Incentives and Discrimination

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- Why do we see hierarchies in organizations where authority plays no role at all?
- Why do we see major differences in bonuses across top level managers of the same firm?
- Why should we expect more inequality among workers who are complements (in the production technology) than among ones who are substitutes?

The Model

- The organizational project involves n tasks each performed by a different individual (agent).
  - Agents decide whether to invest on the task they perform.
- c is the cost of investment (constant across agents).
  - If i invests his task ends successfully with probability 1
  - If he doesn't invest this probability is only α.
  - The project succeeds if and only if all tasks end successfully.
- If the project succeeds agents are paid $v = (v_1,\ldots,v_n)$.
  - They are paid 0 if it fails.

The Game $G(v)$

- Strategies:
  - $d_i = 1$ investment,
  - $d_i = 0$ non-investment.
- Payoffs: $f_i(d) = v_i\, \alpha^{s(d)} - cI_{\{d_i = 1\}}$
- where
  - $I_{\{d_i = 1\}} = 1$ if $d_i = 1$ and 0 otherwise.
  - $s(d) = |\{j \mid d_j = 0\}|$ is the number of individuals who choose to shirk.

The Mechanism

- We say that a mechanism $v = (v_1, \ldots, v_n)$ is incentive inducing (INI) if
- v induces all players to invest in every equilibrium,
  - i.e., $d = (1, 1, \ldots, 1)$ is the only Nash equilibrium of the game $G(v)$.
- We will say that an INI mechanism v is optimal if $\sum_{i \in N} v_i' \geq \sum_{i \in N} v_i$ for every other INI mechanism v'.

Proposition 1:

- Let $v^* = \left(\frac{c}{1-\alpha}, \frac{c}{\alpha(1-\alpha)}, \cdots, \frac{c}{\alpha^{n-1}(1-\alpha)} \right)$
- A mechanism v is an optimal *INI* mechanism iff
- $v = \theta(v^*)$ for some permutation θ,
  - i.e. all optimal mechanisms are discriminatory.

Why Discrimination?

- Define $v(k)$ the reward that will make an agent indifferent between investing and shirking given that k other agents invest.
- $v(k)$ solves $v\alpha^{n\text{-}k} = v\alpha^{n\text{-}k\text{-}1} - c$, or: $v(k) = \dfrac{c}{\alpha^{n-k-1}(1-\alpha)}$
  - $v(k) > v(k+1)$.
  - In an optimal mechanism the principal pays $v(0)$ to some player making it a dominant strategy for him/her to invest.
  - Then $v(1)$ to another, $v(2)$ to a third, etc.

General Success Technologies p

- p is a function from $\{0, 1\}^N$ to $[0, 1]$.
  - With symmetry,
- p: $\{0,1, \ldots, n\} \to [0, 1]$, with
  - $p(k) =$ the probability of success if k agents contribute.
  - p is strictly increasing.
- In the benchmark model $p(k) = \alpha^{n\text{-}k}$.
- Proposition 2: A symmetric *INI* mechanism exists iff $p(n) - p(n-1) \leq p(k+1) - p(k)$ for all $k < n - 1$.

Returns to Scale and Discrimination

- We say that a technology p has increasing returns to scale if $p(k+1) - p(k)$ is increasing in k.
- We say that an *INI* mechanism v is fully discriminating if $v_j \neq v_i$ for all pairs i, j
- Proposition 3: The technology p has increasing returns to scale iff all optimal *INI* mechanisms are fully discriminating.
  - Rewards: $\left(\frac{c}{p(1)-p(0)}, \frac{c}{p(2)-p(1)}, \cdots, \frac{c}{p(n)-p(n-1)}\right)$

Different Costs

- Proposition 4: Let p be an increasing returns to scale technology, and
- Let $c_1 < c_2 < \ldots < c_n$ denote agents' effort costs, then
- the (unique) optimal mechanism pays player j

$$v_j^* = \frac{c_j}{p(j) - p(j-1)}$$

  - i.e. v_j^*/c_j is decreasing with j.

What If Agents Are "Almost" Identical?

- $c_1 = 1$
- $c_2 = 1.00001$
- $c_3 = 1.00002$
- $\alpha = 1/2,$
- $v^* = (8,\ 4.00004,\ 2.00004)$

Differential Tasks

- Proposition 5: Consider the benchmark model in Section 2 and assume that $\alpha_1 < \alpha_2 < \ldots < \alpha_n$ and that c is the constant effort cost.
- Then the optimal mechanism is unique and pays

$$v_i = \frac{c}{\prod_{j=i+1}^{n} \alpha_j (1 - \alpha_i)}$$

- For $i < n$ and $v_n = c/(1 - \alpha_n)$
- Furthermore, negligible differences in the values of α_j's result in major differences in rewards.

Tasks of Almost Equal Importance

- $c_i = 1$
- $\alpha_1 = .5$
- $\alpha_2 = .5001$
- $\alpha_3 = .5002$
- $v^* = (7.995, 3.999, 2.001)$

Coordination Strong Equilibrium/Coalition Proof

- Proposition 5: If p has increasing returns to scale then
- $\left(v(n-1), ..., v(n-1)\right)$ is an optimal INI mechanism with respect to Strong Nash Implementation.
- where $v(n-1) = \dfrac{c}{p(n) - p(n-1)}$

Single Agent

- Assume that a single agent can handle all tasks and his effort cost is c for each task.
  - The principal offers to pay the agent v if the project succeeds and 0 otherwise.
- The incentive constraints that the agent will find it better to invest all tasks to investing on none:

$$v\alpha^n > v - nc,$$

- which is $v = \dfrac{nc}{1-\alpha^n}$.

Sequential Moves

- (Winter 2006) The optimal sequential mechanism pays:

$$v_1 = \frac{c}{1-\alpha^n}$$

$$v_2 = \frac{c}{1-\alpha^{n-1}} \leq \frac{c}{1-\alpha^n} = v_1$$

$$\ldots\ldots$$

$$v_n = \frac{c}{1-\alpha} \leq \frac{c}{1-\alpha^2} = v_{n-1} \leq \cdots \leq \frac{c}{1-\alpha^n} = v_1$$

- Hence the sequential mechanism is cheaper.

Increasing Returns to Scale