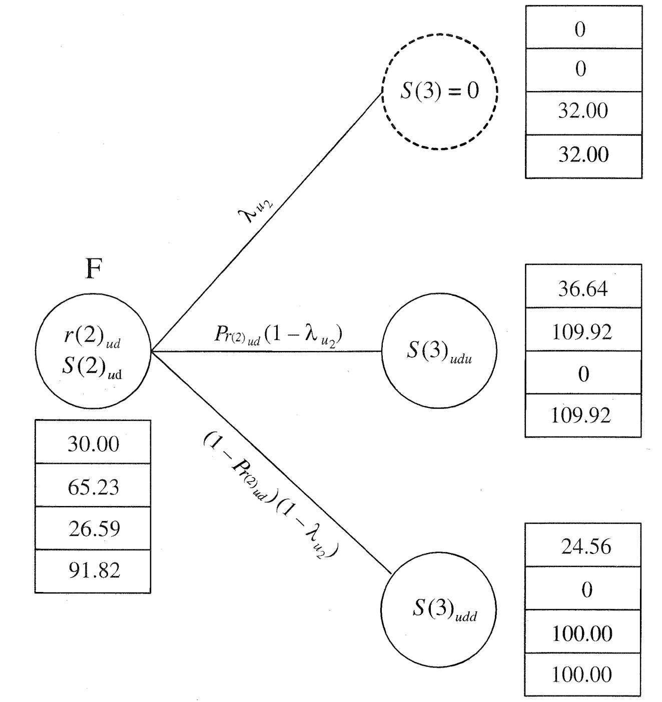
At node K, the stock price $S(3)_{uud}$ is 36.64. If the holder exercises the option to convert the bond to three shares of stock, he can receive 36.64 × 3 = 109.92. If the holder lets the convertible remain a bond, he receives only 100, the face value of the convertible bond. The bondholder will rationally choose to convert the convertible bond to three shares of stock, so the payoff of node K is 109.92.

Because the tree is too complex, we do not display the payoff of each terminal node here, but by the same reasoning, we can derive the payoff of other terminal nodes.

After deciding the payoff of each terminal node, we examine the details of rolling back through the tree more specifically. Take nodes F, B, C, and A as examples.

Let us first examine node F in Exhibit 10A. The three

Exhibit 10A
Rolling Back Through Tree (node F)



$P_{r(2)ud} = 0.7086$, $\lambda_{\mu 2} = 0.0755$, $r(2)_{ud} = 0.099$.

branch nodes of node F are all terminal nodes. The payoffs of these nodes are derived using the same method described above, but we will take the node $S(3)_{udd}$ as an example to explain this process. The bondholder at this node will find that if she exercises the conversion option, she will get only $24.56 \times 3 = 73.68$. But if she waits until the bond matures, she will receive the face value 100, so any rational holder will hold the bond until the maturity date. The equity part of this node is 0, and the debt part is 100.

If we have information on the payoff of each of the successor nodes of node F, the equity part of node F can be derived as follows:

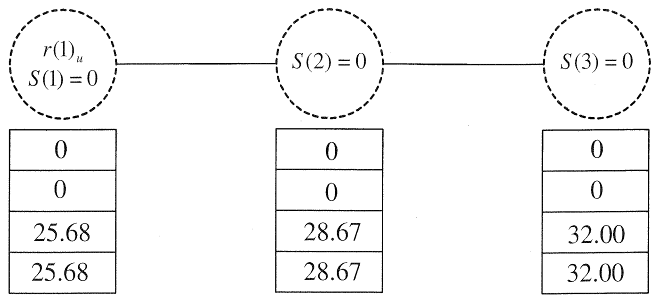
$$(0.0755 \times 0 + 0.7086 \times 0.9245 \times 109.92 + 0.2914 \times 0.9245 \times 0)e^{-0.099} = 65.23$$

And the debt part of node F can be derived as follows:

$$(0.0755 \times 32 + 0.7086 \times 0.9245 \times 0 + 0.2914 \times 0.9245 \times 100)e^{-0.099} = 26.59$$

Note that we use $r(2)_{ud} = 0.099$ as the discount yield for both the equity and the debt component. This is because the default probability and the recovery rate have been combined in construction of the tree. Thus, we can use the same discount yield for both parts. We can compute the total value as $65.23 + 26.59 = 91.82$, which is lower than the call price 105 and higher than the conversion value of $30 \times 3 = 90$. Thus the issuer will not call, and the holder will not exercise the conversion option.

Exhibit 10B
Rolling Back Through Tree (node B)



$r(1)_u = 0.1099$.

After looking at node F, we will show how to roll back through the tree once the company has defaulted. In Exhibit 10B, the default occurs at node B. Simultaneously, the stock price jumps to zero. After that node, the interest rate process as well as the stock price process will not fluctuate again. Finally, the bondholder will receive the recovery rate of the face value.

The debt part of node B is calculated as:

$$32e^{-0.1099 \times 2} = 25.68$$

In this equation, we use $r(1)_u = 0.1099$ as the discount yield. The equity part of node B is obviously equal to 0, so the total value of node B is $0 + 25.68 = 25.68$.

Now let us consider node C, which is the most usual type of node in our model. The equity part of node C is as follows:

$$(0.5 \times 0.0735 \times 0 + 0.5 \times 0.0735 \times 0 + 0.5 \times 0.7387 \times 0.9265 \times 134.25 + 0.5 \times 0.2613 \times 0.9265 \times 69.29 + 0.5 \times 0.7387 \times 0.9265 \times 134.25 + 0.5 \times 0.2613 \times 0.9265 \times 65.23)e^{-0.1099} = 96.91$$

The debt part of node C is as follows:

$$(0.5 \times 0.0735 \times 28.36 + 0.5 \times 0.0735 \times 28.98 + 0.5 \times 0.7387 \times 0.9265 \times 0 + 0.5 \times 0.2613 \times 0.9265 \times 21.03 + 0.5 \times 0.7387 \times 0.9265 \times 0 + 0.5 \times 0.2613 \times 0.9265 \times 26.59)e^{-0.1099} = 7.05$$

EXHIBIT 10C
Rolling Back Through Tree (node C)

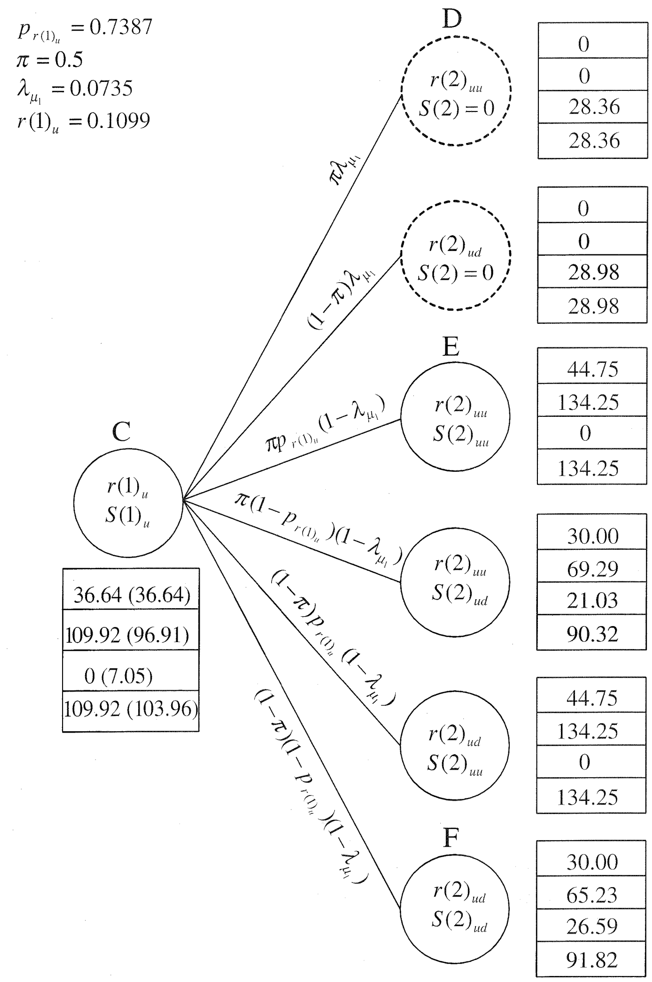
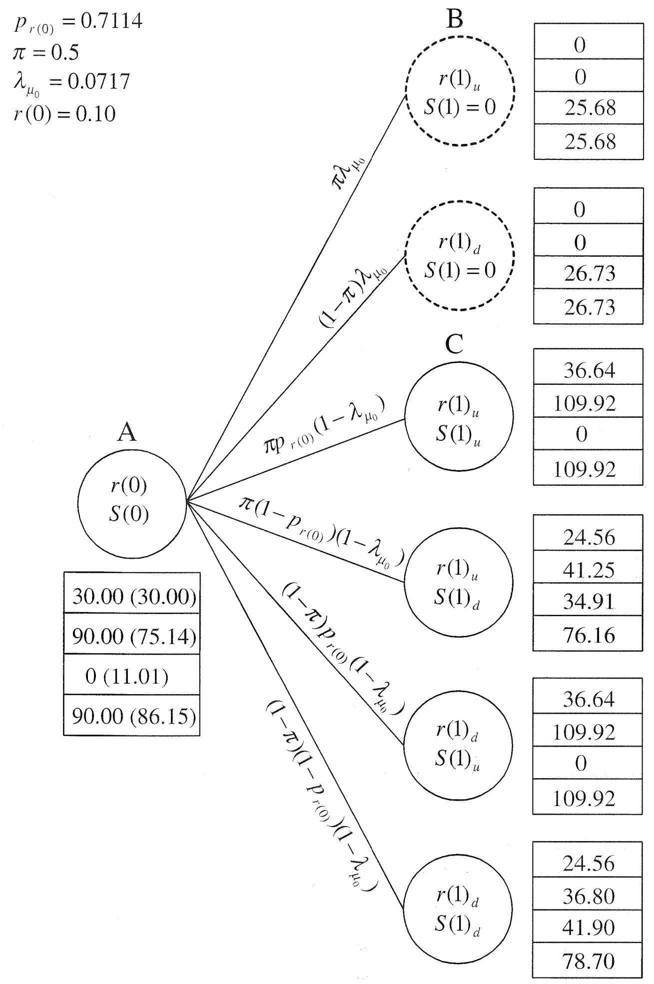


EXHIBIT 10D
Rolling Back Through Tree (node A)



The total value of node C seems to be 96.91 + 7.05 = 103.96. Because this value is lower than the conversion value 36.64 × 3 = 109.92, the rational bondholder will exercise the right to convert the bond. According to the reasoning above, the actual total value of the convertible bond at node C is 109.92, which is equal to three times $S(1)_u$.

Following the rolling back, we can derive the equity and the debt parts of each node. Finally, we reach the last step of our model, which is to determine each part of node A. Exhibit 10D provides the details.

Following the same rule, the equity and debt parts of node A are as follows:

$$(0.5 \times 0.0717 \times 0 + 0.5 \times 0.0717 \times 0 + 0.5 \times 0.7114 \times 0.9283 \times 109.92 + 0.5 \times 0.2886 \times 0.9283 \times 41.25 + 0.5 \times 0.7114 \times 0.9283 \times 109.92 + 0.5 \times 0.2886 \times 0.9283 \times 36.80)e^{-0.10} = 75.14$$

$$(0.5 \times 0.0717 \times 25.68 + 0.5 \times 0.0717 \times 26.73 + 0.5 \times 0.7114 \times 0.9283 \times 0 + 0.5 \times 0.2886 \times 0.9283 \times 34.91 + 0.5 \times 0.7114 \times 0.9283 \times 0 + 0.5 \times 0.2886 \times 0.9283 \times 41.90)e^{-0.10} = 11.01$$

If the bondholder cannot exercise the conversion option on the date of issuance, the value of the convertible will be 75.14 + 11.01 = 86.15. Otherwise, the bondholder will find it more beneficial to convert the bond and obtain three shares of stock, which is worth 30 × 3 = 90.

Explanation of the Differences

Note that the value calculated using our model is a little lower than that derived from the usual model shown in Exhibit 2 (91.74). The difference is attributable to two characteristics of our model.

First, our model takes into account the stochastic characteristic of the risk-free and the risky discount rates. Second, whenever default occurs, the stock price will jump to zero, and bondholders can receive only the recovery rate of the face value. Previous models do not handle this problem, but our model can describe this behavior more accurately.

III. REAL CASE: LUCENT CONVERTIBLE BOND

We use the model here to provide realistic computations for a convertible bond value. First, we show how to calibrate the parameters of the model according to market prices. Then, we compare the result with the real issue price of that security.

Parameters

In 1997, Lucent Technologies, a high-technology network company, issued a six-year zero-coupon convertible bond scheduled to mature on May 15, 2003. The conversion rate of the bond is 50.7524 shares per 1000 nominal, but the bondholder could not convert until July 6, 1997. The call schedule of the bond is in Exhibit 11.

Besides the basic information on the convertible bond, the model needs the prices of default-free zero-coupon bonds $p(0, T)$ and the prices of defaultable zero-coupon bonds of the company $v(0, T)$. We obtain $p(0,T)$ and $v(0,T)$ from the risky and risk-free yield curves on the date of issuance. The corresponding risk-free yield curve is obtained from Bloomberg. The credit rating of this convertible bond is Aa2, so we can take the risky yield curve for this credit class, also from Bloomberg. The risk-free and risky yield curves on the date of issuance are shown in Exhibit 12.

EXHIBIT 11
Call Schedule of 6-Year Zero-Coupon Convertible Bond

Date	5/15/2000	5/15/2001	5/15/2002
Price	94.205	96.098	98.030

EXHIBIT 12
Risk-Free and Risky Yield Curves on Date of Issuance

Maturity T (yrs)	1	2	3	4	5	6
Risk-Free Yield Curve (%)	5.969	6.209	6.373	6.455	6.504	6.554
Risky Yield Curve of Aa2 (%)	6.110	6.460	6.630	6.780	6.830	6.894

EXHIBIT 13
Implied Prices of Default-Free and Defaultable Zero-Coupon Bonds

Maturity T	Prices of Default-Free Zero-Coupon Bonds $p(0, T)$	Prices of Company's Defaultable Zero-Coupon Bonds $v(0, T)$
1	94.2057	94.0729
2	88.3221	87.8798
3	85.5976	81.9632
4	77.2441	76.2464
5	72.2383	71.0703
6	67.4867	66.1239

EXHIBIT 14
Default Probabilities for Lucent in Each Period

Time period	0-1	1-2	2-3	3-4	4-5	5-6
Default Probability λ_{μ_t}	0.0021	0.0053	0.0040	0.0078	0.0049	0.0061

From the risk-free and risky yield curves, we compute the implied prices of default-free zero-coupon bonds $p(0, T)$ and defaultable zero-coupon bonds $v(0, T)$, both shown in Exhibit 13.

From $p(0, T)$ and $v(0,T)$, the algorithm in Jarrow and Turnbull [1995] gives the default probabilities of Lucent at each period as in Exhibit 14. We also need the initial stock price and its volatility. Lucent's closing price on the day before the date of issuance was $15.006 per share. The volatility, estimated from the daily stock prices from May 15, 1995, to May 15, 1997, is 35.3836%. As in the numerical example above, we again assume the standard deviation of the risk-free zero coupon bond yield to be 10%.

Result

The issue price of Lucent's convertible bond was 88.7060. According to our model, when we divide six years into six periods, the value of the convertible bond is 90.4633. The theoretical value is not far from the real issue price, and if we adjust the standard deviation of the rate of return of the stock to match the real issue price, we can find the implied volatility to be 32.2459%. This computation shows that our model is a useful and practical method for pricing convertible bonds.

IV. CONCLUSION

To price convertible bonds subject to default risk, one must consider not only the stochastic characteristic of the stock price, but also stochastic risky and risk-free term structures. We extend the method proposed by Jarrow and Turnbull [1995] to develop a more suitable method to calculate the value of convertible bonds.

Our model incorporates the default probability, and we combine the stock price process, the risky discount yield process, and the risk-free interest rate process into one tree. The model can price not only convertible bonds but also other derivatives, as long as the valuations depend simultaneously on the stochastic risk-free process and the stochastic risky process.

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