Raising Capital from Heterogeneous Investors

Marina Halac   Ilan Kremer   Eyal Winter

May 2019

Introduction

- Firm raises capital from heterogeneous investors to fund project
- Investors face strategic risk: project succeeds only if enough invest
  - Possible outcomes where investors don't invest, expecting others won't

Introduction

- Firm raises capital from heterogeneous investors to fund project
- Investors face strategic risk: project succeeds only if enough invest
  - Possible outcomes where investors don't invest, expecting others won't
- This paper: What is optimal mechanism that guarantees investment?
  - Compensate for strategic risk, which depends on amount invested
  - How does heterogeneity in investor size affect scheme and payoffs?
  - Does firm offer differential returns based on size? Who is favored?

Example (1)

- Firm's project succeeds if capital raised exceeds $I \sim U[0, 30]$
  - Success yields additional surplus
- Agent 1 has 10 units of capital, agent 2 has 20 units
  - Outside option is safe asset with net return of 10%

Example (1)

- Firm's project succeeds if capital raised exceeds $I \sim U[0, 30]$
  - Success yields additional surplus
- Agent 1 has 10 units of capital, agent 2 has 20 units
  - Outside option is safe asset with net return of 10%
- Firm wants to guarantee full investment; offers returns under success
  - If offer 10%, full-investment NE at minimum cost, but also other NE
  - Optimal scheme makes investment dominant for one of the agents

Example (2)

- Suppose firm makes investment dominant for agent 1
  - Must offer agent 1 net return (slightly above) r satisfying $\frac{r}{3} = 10\%$
  - Then offer 10% to agent 2. Cost is $10(30\%) + 20(10\%) = 5$

Example (2)

- Suppose firm makes investment dominant for agent 1
  - Must offer agent 1 net return (slightly above) r satisfying $\frac{r}{3} = 10\%$
  - Then offer 10% to agent 2. Cost is $10(30\%) + 20(10\%) = 5$
- Firm's cost is lower if investment is made dominant for agent 2
  - Must offer agent 2 net return (slightly above) r satisfying $\frac{2r}{3} = 10\%$
  - Then offer 10% to agent 1. Cost is $10(10\%) + 20(15\%) = 4$

Example (2)

- Suppose firm makes investment dominant for agent 1
  - Must offer agent 1 net return (slightly above) r satisfying $\frac{r}{3} = 10\%$
  - Then offer 10% to agent 2. Cost is $10(30\%) + 20(10\%) = 5$
- Firm's cost is lower if investment is made dominant for agent 2
  - Must offer agent 2 net return (slightly above) r satisfying $\frac{2r}{3} = 10\%$
  - Then offer 10% to agent 1. Cost is $10(10\%) + 20(15\%) = 4$
- Result: Larger investor receives higher net return than smaller investor

Example (3)

- Suppose we now transfer 4 units of capital from agent 1 to agent 2
  - Firm offers net return of 12.5% to agent 2 and 10% to agent 1
  - Cost is $6(10\%) + 24(12.5\%) = 3.6$

Example (3)

- Suppose we now transfer 4 units of capital from agent 1 to agent 2
  - Firm offers net return of 12.5% to agent 2 and 10% to agent 1
  - Cost is $6(10\%) + 24(12.5\%) = 3.6$
- Result: Firm benefits from dispersion in investor size
- Result: Dispersion reduces range of net returns

Example (3)

- Suppose we now transfer 4 units of capital from agent 1 to agent 2
  - Firm offers net return of 12.5% to agent 2 and 10% to agent 1
  - Cost is $6(10\%) + 24(12.5\%) = 3.6$
- Result: Firm benefits from dispersion in investor size
- Result: Dispersion reduces range of net returns

What we do: General setting. Identify condition under which results hold

- Condition on distribution of threshold I; implied by log-concavity

Literature

- Contracting with externalities
  - Segal (1999, 2003), Winter (2004), Bernstein-Winter (2012)
  - Departure: Endogenous externalities, heterogeneity
- Prior results on discrimination, exogenous heterogeneous externalities
  - Segal (2003), Winter (2004). Inostroza-Pavan (2017) on persuasion
  - Bernstein-Winter (2012). Sákovics-Steiner (2012) in global game
- Broader literature on capital raising and coordination

Setup

- Firm owns project that requires capital to be implemented/ "succeed"
  - Required capital is uncertain: distributed over $[0, \bar{I}]$ with cdf F
  - Success yields fixed additional surplus $A > 0$

Setup

- Firm owns project that requires capital to be implemented/ "succeed"
  - Required capital is uncertain: distributed over $[0, \overline{I}]$ with cdf F
  - Success yields fixed additional surplus $A > 0$
- Set of $N > 1$ agents. Agent $n \in S = \{1, \ldots, N\}$ has endowment $\overline{x}_n$

Setup

- Firm owns project that requires capital to be implemented/ "succeed"
  - Required capital is uncertain: distributed over $[0, \overline{I}]$ with cdf F
  - Success yields fixed additional surplus $A > 0$
- Set of $N > 1$ agents. Agent $n \in S = \{1, \ldots, N\}$ has endowment $\overline{x}_n$
- Firm proposes compensation contract to each agent
  - Agents decide simultaneously if invest or take safe asset return $\theta > 0$
  - Firm wants to guarantee unique NE outcome

Contracts

- For each n, net returns (r_n, k_n) conditional on investment $x_n \in [0, \overline{x}_n]$
  - r_n if success; k_n if failure

Contracts

- For each n, net returns (r_n, k_n) conditional on investment $x_n \in [0, \overline{x}_n]$
  - r_n if success; k_n if failure
- Denote n's decision by $y_n \in \{0, 1\}$. Firm's budget constraint (BC) is

$$\sum_{n=1}^{N} r_n y_n x_n \leq A \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq 0 \qquad \forall \mathcal{Y} = (y_1, \ldots, y_N)$$

Contracts

- For each n, net returns (r_n, k_n) conditional on investment $x_n \in [0, \overline{x}_n]$
  - r_n if success; k_n if failure
- Denote n's decision by $y_n \in \{0, 1\}$. Firm's budget constraint (BC) is

$$\sum_{n=1}^{N} r_n y_n x_n \leq A \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq 0 \qquad \forall \mathcal{Y} = (y_1, \ldots, y_N)$$

- Analyze firm's problem in two steps:

(i) for fixed $(x_n)_{n \in S}$, find optimal $(r_n, k_n)_{n \in S}$ guaranteeing these investments

(ii) given (i), find optimal $(x_n)_{n \in S}$ with $x_n \in [0, \overline{x}_n]$ for each n

Firm's problem: Step (i)

- Find least-cost $(r_n, k_n)_{n \in S}$ s.t. investments $(x_n)_{n \in S}$ are unique NE
  - Since open set, require unique NE when each r_n increased by any $\varepsilon > 0$

Firm's problem: Step (i)

- Find least-cost $(r_n, k_n)_{n \in S}$ s.t. investments $(x_n)_{n \in S}$ are unique NE
  - Since open set, require unique NE when each r_n increased by any $\varepsilon > 0$

- Let E be set of NE profiles given $(r_n, k_n)_{n \in S}$. Two conditions:

(C1) $\mathcal{Y}^1 \equiv (1, \ldots, 1) \in E$

(C2) $\mathcal{Y} \in E, \mathcal{Y} \neq \mathcal{Y}^1 \implies \exists n : y_n = 0,\ U_n(1, \mathcal{Y}_{-n}) = U_n(0, \mathcal{Y}_{-n})$

Firm's problem: Step (i)

- Find least-cost $(r_n, k_n)_{n\in S}$ s.t. investments $(x_n)_{n\in S}$ are unique NE
  - Since open set, require unique NE when each r_n increased by any $\varepsilon > 0$

- Let E be set of NE profiles given $(r_n, k_n)_{n\in S}$. Two conditions:

(C1) $\mathcal{Y}^1 \equiv (1, \ldots, 1) \in E$

(C2) $\mathcal{Y} \in E, \mathcal{Y} \neq \mathcal{Y}^1 \implies \exists n : y_n = 0,\ U_n(1, \mathcal{Y}_{-n}) = U_n(0, \mathcal{Y}_{-n})$

- Let $X_N \equiv \sum_{n=1}^{N} x_n$. Optimal scheme guaranteeing $(x_n)_{n\in S}$ solves:

$$\max_{(r_n,k_n)_{n\in S}} V = \left(A - \sum_{n=1}^{N} r_n x_n\right) F\left(X_N\right) - \sum_{n=1}^{N} k_n x_n \left(1 - F\left(X_N\right)\right)$$

subject to (BC), (C1), and (C2)

Discussion of assumptions

- Firm cannot coordinate agents to its preferred equilibrium
  - Consistent with experiments (e.g. Devetag-Ortmann 2007)
- Agents make choices simultaneously
  - Extends to sequential moves under solution concepts used in literature
- Firm relies on contracts that are bilateral and simple
  - Simple excludes menus. Without loss if indivisibilities or condition holds
- Budget constraint on and off path
  - Without loss given focus on unique implementation

Characterizing the optimal return schedule

Lemma

(C1)-(C2) $\iff$ $\exists$ *permutation* $\pi = (n_1, \ldots, n_N)$ *of set of agents s.t., for each* i, n_i *is willing to invest if* $(n_1, \ldots, n_{i-1})$ *do, no matter rest*

Characterizing the optimal return schedule

Lemma

(C1)-(C2) $\Longleftrightarrow$ $\exists$ *permutation* $\pi = (n_1, \ldots, n_N)$ *of set of agents s.t., for each* i, n_i *is willing to invest if* $(n_1, \ldots, n_{i-1})$ *do, no matter rest*

($\Rightarrow$)
- By (C2), $\exists n_1$ willing to invest if noone does; by (C1), n_1 willing if all do
- Hence, n_1 willing to invest no matter what others do
- Induction shows n_i willing to invest if $(n_1, \ldots, n_{i-1})$ do, no matter rest

Characterizing the optimal return schedule

Lemma

(C1)-(C2) $\iff \exists$ *permutation* $\pi = (n_1, \ldots, n_N)$ *of set of agents s.t., for each* i, n_i *is willing to invest if* $(n_1, \ldots, n_{i-1})$ *do, no matter rest*

($\Rightarrow$) ◦ By (C2), $\exists n_1$ willing to invest if noone does; by (C1), n_1 willing if all do

◦ Hence, n_1 willing to invest no matter what others do

◦ Induction shows n_i willing to invest if $(n_1, \ldots, n_{i-1})$ do, no matter rest

- Optimal schedule specifies $\pi = (n_1, \ldots, n_N)$ and (r_i, k_i) for each n_i
  - First characterize $(r_i^*, k_i^*)_{i \in S}$ and then solve for $\pi^* = (n_1^*, \ldots, n_N^*)$

Optimal returns

- Given $\pi = (n_1, \ldots, n_N)$, let $X_i \equiv \sum_{j=1}^{i} x_{n_j}$

Proposition

Optimal schedule specifies permutation π and $(r_i^*, k_i^*)_{i \in S}$ s.t., for each i,

- n_i *is indifferent over investing if* $(n_1, \ldots, n_{i-1})$ *invest and others don't*
- *Returns satisfy*

$$r_i^* = \frac{\theta}{F(X_i)} \quad \textit{and} \quad k_i^* = 0$$

Sketch of proof (1)

- By Lemma, $\exists \pi$ and (r_i^*, k_i^*) s.t. $\forall i \in S$ and $\forall j \in \{i, \ldots, N\}$,

$$r_i^* F(X_j) + k_i^* (1 - F(X_j)) \geq \theta$$

Sketch of proof (1)

- By Lemma, $\exists \pi$ and (r_i^*, k_i^*) s.t. $\forall i \in S$ and $\forall j \in \{i, \ldots, N\}$,

$$r_i^* F(X_j) + k_i^* (1 - F(X_j)) \geq \theta$$

- By BC and $\theta > 0$, schedule must set $\forall i$

$$r_i^* > 0 \geq k_i^*$$

Sketch of proof (1)

- By Lemma, $\exists \pi$ and (r_i^*, k_i^*) s.t. $\forall i \in S$ and $\forall j \in \{i, \ldots, N\}$,

$$r_i^* F(X_j) + k_i^* (1 - F(X_j)) \geq \theta$$

- By BC and $\theta > 0$, schedule must set $\forall i$

$$r_i^* > 0 \geq k_i^* \rightarrow \text{strategic complementarities (SC)}$$

Sketch of proof (1)

- By Lemma, $\exists \pi$ and (r_i^*, k_i^*) s.t. $\forall i \in S$ and $\forall j \in \{i, \ldots, N\}$,

$$r_i^* F(X_j) + k_i^* (1 - F(X_j)) \geq \theta$$

- By BC and $\theta > 0$, schedule must set $\forall i$

$$r_i^* > 0 \geq k_i^* \rightarrow \text{strategic complementarities (SC)}$$

- Thus, optimal scheme is "divide and conquer":

$$r_i^* F(X_i) + k_i^* (1 - F(X_i)) = \theta \quad \forall i \in S$$

Sketch of proof (2)

- Given $r_i^* F(X_i) + k_i^* (1 - F(X_i)) = \theta$, set $k_i^* = 0$, $r_i^* = \frac{\theta}{F(X_i)}$
  - If $k_i < 0$, $\uparrow k_i$ by small $\varepsilon > 0$ and $\downarrow r_i$ by $\varepsilon \eta_i$ for $\eta_i \equiv \frac{1 - F(X_i)}{F(X_i)}$
  - Incentives are preserved
  - Firm's payoff V changes by $\varepsilon \frac{(F(X_N) - F(X_i))}{F(X_i)} \geq 0$

Sketch of proof (2)

- Given $r_i^* F(X_i) + k_i^* (1 - F(X_i)) = \theta$, set $k_i^* = 0$, $r_i^* = \dfrac{\theta}{F(X_i)}$
  - If $k_i < 0$, $\uparrow k_i$ by small $\varepsilon > 0$ and $\downarrow r_i$ by $\varepsilon \eta_i$ for $\eta_i \equiv \dfrac{1 - F(X_i)}{F(X_i)}$
  - Incentives are preserved
  - Firm's payoff V changes by $\varepsilon \dfrac{(F(X_N) - F(X_i))}{F(X_i)} \geq 0$

- Intuition: firm conditions on all investing, n_i on only $(n_1, \ldots, n_i)$
  - Hence, firm values r_i relative to k_i more than n_i

Sketch of proof (2)

- Given $r_i^* F(X_i) + k_i^* (1 - F(X_i)) = \theta$, set $k_i^* = 0$, $r_i^* = \dfrac{\theta}{F(X_i)}$
  - If $k_i < 0$, $\uparrow k_i$ by small $\varepsilon > 0$ and $\downarrow r_i$ by $\varepsilon \eta_i$ for $\eta_i \equiv \dfrac{1 - F(X_i)}{F(X_i)}$
  - Incentives are preserved
  - Firm's payoff V changes by $\varepsilon \dfrac{(F(X_N) - F(X_i))}{F(X_i)} \geq 0$
- Intuition: firm conditions on all investing, n_i on only $(n_1, \ldots, n_i)$
  - Hence, firm values r_i relative to k_i more than n_i

Remark

Optimal scheme yields unique rationalizable outcome

Finding the optimal permutation

- $(r_i^*, k_i^*)_{i \in S}$ maximally relaxes BC. Firm can thus guarantee $(x_n)_{n \in S}$ iff

$$A \geq \sum_{i=1}^{N} \frac{\theta}{F\left(X_i\right)} x_{n_i} \text{ for some } \pi$$

Finding the optimal permutation

- $(r_i^*, k_i^*)_{i \in S}$ maximally relaxes BC. Firm can thus guarantee $(x_n)_{n \in S}$ iff

$$A \geq \sum_{i=1}^{N} \frac{\theta}{F(X_i)} x_{n_i} \text{ for some } \pi$$

- Firm's payoff is

$$V = \left(A - \theta \sum_{i=1}^{N} \frac{x_{n_i}}{F(X_i)} \right) F(X_N)$$

- Optimal permutation π^* minimizes firm's costs under success:

$$\theta \sum_{i=1}^{N} \frac{x_{n_i}}{F(X_i)}$$

Optimal permutation

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$

For any investments $(x_n)_{n \in S}$ *with* $X_N \leq X$*,* $\pi^* = (n_1^*, \ldots, n_N^*)$ *satisfies*

$$x_{n_1^*} \geq \ldots \geq x_{n_N^*}$$

Hence, larger investors receive higher net returns than smaller investors

Optimal permutation

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$

For any investments $(x_n)_{n \in S}$ *with* $X_N \leq X$, $\pi^* = (n_1^*, \ldots, n_N^*)$ *satisfies*

$$x_{n_1^*} \geq \ldots \geq x_{n_N^*}$$

Hence, larger investors receive higher net returns than smaller investors

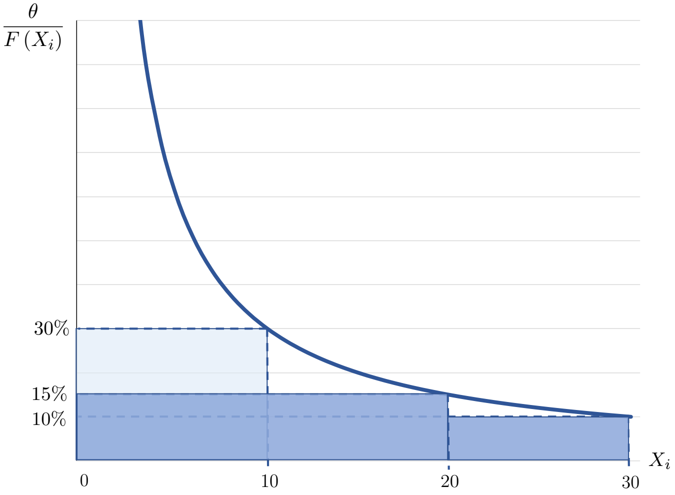
Remark

$F(x)$ *log-concave* $\implies 1/F(x)$ *convex*

- Most commonly used distributions are log-concave

Example

- F uniform over $[0, 30]$, $\theta = 10\%$, $(x_1, x_2) = (10, 20)$
  - Optimal permutation is $\pi^* = (2, 1)$



Intuition

- Agent n_i paid on marginal unit invested: $r_i^* = \theta / F(X_i)$
- Thus, if $1/F(x)$ is convex, decreasing order minimizes costs
  - I.e., optimal to move down the return curve $\theta / F(X_i)$ "quickly"
- Intuitively, large x_n self-insures agent, reduces required risk premium
  - Place large x_n when risk premium drops most sharply with investment

Intuition

- Agent n_i paid on marginal unit invested: $r_i^* = \theta/F(X_i)$
- Thus, if $1/F(x)$ is convex, decreasing order minimizes costs
  - I.e., optimal to move down the return curve $\theta/F(X_i)$ "quickly"
- Intuitively, large x_n self-insures agent, reduces required risk premium
  - Place large x_n when risk premium drops most sharply with investment

Remark

$1/F(x)$ *convex (over range) not only sufficient but also necessary for result*

Characterizing the optimal investments

- So far $(x_n)_{n \in S}$ as given. What are the optimal capital amounts?

Characterizing the optimal investments

- So far $(x_n)_{n \in S}$ as given. What are the optimal capital amounts?

Definition

For two N-vectors $\mathbf{x}$ and $\widehat{\mathbf{x}}$, $\widehat{\mathbf{x}}$ majorizes $\mathbf{x}$ if

- *components of $\widehat{\mathbf{x}}$ and $\mathbf{x}$ have same total sum, and*
- *$\forall m$, sum of m smallest components is weakly smaller in $\widehat{\mathbf{x}}$ than in $\mathbf{x}$*

Optimal investments

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$*. Take investments* $(x_n)_{n \in S}$, $X_N \leq X$

Let investments $(\widehat{x}_n)_{n \in S}$ *majorize* $(x_n)_{n \in S}$

Firm's expected payoff under $(\widehat{x}_n)_{n \in S}$ *is higher than that under* $(x_n)_{n \in S}$

Optimal investments

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$. *Take investments* $(x_n)_{n \in S}$, $X_N \leq X$

Let investments $(\widehat{x}_n)_{n \in S}$ *majorize* $(x_n)_{n \in S}$

Firm's expected payoff under $(\widehat{x}_n)_{n \in S}$ *is higher than that under* $(x_n)_{n \in S}$

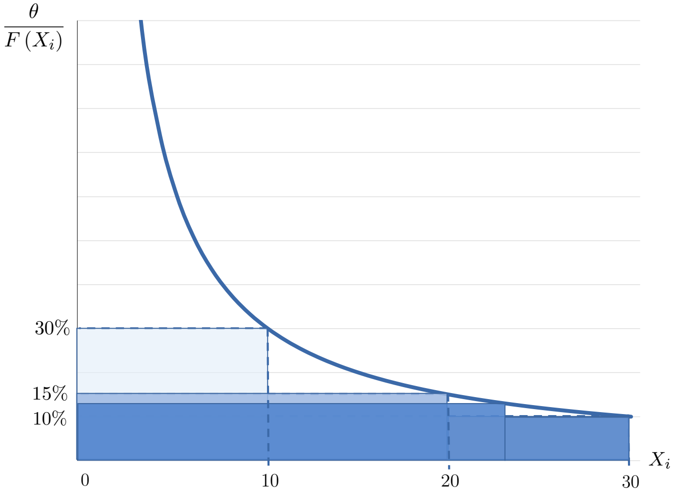
Corollary

Given $(\overline{x}_n)_{n \in S}$, *firm raises capital from agents with largest endowments*

- If $X_N < \overline{X}_N$, not only preferential returns but also preferential access

Example

- If $(x_1, x_2, x_3) = (10, 10, 10)$, cost is $10(30\% + 15\% + 10\%) = 5.5$
- If $(x_1, x_2, x_3) = (10, 20, 0)$, cost is 4. Further reduce w/transfer 1→2



Intuition

- Aggregating capital of subset reduces strategic uncertainty
  - Self-insurance: single agent knows she will invest the whole amount
- More generally, derive $(\widehat{x}_n)_n$ from $(x_n)_n$ by finite sequence of transfers
  - From small to large (Hardy-Littlewood-Polya 1934)
- We show any such transfer lowers firm's costs
  - Move down return curve $\theta/F(X_i)$ "more quickly" given original π^*
  - Changing to optimal π^* can only raise firm's payoff further

Distribution of returns

- Given $\pi^* = (n_1^*, \ldots, n_N^*)$, range of net returns is $F(X_N)\,(r_1^* - r_N^*)$

Distribution of returns

- Given $\pi^* = (n_1^*, \ldots, n_N^*)$, range of net returns is $F(X_N)\,(r_1^* - r_N^*)$

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$*. Take investments* $(x_n)_{n\in S}$, $X_N \leq X$

Let investments $(\widehat{x}_n)_{n\in S}$ *majorize* $(x_n)_{n\in S}$

Range of net returns under $(\widehat{x}_n)_{n\in S}$ *is smaller than that under* $(x_n)_{n\in S}$

Distribution of returns

- Given $\pi^* = (n_1^*, \ldots, n_N^*)$, range of net returns is $F(X_N)\,(r_1^* - r_N^*)$

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$*. Take investments* $(x_n)_{n \in S}$*,* $X_N \leq X$

Let investments $(\widehat{x}_n)_{n \in S}$ *majorize* $(x_n)_{n \in S}$

Range of net returns under $(\widehat{x}_n)_{n \in S}$ *is smaller than that under* $(x_n)_{n \in S}$

- Dispersion lowers largest investor's return; keeps smallest unchanged
  - As a result, range of final capital can decrease with dispersion

Discussion of results

- Differential net returns: larger investors get more per unit invested
  - Consistent with evidence from private equity
  - Suggests "winner-takes-all dynamics": large investors become larger
- Distribution of capital: larger investments from wealthier investors
  - Dispersion in investor size increases firm's payoff
  - Dispersion thus also increases feasibility of investment
- Return advantage of large investors depends on capital distribution
  - Scheme is less discriminatory when investments are more unequal
  - To the extent that final capital may become more equal with dispersion

Firm's initial capital (1)

- Suppose firm has capital $W > 0$, with $W < \theta X_N$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq W + A \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq W$$

Firm's initial capital (1)

- Suppose firm has capital $W > 0$, with $W < \theta X_N$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq W + A \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq W$$

- By Lemma: n_i willing to invest if $(n_1, \ldots, n_{i-1})$ do, no matter rest

Firm's initial capital (1)

- Suppose firm has capital $W > 0$, with $W < \theta X_N$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq W + A \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq W$$

- By Lemma: n_i willing to invest if $(n_1, \ldots, n_{i-1})$ do, no matter rest
- Firm can induce strategic substitutability.

Firm's initial capital (1)

- Suppose firm has capital $W > 0$, with $W < \theta X_N$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq W + A \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq W$$

- By Lemma: n_i willing to invest if $(n_1, \ldots, n_{i-1})$ do, no matter rest
- Firm can induce strategic substitutability. Show $r_i \geq k_i \ \forall i$ is optimal
  - If $k_i > r_i$ for some i, then by BC $k_j < r_j$ for some $j \neq i$
  - n_i indifferent when all others invest; n_j when only $(n_1, \ldots, n_{j-1})$ do
  - Perturbation with $\downarrow k_i$, $\uparrow r_i$, $\uparrow k_j$, $\downarrow r_j$ (weakly) increases firm's payoff

Firm's initial capital (2)

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$. *Take* $(x_n)_{n\in S}$, $W + X_N \leq X$

- $\pi^* = (n_1^*, \ldots, n_N^*)$ *satisfies* $x_{n_1^*} \geq \ldots \geq x_{n_N^*}$
- $(r_i^*, k_i^*)_{i\in S}$ *satisfy*

$$k_i^* = \frac{\min\{\theta x_{n_i^*}, W_i\}}{x_{n_i^*}} \quad \textit{and} \quad r_i^* = \frac{\theta - k_i^*(1 - F(W + X_i))}{F(W + X_i)}$$

where $W_N \equiv W$, $W_i \equiv \max\{W - \sum_{j=i+1}^{N} k_j^* x_{n_j^*}, 0\}$ *for* $i \in \{1, \ldots, N-1\}$

Firm's initial capital (2)

Proposition

Suppose $1/F(x)$ *convex over* $[0, X]$. *Take* $(x_n)_{n \in S}$, $W + X_N \leq X$

- $\pi^* = (n_1^*, \ldots, n_N^*)$ *satisfies* $x_{n_1^*} \geq \ldots \geq x_{n_N^*}$
- $(r_i^*, k_i^*)_{i \in S}$ *satisfy*

$$k_i^* = \frac{\min\{\theta x_{n_i^*}, W_i\}}{x_{n_i^*}} \quad \textit{and} \quad r_i^* = \frac{\theta - k_i^*(1 - F(W + X_i))}{F(W + X_i)}$$

where $W_N \equiv W$, $W_i \equiv \max\{W - \sum_{j=i+1}^{N} k_j^* x_{n_j^*}, 0\}$ *for* $i \in \{1, \ldots, N-1\}$

- Benchmark results extend, plus predictions on risk profile
  - Smallest investors fully insured, until W depleted
  - Then order investors in decreasing size order

Proportional surplus

- Suppose project success yields surplus Rx if x invested, for $R > 0$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq \sum_{n=1}^{N} R y_n x_n \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq 0$$

Proportional surplus

- Suppose project success yields surplus Rx if x invested, for $R > 0$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq \sum_{n=1}^{N} R y_n x_n \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq 0$$

- Problem with fixed surplus $A_R \equiv RX_N$ is relaxed version
  - BC with proportional surplus adds restriction: $\max_{n \in S} r_n \leq R$

Proportional surplus

- Suppose project success yields surplus Rx if x invested, for $R > 0$
  - BC: $\forall \mathcal{Y} = (y_1, \ldots, y_N)$,

$$\sum_{n=1}^{N} r_n y_n x_n \leq \sum_{n=1}^{N} R y_n x_n \quad \text{and} \quad \sum_{n=1}^{N} k_n y_n x_n \leq 0$$

- Problem with fixed surplus $A_R \equiv R X_N$ is relaxed version
  - BC with proportional surplus adds restriction: $\max_{n \in S} r_n \leq R$
- Benchmark results extend. Can guarantee $(x_n)_{n \in S}$ iff $r^*_{n_1} \leq R$
  - Solution to relaxed problem minimizes highest return given constraints

Concluding remarks

- Capital raising for new projects must address strategic risk
  - We characterize firm's optimal unique-implementation scheme
- Broad insight: strategic risk may be a driver of inequality
  - Profit-max mechanism favors certain agents to lower risk on others
  - Under condition, favorable terms to those already in favorable position
- Further applications
  - Monopolist offers exclusive contracts to buyers w/different demand size
  - Firm offers rewards to team of workers with different ability
  - Bank offers collateral and interest to depositors of different size

Thank you!