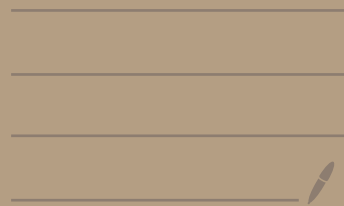


Online Math Camp (24F)

Week 1 (8/12)



Set: A collection of objects, e.g. $S_1 = \{ \triangle, \square, x, \text{😊} \}$

$$S_2 = \{ 1, 2, 3, 4, 5 \} = \{ x : x \text{ is an integer, and } 1 \leq x \leq 5 \}$$

↑
list all elements in the set

↑
describe properties of elements in the set

Def: $x \in A$: x is in A .

$x \notin A$: x is NOT in A .

$\{ x, y, z, \dots \}$: Elements
 $\{ A, B, \dots \}$: Sets

$A \subset B$: A is a subset of B .

Def: $A \cap B = \{ x : x \in A \text{ and } x \in B \}$: the intersection of A and B

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$: the union of A and B

$A^c = \{ x : x \notin A \}$: the complement of A

$A \setminus B = \{ x : x \in A \text{ and } x \notin B \}$: the set difference of A and B

$A \times B = \{ (a, b) : a \in A, b \in B \}$: the product of A and B .

Def: A relation defined on a set S is a subset of $S \times S$

e.g. For $S = \mathbb{R}$, a " $<$ " b is a relation.

Def: A equivalence relation R on S is a relation such that:

① $a R a$ (reflexive)

② $a R b \Rightarrow b R a$ (symmetry)

③ $a R b, b R c \Rightarrow a R c$ (transitive)

e.g. " $=$ ": ① $a = a$, ② $a = b \Rightarrow b = a$, ③ $a = b, b = c \Rightarrow a = c$.

e.g. " \geq " is not an equivalent relation since $3 \geq 1$ ~~\wedge~~ $1 \geq 3$.

Rational Numbers

$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z} \right\}$ is the set of rational numbers, and can be expressed as a relation (m, n) .

However, the representation is NOT unique: $\frac{2}{6} = \frac{1}{3}$ (When is it unique?)

In fact, $\frac{a}{b} = \frac{c}{d} \iff ad = bc$

Define $(a, b) \sim (c, d)$ if $ad = bc$. This is an equivalence relation.

(pf) Exercise.

(So it is unique up to the equivalence relation.)

Addition and Multiplication

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Note that if $\frac{a'}{b'} = \frac{a}{b}$, $\frac{c'}{d'} = \frac{c}{d}$, $\Rightarrow \frac{a'}{b'} + \frac{c'}{d'} = \frac{a}{b} + \frac{c}{d}$ & $\frac{a'}{b'} \cdot \frac{c'}{d'} = \frac{a}{b} \cdot \frac{c}{d}$

i.e. Addition & Multiplication is invariant across the equivalence relation
 $(a', b') \sim (a, b)$

(pt) Addition \Rightarrow Quiz question!

Multiplication: $\left. \begin{array}{l} \frac{a'}{b'} = \frac{a}{b} \Rightarrow \underline{a'b} = \underline{ab'} \\ \frac{c'}{d'} = \frac{c}{d} \Rightarrow \underline{\underline{c'd}} = \underline{\underline{cd'}} \end{array} \right\} \Rightarrow \underline{(a'b)} \underline{(c'd)} = \underline{(ab')} \underline{(cd')}$

$$\Rightarrow \frac{a'c'}{b'd'} = \frac{ac}{bd}$$

$$\Rightarrow \frac{a'}{b'} \cdot \frac{c'}{d'} = \frac{a}{b} \cdot \frac{c}{d} \neq$$