Mixed-Strategy Equilibrium (MSE)

Rock-Scissor-Paper
Searches of passengers on board after 911
Military attack
Sport events

Games with MSE are good places to test theory because

(1) Unique equilibrium
(2) Constant sum so social preference plays no role
(3) Maximin leads to Nash in zero sum and maximin is a simple decision rule.

	Head	Tail
Head	1	-1
Tail	-1	1

Maximin: Row player thinks
Head: Tail (-1)
Tail: Head (-1)
(1/2, 1/2): (0)*

Two views about MSE

Classical view: deliberate randomization of players to conceal (matching pennies)

Bayesian view: ignorance of the other players (battle of sexes)

O'Neill (1987)

Risk aversion plays no role when there are only two possible outcomes.

	1	2	3	J	MSE	Actual	QRE
1	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4			
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

Actual frequencies are quite close to MSE.

Brown and Rosenthal (1990) criticized O'Neill.
They argue that aggregate tests are not good enough.

They run (temporal dependence)

$$J_{t+1}=a_0+a_1J_t+a_2J_{t-1}+b_0J^*_{t+1}+b_1J^*_t+b_2J^*_{t-1}$$
$$+c_1J_tJ^*_t+c_2J_{t-1}J^*_{t-1}$$

We expect that only a_0 is nonzero.

Guessing: b_0 8%

Previous opp choices: b_1, b_2 30%

Previous outcomes: c_1, c_2 38%

Previous choices and outcome: b_1, b_2, c_1, c_2 44%

Previous own choices: a_1, a_2 48%

All effects: 62%

Run: 2JJJJ1233

2 JJJJ 1 2 33

Subjects play J twice too rarely (too short runs)

Players are not able to use the temporal dependence to guess what their opponent will do in this period.

Each player may deviate from MSE, but they cannot detect them, so equilibrium-in-beliefs is somewhat supported.

Psychology studies use production task in which subjects generate random sequences.

Subjects produce sequences whose features resemble the underlying statistical process more closely than short random sequences actually do.

Typically, too balanced, too many runs, the longest run is too short.

Children don't seem to learn this misconception until after 5th grade.

Rapport and Budescu (1992, 1994, 1997) compare sequences from a production task to strategies in a constant-sum game.

Condition D: matching pennies 150 times (trial by trial)

Condition S: give the entire sequence of 150 plays at once

Condition R: produce the random outcome of tossing an unbiased coin 150 times

iid is rejected 40%, 65% and 80%. Game playing seems to reduce deviations from randomness.

Is it because subjects are better motivated or they are interfered their misrandomizing is inhibited?

DR (SR, 1-2) vs (LR, 3-4)

A theory model like this: subjects only remember m elements in their sequence and they chose the m+1st to balance the number of H and T choices in the last m+1 flips. If m is not very large, we will see they alternate choices too fequently.

Subjects estimate:

P(H|H)=0.42

P(H|HH)=0.32

P(H|HHH)=0.21

Magic 7?

Explicit randomization

Subjects control randomization by allocating a total of say 100 choices to either of their say two strategies. These choices are shuffled and the computer select the top choice as the realized strategy.

Deviations cannot be due to cognitive limit.

This allows experimenters to observe the randomization subjects really want to play.

Result: Deviations from MSE are small but still significant.

There are about 10 percent who are purists.

Patent races games and location games (hotelling model with 3 firms)

Result: Though MSE is difficult to solve in these games, but frequency distribution is quite consistent with the counterintuitive MSE predictions.

Two field studies

Walker and Wooders: serve decisions of tennis players in 10 Grand Slam matches.

Result: Win rates across two different directions are not statistically different.

Players still exhibit some over-alteration in serve choices though the temporal dependence is weaker, compared to lab subjects.

Palacios-Huerta (2001) look at penalty kicks in soccer.

Can code both kicker and goalie's choices.

No selection bias.

Win rates are equal and no serial dependence (not surprising since penalty kicks are few and are often done by different players).

The take-home message is:

Aggregate frequencies of play are close to MSE but the deviations are statistically significant.

QRE seems to fit behaviors well.

Temporal dependence is frequently observed.

With explicit randomization, the existence of purists hint on equilibrium in beliefs (players cannot guess what opponents are doing and their beliefs about opp are correct on average, but they may not be randomizing themselves).

field vs. lab