Estimation of Risk Aversion Parameters: Analyzing Ultimatum Game Data
估計風險偏好: 分析最後通牒談判實驗結果

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EE-BGT, Lecture 3b (Experimetrics Module 3b)

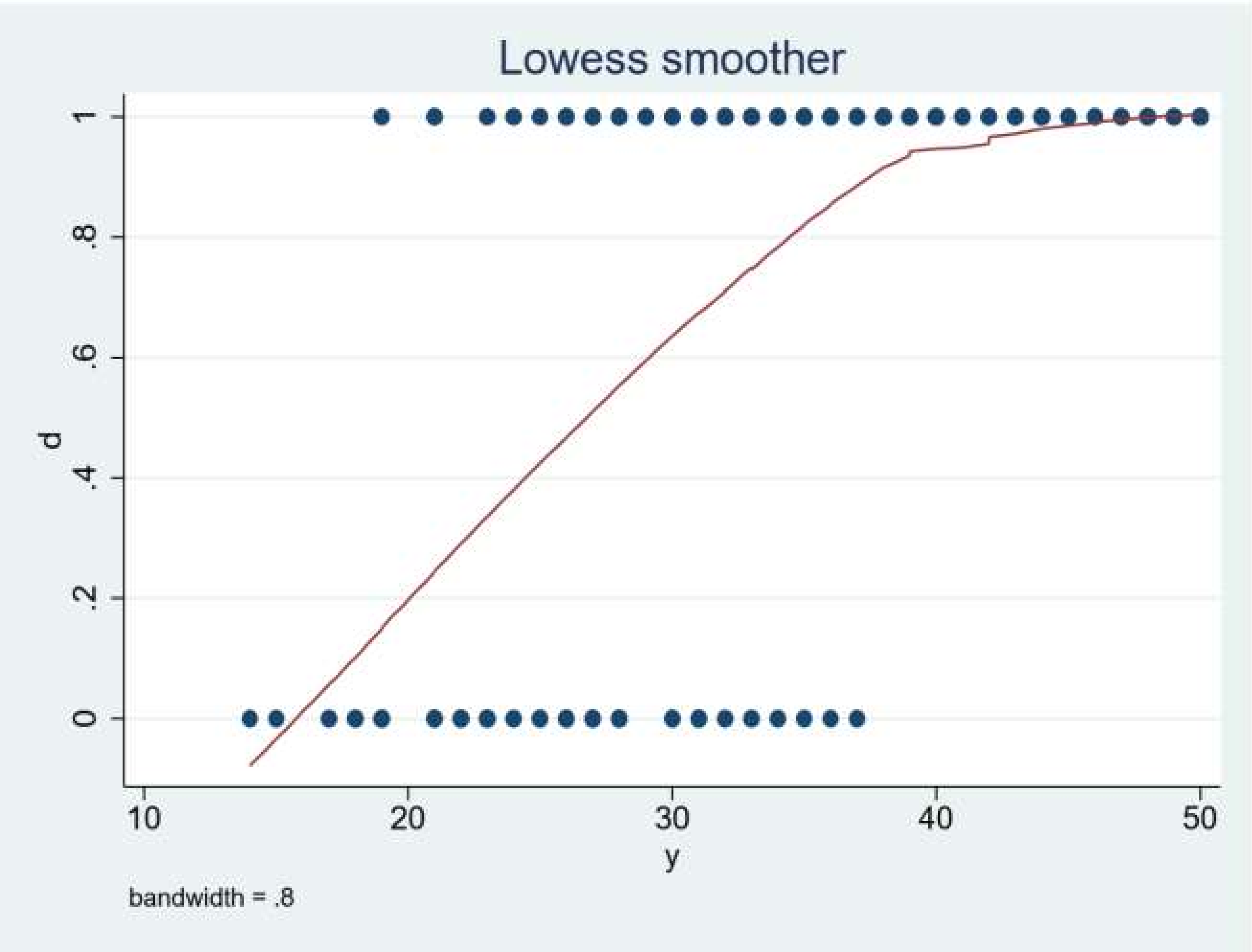
Ultimatum Game Data

- 200 subjects splitting a pie of $100
  - Play ultimatum game twice with different opponents
  - Be Proposer and Respondent each once
- Simulated Experiment Data: `ug_sim.dta`
  - Proposer i Offer: y
  - Respondent j Reaction: d (=1 if Accept; =0 if Reject)
  - `male_i`: Gender dummy for Proposer i to be male
  - `male_j`: Gender dummy for Respondent j to be male

Respondents Accept or Reje

- STATA: `tab d`

d	Freq.	Percent	Cum.
0	51	25.50	25.50
1	149	74.50	100.00
Total	200	100.00	



- Plot acceptance function:
  - STATA: `lowess d y`
  - Locally Weighted Scatter-plot Smoother (non-parametric)
  - Mean value of d conditional on different values of y
    - Want to jitter? Try: `lowess d y jitter(5) msize(3)`

Probit Model for Choosing Accept

- Model this as Probit: $\Pr(d = 1|y) = \Phi(\beta_0 + \beta_1 y)$

  where $\Phi(z) = \Pr(Z < z) = \int_{-\infty}^{z} \phi(z)dz$ is standard Normal cdf

  - This is because:
- Propensity to accept: $d^* = \beta_0 + \beta_1 y + \epsilon, \ \epsilon \sim N(0,1)$
  - Accept if great than 0: $d = 1 \Leftrightarrow d^* = \beta_0 + \beta_1 y + \epsilon > 0$

    $\Leftrightarrow \epsilon > -\beta_0 - \beta_1 y$
- So, $\Pr(d = 1) = \Pr(\epsilon > -\beta_0 - \beta_1 y) = \Phi(\beta_0 + \beta_1 y)$

Probit Model for Choosing Accept

- STATA Results: `probit d y`

```
Iteration 0:   log likelihood = -113.55237
Iteration 1:   log likelihood = -70.230335
Iteration 2:   log likelihood = -66.806698
Iteration 3:   log likelihood = -66.738058
Iteration 4:   log likelihood = -66.738049
Iteration 5:   log likelihood = -66.738049

Probit regression                                 Number of obs   =        200
                                                  LR chi2(1)      =      93.63
                                                  Prob > chi2     =     0.0000
                                                  Pseudo R2       =     0.4123

------------------------------------------------------------------------------
           d |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .1439157   .0212804     6.76   0.000      .1022069    .1856244
       _cons |  -3.855266    .631443    -6.11   0.000     -5.092872   -2.617661
------------------------------------------------------------------------------
```

Predicted Probability

$$\Pr(d=1|y)=\Phi(-3.855+0.144y)$$

Predict Minimum Acceptable Offer (MAO)

- With Propensity to Accept $d^* = \beta_0 + \beta_1 y + \epsilon$
- Can Calculate Minimum Acceptable Offer (MAO):
  - Indifferent Between Accept/Reject if $\hat{d}^* = \hat{\beta}_0 + \hat{\beta}_1 y = 0$
  - So, $y^{MAO} = -\frac{\hat{\beta}_1}{\hat{\beta}_0} = -\frac{3.855}{0.144} = \underline{\underline{26.79}}$
- STATA: `nlcom MAO: -_b[_cons]/_b[y]`
  - Get s.e./CI via Delta Method

Accept if > 26.8!

```
         MAO:  -_b[_cons]/_b[y]

------------------------------------------------------------------------------
           d |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         MAO |   26.78837   .9268278    28.90   0.000     24.97182    28.60492
------------------------------------------------------------------------------
```

Strategy Method vs. Direct Response

- In addition to ask Proposer to make offer y
- Solnick (2001) and others ask responders to state
  - MAO (Minimum Acceptable Offer) y^{MAO}
- Then play out the decision:
  - Accept if $y \geq y^{MAO}$
  - Reject if $y < y^{MAO}$
- Is stating your true MAO incentive compatible? Yes!
  - MAO $=$ entire strategy (more informative than "Accept 50")

Ultimatum Game Data: Strategy Method

- 200 subjects splitting a pie of 100
  - Play ultimatum game twice with different opponents
  - Be Proposer and Respondent each once
- Simulated Strategy Method Data: `ug_sm_sim.dta`
  - Proposer i Offer: y
  - Respondent j MAO: y^{MAO}
  - `male_i`: Gender dummy for Proposer i to be male
  - `male_j`: Gender dummy for Respondent j to be male
  - Outcome: d (=1 if $y \geq y^{MAO}$; $= 0$ f $y < y^{MAO}$)

Confidence Interval of MAO

- 95% Confidence Interval in STATA: `ci means MAO` (`ci MAO`)
  - STATA Results:

Variable	Obs	Mean	Std. Err.	[95% Conf.	Interval]
MAO	200	31.375	.6666664	30.06036	32.68964

- Narrower than direct response: (24.97, 28.60)
  - But 5 units higher (31.4 vs. 26.8), to:
    - Signal Toughness (Eckel and Grossman, 2001)
    - Hypothetical (Cold)
  - So more rejections: `tab d`

d	Freq.	Percent	Cum.
0	87	43.50	43.50
1	113	56.50	100.00
Total	200	100.00	

Test of Gender Effects

- Gender as a treatment in Ultimatum game: `ug_sim.dta`
- Conduct regression analysis

$$y_i = \beta_0 + \beta_1 \texttt{male\_i} + \beta_2 \texttt{male\_j} + \beta_3 \texttt{m\_to\_f} + \epsilon_i$$

  - STATA: `regress y male_i male_j m_to_f`
    - With dummies:
  - `male_i`: Gender dummy for Proposer i to be male
  - `male_j`: Gender dummy for Respondent j to be male
  - `m_to_f`: Dummy for male proposing to female responder

```
gen m_to_f=male_i*(1-male_j)
```

Test of Gender Effects

- STATA Results: `regress y male_i male_j m_to_f`

```
      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  3,   196) =    3.37
       Model |  976.185392     3  325.395131           Prob > F      =  0.0195
    Residual |  18901.4946   196   96.436197           R-squared     =  0.0491
-------------+------------------------------           Adj R-squared =  0.0346
       Total |    19877.68   199   99.8878392          Root MSE      =  9.8202
------------------------------------------------------------------------------
           r |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      male_i |  -4.519608   1.885099    -2.40   0.017     -8.23729   -.8019261
      male_j |   3.744608   2.074081     1.81   0.073    -.3457722    7.834988
      m_to_f |   2.381863    2.80275     0.85   0.396    -3.145557    7.909282
       _cons |     35.275   1.552709    22.72   0.000     32.21284    38.33716
------------------------------------------------------------------------------
```

Male offer \$4.52 less ($z =$ -2.40)

Offer male \$3.74 more ($z =$ -1.81)

Male offer female \$2.38 more ($z = 0.85$)

- Chivalry Effect: Eckel and Grossman (2001)

Do Male Responders Reject Offers More Often?

- STATA Results: `probit d y male_j`

Male reject same offer more than female (z = -2.24)

```
Iteration 0: log likelihood = -113.55237
Iteration 1: log likelihood = -68.373743
Iteration 2: log likelihood = -64.187937
Iteration 3: log likelihood = -64.116934
Iteration 4: log likelihood = -64.116904
Iteration 5: log likelihood = -64.116904
Probit regression                          Number of obs   =        200
                                           LR chi2(2)      =      98.87
                                           Prob > chi2     =     0.0000
Log likelihood = -64.116904                Pseudo R2       =     0.4354
------------------------------------------------------------------------------
           r |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .1567836   .0231961     6.76   0.000       .11132    .2022472
      male_j |  -.5976406   .2668131    -2.24   0.025    -1.120585   -.0746966
       _cons |  -3.933341   .6589175    -5.97   0.000    -5.224796   -2.641886
------------------------------------------------------------------------------
```

Offer how much more for same acceptance rate? 0.598/0.157!!

Offer How Much More For Same Acceptance Rate?

- STATA `nlcom more_to_male: -_b[male_j]/_b[y]`

  Results:

```
more_to_male: -_b[male_j]/_b[y]
------------------------------------------------------------------------------
           r |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
more_to_male |   3.811882   1.612015     2.36   0.018     .6523915    6.971373
------------------------------------------------------------------------------
```

  - 95% Confidence Interval is very wide
- $0.598/0.157 = \$3.81$ is very close to $\hat{\beta}_2 = \$3.74$
  - Estimated coefficient of `male_j`
- Proposers rationally react to tough male responders
  - By proposing $\$3.74$ more (compared to female responders)

Recall: Probit Model for Choosing Accept

- STATA Results: `probit d y`

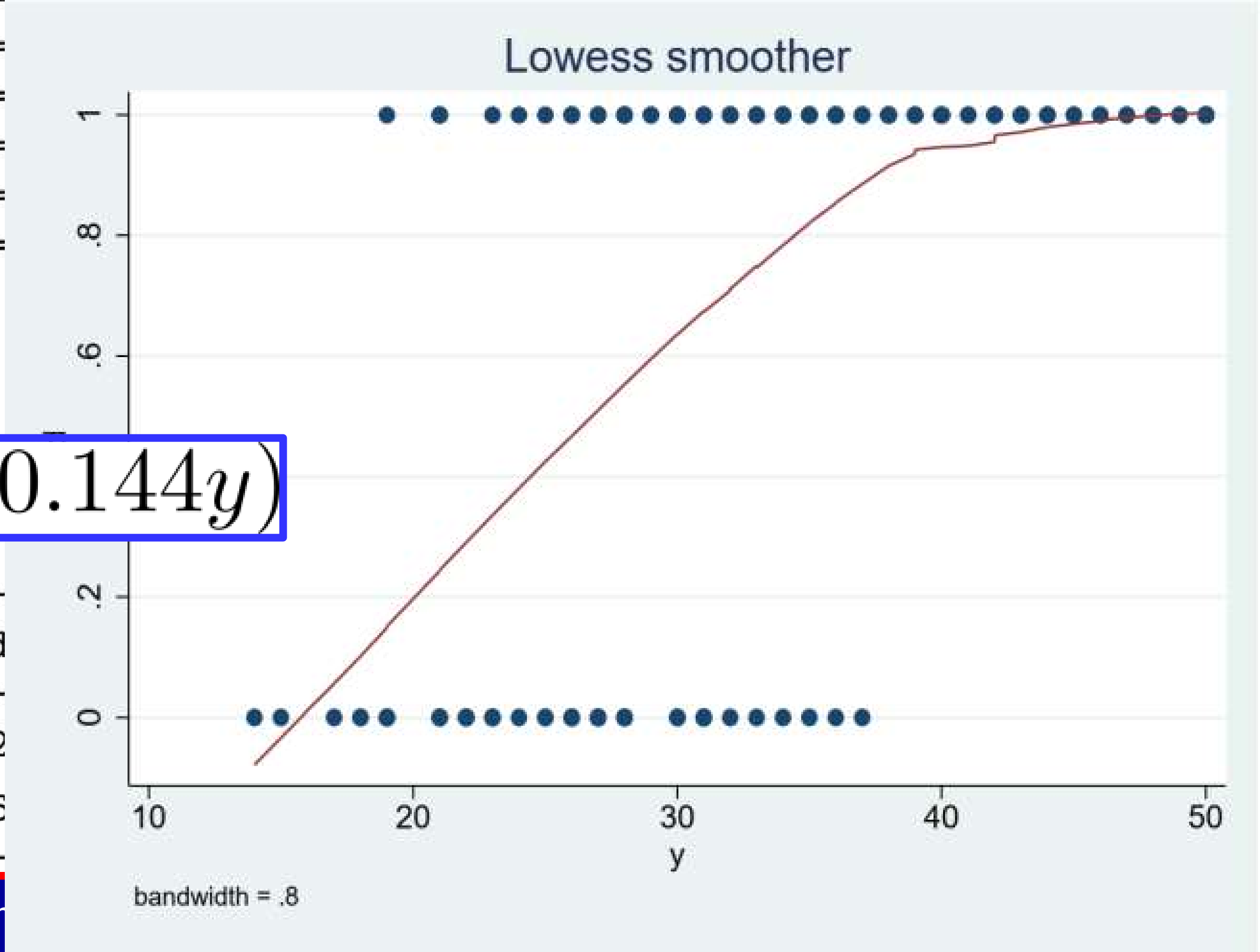
```
Iteration 0:   log likelihood = -113.55237
Iteration 1:   log likelihood =
Iteration 2:   log likelihood =
Iteration 3:   log likelihood =
Iteration 4:   log likelihood =
Iteration 5:   log likelihood =

Probit regression

------------------------------------
           d |      Coef.    Std
-------------+----------------------
           y |   .1439157    .02
       _cons |  -3.855266     .6
------------------------------------
```

Predicted Probability

$$\Pr(d=1|y)=\Phi(-3.855+0.144y)$$



Proposer Decision as Risky Choice

- Rational Expectations: Know Acceptance probability is

$$\Pr(d=1|y) = \Phi(-3.855 + 0.144y)$$

  - What should proposers do? Roth et al. (AER 1991) propose:
- Offer 50: Get $\$50$ for sure (50-50 are 100% accepted) or
- Offer $y = \$40$: Get $\$(100\text{-}y) = \$60$ with uncertainty
  - Normalize pie size $100 to 1: `proposer decision.xlsx`
  - $r = 0.4$ yield: $EU(y) = \Phi(-3.855 + 0.144y) \times \frac{\left(\frac{100-y}{100}\right)^{1-0.4}}{1-0.4}$

Proposer Decision as Risky Choice

- Plot EU = EU(y):

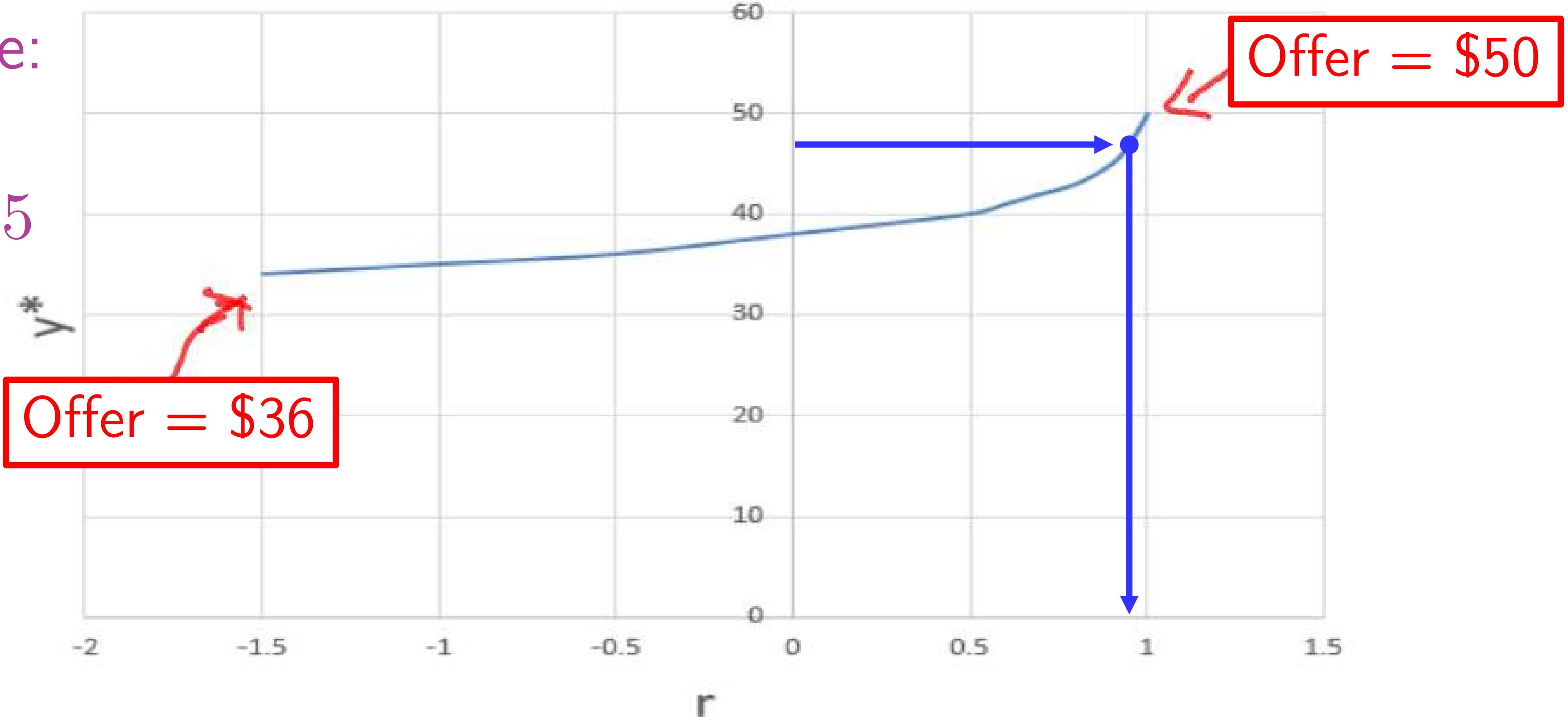
$$EU(y) = \Phi(-3.855 + 0.144y) \times \frac{\left(\frac{100-y}{100}\right)^{1-0.4}}{1-0.4}$$

Optimal offer $y* = \$40$ for $r = 0.4$

- Try other r
  - Find $y^*(r)$

Proposer Decision as Risky Choice

- Plot $y^*(r)$:
  - Example: $y = 47$
  - $r = 0.95$



Toward a Mixture Model of Social Preference

- 18% (36 out of 200) of the proposer offer $y = 50$
  - Are all of them extremely risk averse?
- No! People can offer 50-50 because they think it is fair
  - Need a mixture model to analyze this (see next lecture):
  - 18% of the population motivated by fairness offer 50-50
  - 82% of the population motivated by self-interest
    - make risky choices according to their risk preferences
- Who are these equal-splitters/egalitarians?

Who Are These Equal-Splitters/Egalitarian?

- STATA Command: `gen egal=y==50` `tab egal`
- STATA Results:

egal	Freq.	Percent	Cum.
0	164	82.00	82.00
1	36	18.00	100.00
Total	200	100.00	

- Is Egalitarianism is Related to Gender? Use χ^2 test!
  - STATA Command: `tab male_i egal , chi2`
  - STATA Results:

male_i	egal 0	1	Total
0	66	25	91
1	98	11	109
Total	164	36	200

Pearson chi2(1) = 10.1506 Pr = 0.001

25 of 91 female are egalitarian

11 of 109 male are egalitarian

Female more likely to be egalitarian ($p = 0.001$)

Acknowledgment

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