Introduction to Quantitative Methods, Quiz 8

1. Let $\{p_n\}$ be a sequence.

   (a) (15 points) What is the meaning of ' $\lim\limits_{n\to\infty} p_n = p$'?

   (b) (15 points) What is the meaning of '$\{p_n\}$ being a Cauchy sequence'?

2. Find the limit of each of the following sequences. You should provide an $N - \epsilon$ argument rather than just writing out the answers.

   (a) (10 points) $a_n = \frac{2n^2}{1+n^2}$

   (b) (10 points) $b_n = (\frac{1}{n}, \frac{n+2}{3n})$

   (c) (10 points) $c_n = 1 + \frac{1}{3} + \ldots + \frac{1}{3^{n-1}}$

3. (20 points) Let $x_1, x_2, \ldots, x_n$ be a sequence in $\mathbb{R}$ that converges to a real number d, prove that

$$\lim_{n\to\infty} (2x_n^2 + 3x_n + 4) = 2d^2 + 3d + 4.$$

4. (a) (10 points) Give a sequence $\{a_n\}$ such that there is a subsequence of $\{a_n\}$ that converges, does $\{a_n\}$ converge? Explain your answer.

   (b) (10 points) If we further assume $\{a_n\}$ is a Cauchy sequence, does $\{a_n\}$ converge? Explain your answer.