Introduction to Quantitative Methods, Quiz 1

1. Let $A = \{2, 3, 4\}$, $B = \{1, 4\}$

   a. (10 points) List all the subset of A (including the empty set).

   b. (20 points) What is $A \cap B, A \cup B, A \setminus B,\ A \times B$

   c. (10 points) Let $C = \{1, 2, \cdots n\}$ for a positive integer n. How many subsets are there in C.

2. The ***rational numbers*** $\mathbb{Q}$ are the set of numbers that can expressed by fractions. Formally,

$$\mathbb{Q} \coloneqq \{\frac{m}{n} \,|\, m, n \in \mathbb{Z}, n \neq 0\}$$

   , where $\mathbb{Z}$ is the set of integers.

   a. (20 points) Define a relation $=$ between on rational numbers, such that $\frac{a}{b} = \frac{c}{d}$ if $ad = cb$. Show that this relation is an equivalence relation. (You will need to state what conditions should be satisfied if a relation is an equivalence relation.)

   b. (30 points) Define the addition of rational numbers and check that your definition is well defined, that is, $\frac{a}{b} = \frac{a'}{b'}, \frac{c}{d} = \frac{c'}{d'}$, then $\frac{a}{b} + \frac{c}{d}$ is a rational number and $\frac{a}{b} + \frac{c}{d} = \frac{a'}{b'} + \frac{c'}{d'}$.

   c. (10 points) Define 0 to be the equivalence class $\mathbb{Q} \coloneqq \{\frac{0}{n} \,|\, n \in \mathbb{Z}, n \neq 0\}$. For any rational number $\frac{a}{b}$, show that there exist a unique rational number $\frac{c}{d}$, such that $\frac{a}{b} + \frac{c}{d} = 0$.