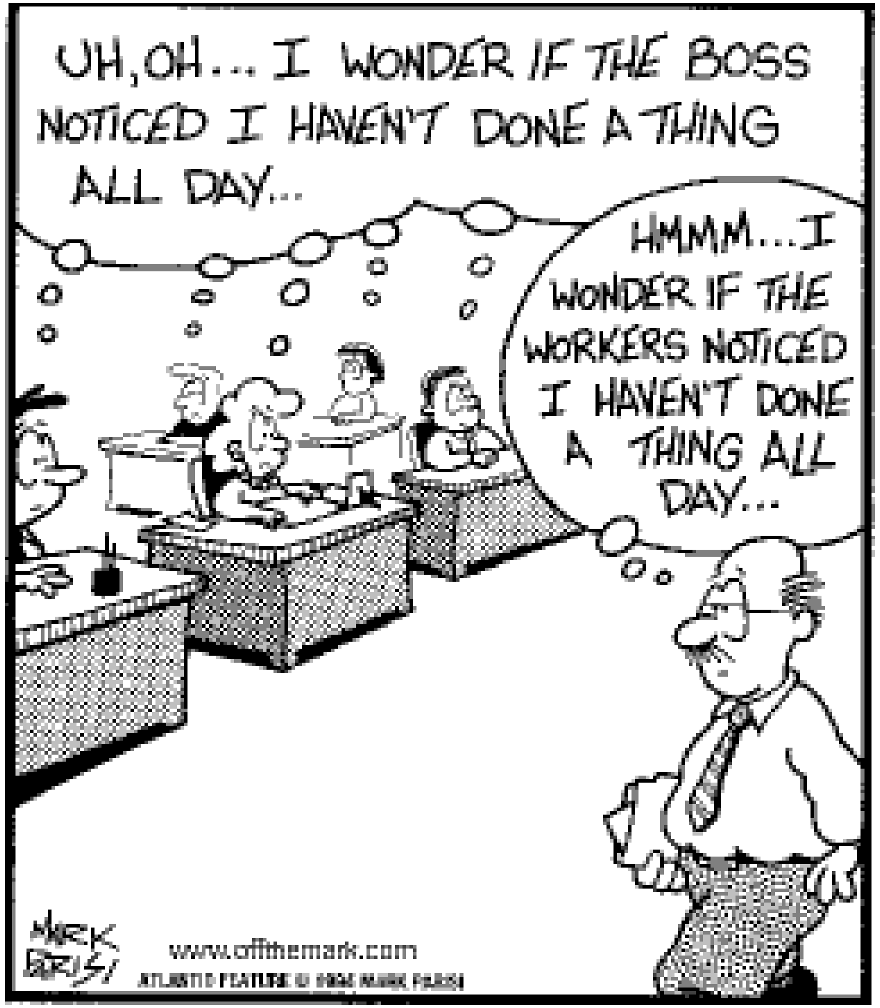
Information among Peers and Incentives

Eyal Winter
The Center for the Study of Rationality, HU
NTU, April 2024



Multi-Agent PA Models

- Holmstrom, *RAND* 1982
- Holmstrom and Milgrom, *JITE* 1990
- This literature primarily deals with the issue of risk allocation.
- Winter, *AER* 2004
- Winter, *RAND* (forthcoming)

Networks in Games

- Jackson and Wolinsky, *JET* 1996
- Bala and Goyal, *Econometrica* 2000
- Mutuswami and Winter, *JET* 2002
  - There the game is in forming the network.

- Here the game is played after the network is formed and the principal designs the network.

Empirical Evidence

- Heywood and Jirjahn (2004) show that blue-collar workers who work jointly in small teams have a lower absentee rate
  - than other similar workers who work alone.
- Teasley et al. (2002) evaluated workers' productivity using measures commonly used in software development.
  - Teams in war rooms are twice as productive as similar teams working in closed offices.

Example

- 2-agent organization. Each agent deals with a single task.
- c is the cost of effort.
  - Effort increases the success probability of a task from α to 1.
- The project succeeds iff all tasks are successful.
  - The principal can observe only the outcome of the project.
- A mechanism is a pair of payoffs (v_1, v_2) paid to the agents if the project succeeds (paid 0 if fails).
  - The principal wants to induce both agents to invest in an equilibrium, and he wants to achieve it with minimal rewards.

The Effect of Peer Information

1. Agents move simultaneously:
   - An agent BR to an effort by his peer should be effort
   - So, his reward v should satisfy: $v - c \geq \alpha v$
   - Or, $v \geq c/(1-\alpha)$.
2. Agents act sequentially:
   - Agent 2 should get $c/(1-\alpha)$.
   - It is enough to pay agent 1 $v_1 = c/(1-\alpha^2)$, which solves: $v_1 - c = \alpha^2 v_1$.

- Less rewards when agent 2 sees agent 1.

General Model

- The organizational project involves n agents that collectively manage a project.
- For a group $S \subset N$ of investing agents the probability that the project succeeds is $p(S)$.
- A mechanism $v = (v_1, \ldots, v_n)$ pays agent i the payoff v_i if the project succeeds and zero otherwise.

Internal Information about Effort (IIE)

- IIE: Binary order k over the set of agents N, where
- $i\ k\ j$, stands for agent i knowing the effort decision of agent j before making his own decision.
- We require that k is acyclic: $i_1\ k\ i_2\ k \ldots k\ i_r$ with $r \leq n$ implies that $i_1, \ldots, i_r$ are distinct.
- $K_i = \{j\colon i\ k\ j\}$ are the set of agents i sees.

The Game $G(k, v)$

- Given a reward vector v and an IIE k:
- A strategy for player i is a function s_i: $2^{K_i} \to \{0, 1\}$
- For a strategy profile $s = (s_1, \ldots, s_n)$ we denote by $M(s)$ the set of agents who exert effort under s.
- Payoffs:
  - $f_i(s) = v_i\, p(M(s)) - c$ if $i \in M(s)$, and
  - $f_i(s) = v_i\, p(M(s))$ if $i \notin M(s)$.

Incentive Inducing Mechanisms

- v is an *INI* mechanism with respect to k if there exists an equilibrium s of $G(k, v)$ with $M(s) = N$.
- v is an optimal INI mechanism if it's *INI* and has minimal total reward among those.
- For an IIE k we denote by $v^*(k)$ the total reward in an optimal INI mechanism with respect to k.

Comparing Information Structures

- We say that k_1 is richer than k_2 if
- for all i, j in N, we have i k_2 j implies i k_1 j.

  - Transparency among peers is good for the principal:
- Proposition 1: If k_1 is richer than k_2, then $v^*(k_1) \leq v^*(k_2)$.

Indirect Information About Effort

- For an IIE k we denote by $t(k)$ the IIE obtained from the transitive closure of k.
- $i \; t(k) \; j$ if and only if there exists a sequence of agents $i_1, i_2, \ldots, i_r$ with $i_1 = i$ and $i_r = j$ and $i_m \; k \; i_{m+1}$.
  - Agent i indirectly knows the effort decision of agent j
- Proposition 2: If $t(k_1)$ is richer than $t(k_2)$, then $v^*(k_1) \leq v^*(k_2)$, with strict inequality iff $t(k_1) \neq t(k_2)$.

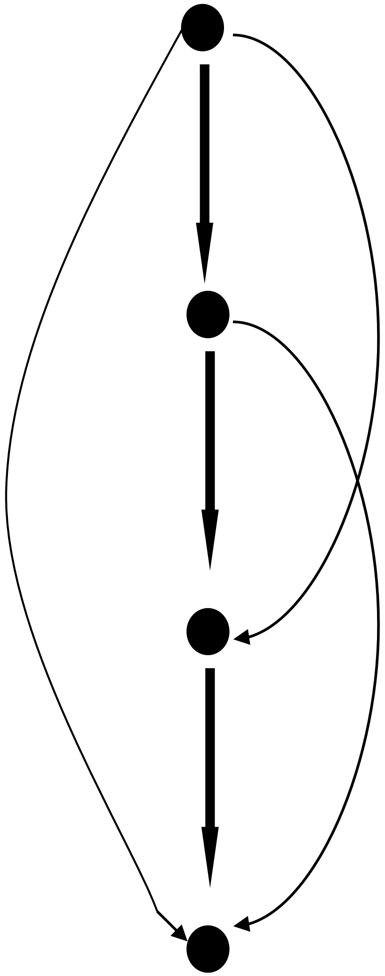
- If i shirks he triggers the shirking of

$$C(i,\ k) = \{j \in N \mid j\ t(k)\ \ i\}.$$

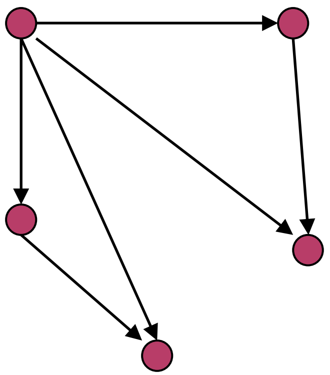
- Agent i is incentivized if:

$$p(N) v_i - c = v_i\, p(N - [C(i,\ k)\, U\{i\}]).$$

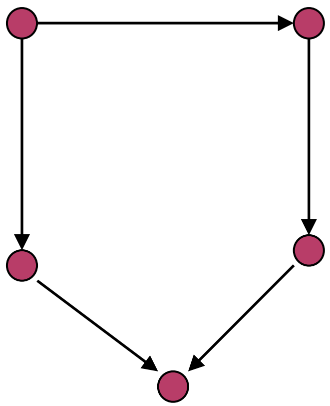
- Corollary: the IIE k_s corresponding to a chain yields the minimal cost for the principal and empty IIE yields the maximal cost.



A

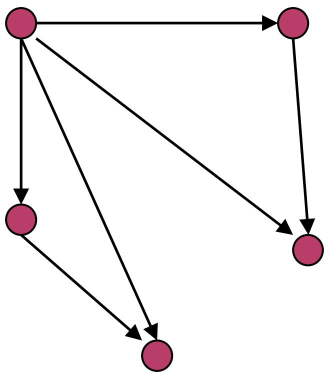


B

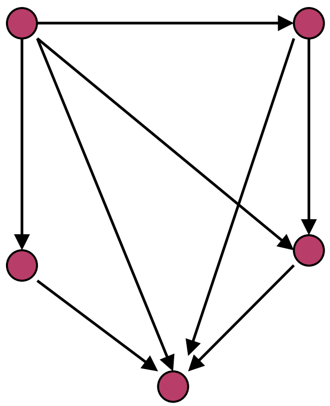


Transitive Closures

A



B



Noisy Peer Monitoring

- A random IIE is a pair (w, k_q), where
  - w is an order of the agents and k_q is a random directed graph on the set of agents with arcs emerging randomly
  - according to an IID Bernoulli with probability $0 < q < 1$.
- When agent j acts (at stage j), he is informed about the effort decision of all his predecessors to which he has an arc.
  - But he is not informed about who will observe his effort decision.

Noisy Peer Monitoring

- For a random IIE (w, k_q) we denote by $v*(w, k_q)$ the total reward of the optimal incentive-inducing mechanism under k_q.

- Proposition 3: If $q^0 > q$, then $v^*(q) > v^*(q^0)$.

Substitution vs. Complementarity

- Complementarity: for every S, T with $T \in S$ and every agent i in $N \backslash S$ we have:

$$p(S \cup \{i\}) - p(S) > p(T \cup \{i\}) - p(T).$$

- Substitutability:

$$p(S \cup \{i\}) - p(S) \leq p(T \cup \{i\}) - p(T).$$

- In our early example $p(S) = \alpha^{2-|S|}$ and complementarity applies.

Proposition 4

- Under Perfect Bayesian Equilibrium:

1. If p satisfies complementarity, then the mechanism derived in Proposition 2 remains the optimal mechanism here.
2. If p satisfies substitution, then the optimal mechanism is $$v_i^* = \frac{c}{p(N) - p(N \setminus \{i\})}$$

   and is identical for all IIEs.

Function-Based vs. Process-Based Teams

- i and j involve complementarity if
- for all S with $i, j \in N \backslash S$ we have:

$$p(S \cup \{i, j\}) - p(S \cup \{j\}) > p(S \cup \{i\}) - p(S).$$

- Substitution:

$$p(S \cup \{i, j\}) - p(S \cup \{j\}) \leq p(S \cup \{i\}) - p(S).$$

2x2 Organization

- Two products A and B. Two production stages u and d.
- Four agents: a_d, a_u, b_d, b_u
- Complementarity across different stages of the same product and substitution across different products at the same stage.
- Complementarity between a_d and a_u as well as between b_d and b_u.
- a_d and b_d are substitutes and so are a_u and b_u.

2x2 Organization

  - Example: each stage of production succeeds with probability α without effort and with probability $\beta > \alpha$ with effort.
- The product succeeds iff both stages are successful.
- The project's goal is to successfully produce at least one of the two goods.
  - Process-based: $\{a_d, a_u\}$ and $\{b_d, b_u\}$
    - (x_d acts before x_u, $x = a, b$)
  - Function-based: $\{a_d, b_d\}$ and $\{a_u, b_u\}$
    - (a_y acts before b_y, $y = u, d$)

Proposition 5

- Suppose that the principal wants to sustain full effort as a Nash equilibrium with undominated strategies.
- Then the optimal mechanism in the process-based structure costs less than the optimal mechanism in the function-based structure.

	Process-Based	Function-Based
	$a_d : c/[p(N) - p(b_d, b_u)]$,	$c/[p(N) - p(b_d, a_u, b_u)]$
	$a_u : c/[p(N) - p(a_d, b_d, b_u)]$,	$c/[p(N) - p(a_d, b_d, b_u)]$
	$b_d : c/[p(N) - p(a_d, a_u)]$,	$c/[p(N) - p(a_d, a_u, b_u)]$
	$b_u : c/[p(N) - p(a_d, b_d, a_u)]$,	$c/[p(N) - p(a_d, b_d, a_u)]$

Extensions

Information Structures

- Agents are moving sequentially in deciding about their effort.
- Player m has $2^{m\text{-}1}$ decision nodes, each of which is a binary vector x of size $m - 1$ specifying the actions taken by m's predecessors.
- An information structure I is now given by a sequence of partitions $I = (P_1, \ldots, P_n)$, where P_m is a partition of his set of nodes.
  - The information structure I_1 is more transparent than I_2 if every partition in I_1 is a refinement of a partition of I_2

- Denote by $v^*(I, p)$ the cost of the optimal mechanism under the information structure I and the technology p

- Proposition 1 Let I_1 and I_2 be two information structures such that I_1 is more transparent than I_2.
- Then under Nash implementation for any technology p we have $v^*(I_1, p) \leq v^*(I_2, p)$

Example

- I_1: player 2 sees the effort decision of player 1, and
  - Players 3, 4, ..., n are informed only when both player 1 and player 2 are shirking
- I_2: player 2 is again informed about the effort decision of player 1, but
  - players 3, 4, ..., n receive no information whatsoever.

- I_1 is more transparent than I_2. Still under perfect Bayesian implementation we have $v^*(I_1, p) > v^*(I_2, p)$.
  - Lack of Informational Substitution:
- Let x and x' be two decision nodes of m, and
- Let $x \wedge x' = y$ where $y(j) = \min\{x(j), x'(j)\}$.
- We denote by $\mathbf{1}$ the decision point which corresponds to the path in which all players exerted effort
- (LIS): $\mathbf{1} \in P_m(x) \cap P_m(x')$ implies $\mathbf{1} \in P_m(x \wedge x')$

∩

Proposition 2

- Let I_1 and I_2 be two information structures such that I_1 is more transparent than I_2 and such that
- both satisfy lack of informational substitution.
- Then under Perfect Bayesian implementation, for any technology p with increasing returns to scale we have $v^*(I_1, p) \leq v^*(I_2, p)$.

Dan Hamermesh

- April 3, 2006 - Peer pressure can generate externalities, both positive and negative ones.
- A pair of Israeli economists has examined this idea using data on baseball. They argue that whether the externalities within an organization are positive or negative depends on the incentives that are created by one's peers' activities.
- If they create incentives that give you a reason to be more productive, you will work harder and be more productive;
- If they create incentives that make life cushy for you, you will slack off.

Dan Hamermesh (Blog Post)

- They show that players’ batting averages are higher when their teammates are batting better.
  - This makes sense, since improvements in my batting coupled with my teammates' will help achieve victory.
- When a team's pitchers are performing better, however, its batters do not do so well-their batting averages are lower.
  - After all, if the pitcher has a no-hitter going, there is no need to try to score huge numbers of runs to win the game.

Continental Airlines by Knez and Simester

- How come the "Go Forward Plan" did so well for CA?
- After all CA is a large company and employees are scattered across different airports.
- Authors claim – peer monitoring is the answer.
- Two characteristics facilitate peer monitoring:
  1. Autonomous work groups (several in each airport), and
  2. The availability of eye contact between workers.