Online Math Camp (24F)

Week 1 (8/12)

Set: A collection of objects, e.g. $S_1 = \{ \triangle, \text{▱}, X, \text{☺} \}$

$$S_2 = \{ 1, 2, 3, 4, 5 \} = \{ x : x \text{ is an integer, and } 1 \le x \le 5 \}$$

list all elements in the set

describe properties of elements in the set

Def: $x \in A$: x is in A.

$x \notin A$: x is NOT in A.

$A \subset B$: A is a subset of B.

$\left\{ \begin{array}{l} x, y, z, \ldots : \text{Elements} \\ A, B, \ldots : \text{Sets} \end{array} \right.$

Def: $A \cap B = \{ x : x \in A \text{ and } x \in B \}$: the intersection of A and B

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$: the union of A and B

$A^c = \{ x : x \notin A \}$: the complement of A

$A \setminus B = \{ x : x \notin A \text{ and } x \notin B \}$: the set difference of A and B

$A \times B = \{ (a, b) : a \in A,\ b \in B \}$: the product of A and B.

Def: A relation defined on a set S is a subset of $S \times S$

e.g. For $S = \mathbb{R}$, a "$<$" b is a relation.

Def: A equivalence relation R on S is a relation such that:

① $a R a$ (reflexive)

② $a R b \Rightarrow b R a$ (symmetry)

③ $a R b,\ b R c \Rightarrow a R c$ (transitive)

e.g. "$=$": ① $a = a$, ② $a = b \Rightarrow b = a$, ③ $a = b,\ b = c \Rightarrow a = c$.

e.g. "$\geq$" is not an equivalent relatin since $3 \geq 1 \not\Rightarrow 1 \geq 3$.

Rational Numbers

$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z} \right\}$ is the set of rational numbers, and can be expressed as a relation (m, n).

However, the representation is <u>NOT</u> unique: $\frac{2}{6} = \frac{1}{3}$ (When is it unique?)

In fact, $\frac{a}{b} = \frac{c}{d} \iff ad = bc$

Define $(a, b) \sim (c, d)$ if $ad = bc$. This is an equivalent relation.

(pf) Exercise.

(So it is unique up to the equivalence relation.)

Addition and Multiplication

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}, \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

Note the if $\frac{a'}{b'}=\frac{a}{b}$, $\frac{c'}{d'}=\frac{c}{d}$, $\Rightarrow \frac{a'}{b'}+\frac{c'}{d'}=\frac{a}{b}+\frac{c}{d}$ & $\frac{a'}{b'}\cdot\frac{c'}{d'}=\frac{a}{b}\cdot\frac{c}{d}$

i.e. Addition & Multiplication is invariant across the equivalence relation $(a',b')\sim(a,b)$

(pf) Addition $\Rightarrow$ Quiz question!

Multiplication: $\frac{a'}{b'}=\frac{a}{b} \Rightarrow \underline{a'b}=\underline{ab'}$
$\frac{c'}{d'}=\frac{c}{d} \Rightarrow \underline{\underline{c'd}}=\underline{\underline{cd'}}$

$$\Rightarrow (a'b)(c'd)=(ab')\cdot(cd')$$

$$\Rightarrow \frac{a'c'}{b'd'}=\frac{ac}{bd}$$

$$\Rightarrow \frac{a'}{b'}\cdot\frac{c'}{d'}=\frac{a}{b}\cdot\frac{c}{d} \quad \#$$