Coordination
協調賽局

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Lecture 10, EE-BGT

Why is Coordination Important?

- Which Equilibrium to Select Among Many?
  - This requires Coordination!
- Examples of Coordination in Daily Life:
  - Language
  - Trading in Markets (Liquidity)
  - Industry Concentration

Why is Coordination Important?

- Equilibrium Selection in Game Theory

1. Desirable Features Approach:
   - Payoff-Dominance, Risk Dominance, etc.
2. Convergence via Adaptation/Learning
   - Weibull (1995), Fudenberg and Levine (1998)
3. Empirical Approach: Infer Principles by
   - Putting people in experiments and observe actual behavior/outcome

Why is Coordination Important?

- Possible "Selection Principles":
  - Precedent, focal, culture understanding, etc.
- Why are observations useful?
- Schelling (1960, p.164):
  - "One cannot, without empirical evidence, deduce what understandings can be perceived in a nonzero-sum game of maneuver any more than one can prove,
  - by purely formal deduction, that a particular joke is bound to be funny."

Why is Coordination Important?

- Can't Communication Solve This?
  - Not always... (See Battle of Sexes below)
- Sometimes communication is not feasible:
  - Avoiding Traffic Jams
  - Speed Limits (useful because they reduce speed "variance," and hence, enhance coordination!)
- Miscommunication can have big inefficiency!

Examples of Coordination Impact

- US railroad tracks is 4 feet and 8.5 inch
  - Because English wagons were about 5 feet (width of two horses), and lead to
- Space Shuttle Rockets smaller than ideal
  - since they need to be shipped back by train...
- Industries are concentrated in small areas
  - Silicon Valley, Hollywood, Hsinchu Science Park
- Urban Gentrification
  - I want to live where others (like me) live

Examples of Coordination Impact:
Drive on Left/Right side of the Road

- Right: Asia, Europe (Same continent!)
- Left: Japan, UK, Hong Kong (Islands!)
- Sweden switched to Right (on Sunday morning)
- What about America? Right, to avoid
  - Hitting others with the whip on your right hand!
- Bolivians switch to Left in mountainous area
  - To see outer cliffside from (left) driver seat
- Pittsburgh left: 1^{st} left-turner goes 1^{st} at green
  - on two-lane streets to avoid blocking traffic

3 Types of Coordination Games

- Matching Games
  - Pure Coordination Game; Assignment Game
- Games with Asymmetric Payoffs
  - Battle of Sexes, Market Entry Game
- Games with Asymmetric Equilibria
  - Stag Hunt, Weak-Link Game
- Applications: Market Adoption and Culture

Examples of Coordination Impact

- Categorizing Products
  - Where should you find MCU? Disney or Action?
  - Find your favorite item at a new Costco store
- Common Language:
  - Internet promotes English
  - Some Koreans even get surgery to loosen their tongues, hoping to improve their pronunciation
- Key: Agreeing on something is better than not; but some coordinated choices are better

Matching Game: GAMES magazine (1989)

- Pick one celebrity (out of 9) for President, another for Vice-President:
  - Oprah Winfrey, Pete Rose,
  - Bruce Springsteen, Lee Iaccoca,
  - Ann Landers, Bill Cosby,
  - Sly Stallone, Pee-Wee Herman,
  - Shirley MacLaine
- One person is randomly awarded prize among those who picked most popular one

Matching Game: Taiwanese Version in Spring 2025

- For 2028 Taiwan Presidential Election:
  - 謝淑薇、陳傑憲、林郁婷、蔣萬安、黃國昌、侯友宜、八炯、鍾明軒、黃子佼、蕭美琴
- Prize?
- Results...

Matching Game: GAMES magazine (1989)

- US Results:

1. Bill Cosby (1489): successful TV show
2. Lee Iacocca (1155): possible US candidate
3. Pee-Wee Herman (656): successful TV show
4. Oprah Winfrey (437): successful TV show

   ...

9. Shirley MacLaine (196): self-proclaimed reincarnate

Pure Coordination Game

- Both get 1 if pick the same;
- Both get 0 if not
- Two pure NE,
  - (A, A) and (B, B)
- One mixed NE
  - (½A + ½B, ½A + ½B)
- Which one will be played empirically?

	A	B
A	1, 1	0, 0
B	0, 0	1, 1

Pure Coordination Game

- Mehta, Starmer and Sugden (AER 1994)
- Picking Condition (P): Just pick a strategy
- Coordinating Condition (C):
  - Win $1 if your partner picks the same as you
- Difference between P and C = How Focal
- Choices: Years, Flowers, Dates, Numbers, Colors, Boy's name, Gender, etc.

Category	Pick a... (n=88)		Coordinate (n=90)	
	Response	%	Response	%
Years	1971	8.0	1990	61.1
Flowers	Rose	35.2	Rose	66.7
Dates	Dec. 25	5.7	Dec. 25	44.4
Numbers	7	11.4	1	40.0
Colors	Blue	38.6	Red	58.9
Boy's Name	John	9.1	John	50.0
Gender	Him	53.4	Him	84.4

Pure Coordination Game: Follow-up 1

- Bardsley, Mehta, Starmer, Sugden (EJ 2010)
  - Incorporate (Replace?) Bardsley, et al. (wp 2001)
- 14 Games: One in choice set is distinctive
  - EX: {Bern, Barbodos, Honolulu, Florida}
- Add Guess Condition (G) to P/C: Guess partner's pick
- Design question: How do you avoid focality of physical location (first/last/top-left)?
  - Have things swim around the computer screen...

Pure Coordination Game: Follow-up 1

- EX: {Bern, Barbodos, Honolulu, Florida}

1. Could choose Bern in C since Bern in P and G
   - Derivative Salience: P=G=C (via Cognitive Hierarchy Model!)
2. Could choose Bern in C, but Florida in P and G
   - Schelling Salience: P=G≠C
   - Team Reasoning: Pick distinctive choice only in C

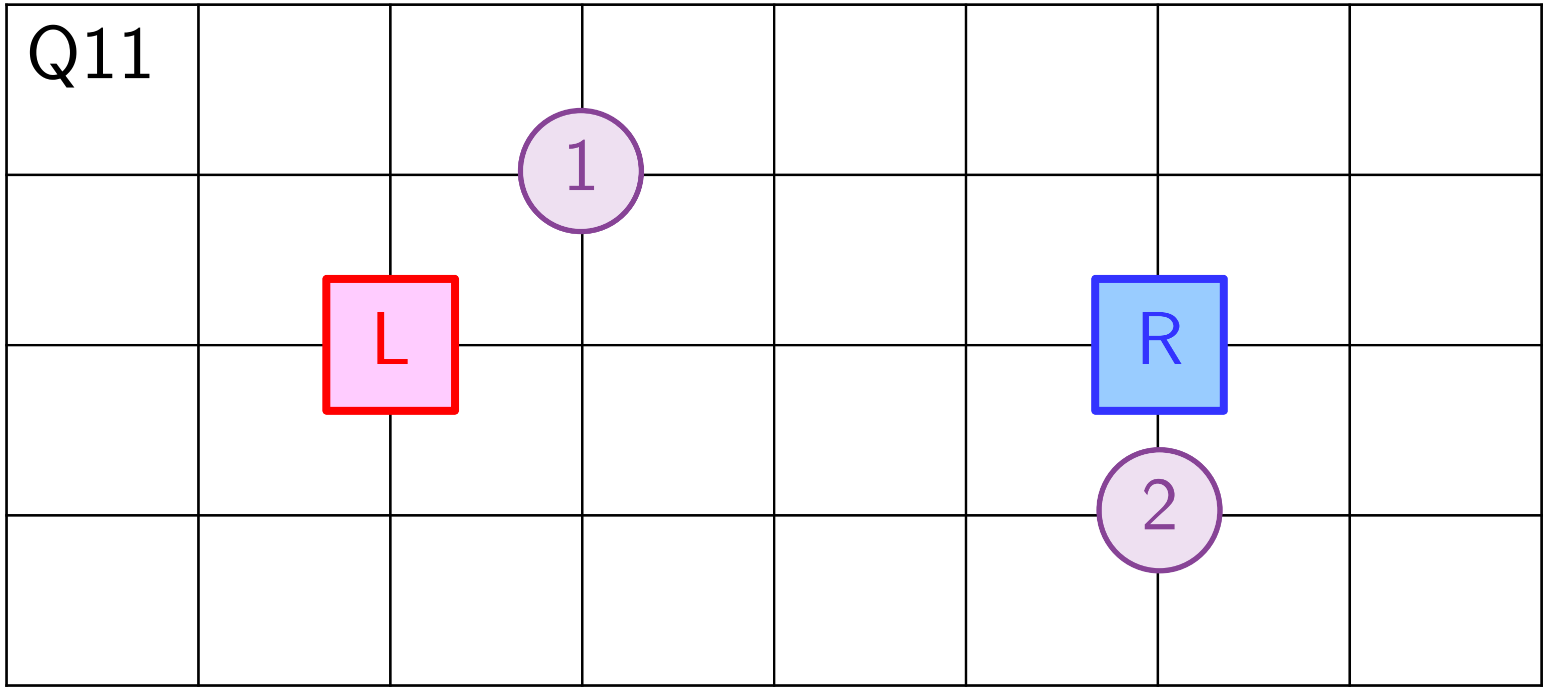
- Coordinate on this: Even though I would not pick this and I know you would not pick this!

Pure Coordination Game: Follow-up 1

  - Derivative Salience: P=G=C vs. Schelling Salience: P=G≠C
- Schelling Salience wins here!
  - In 12 games (out of 14):
- Chose distinctive choice 60% in C (modal!)
  - But less often in P and G
- EJ 2010: Follow-up with Nottingham subjects
  - Both saliences rejected with subtle design differences (used to coordinate)

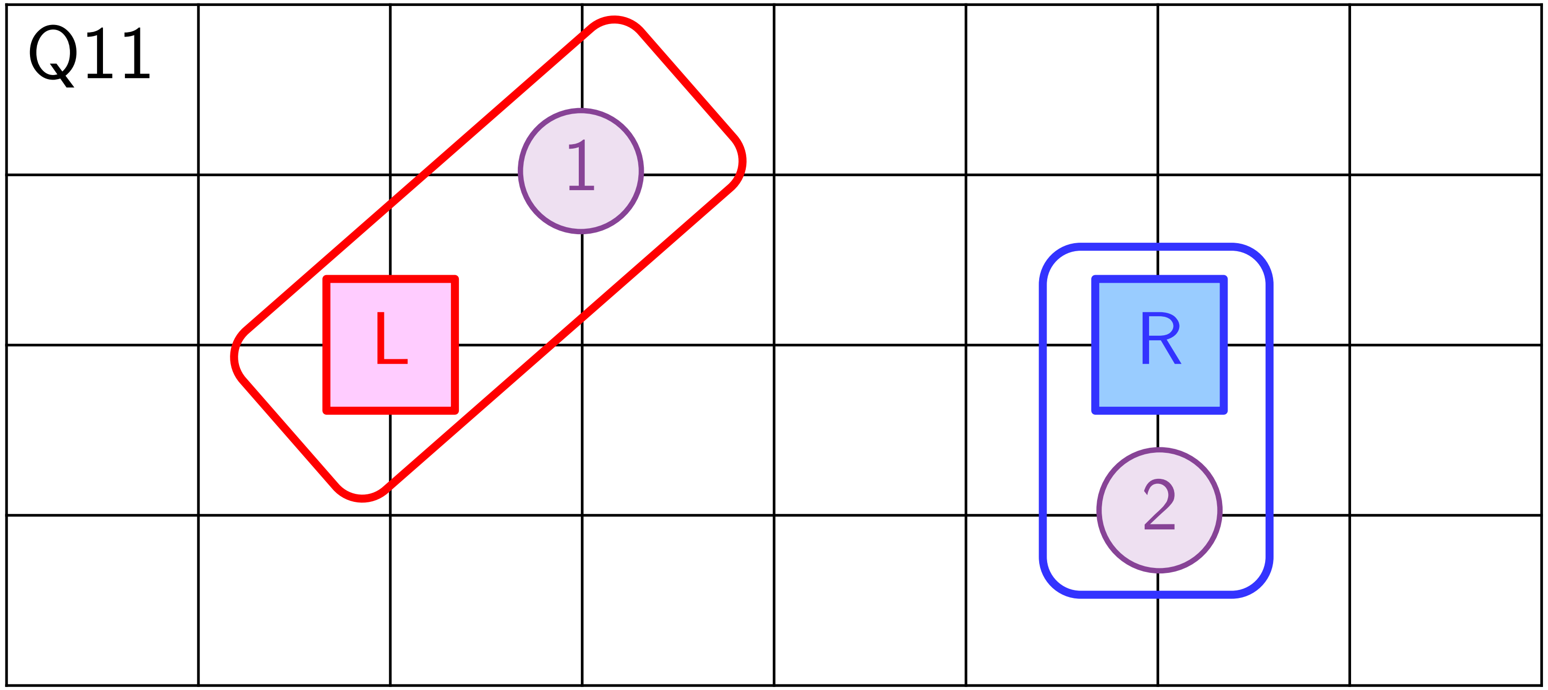
Assignment Game and Visual Selection (Follow-up 2)

- Hume (1978/1740) - Ownership conventions: spatial/temporal proximity, cultural, etc.
  - Mehta, Starmer and Sugden (T&D 1994)



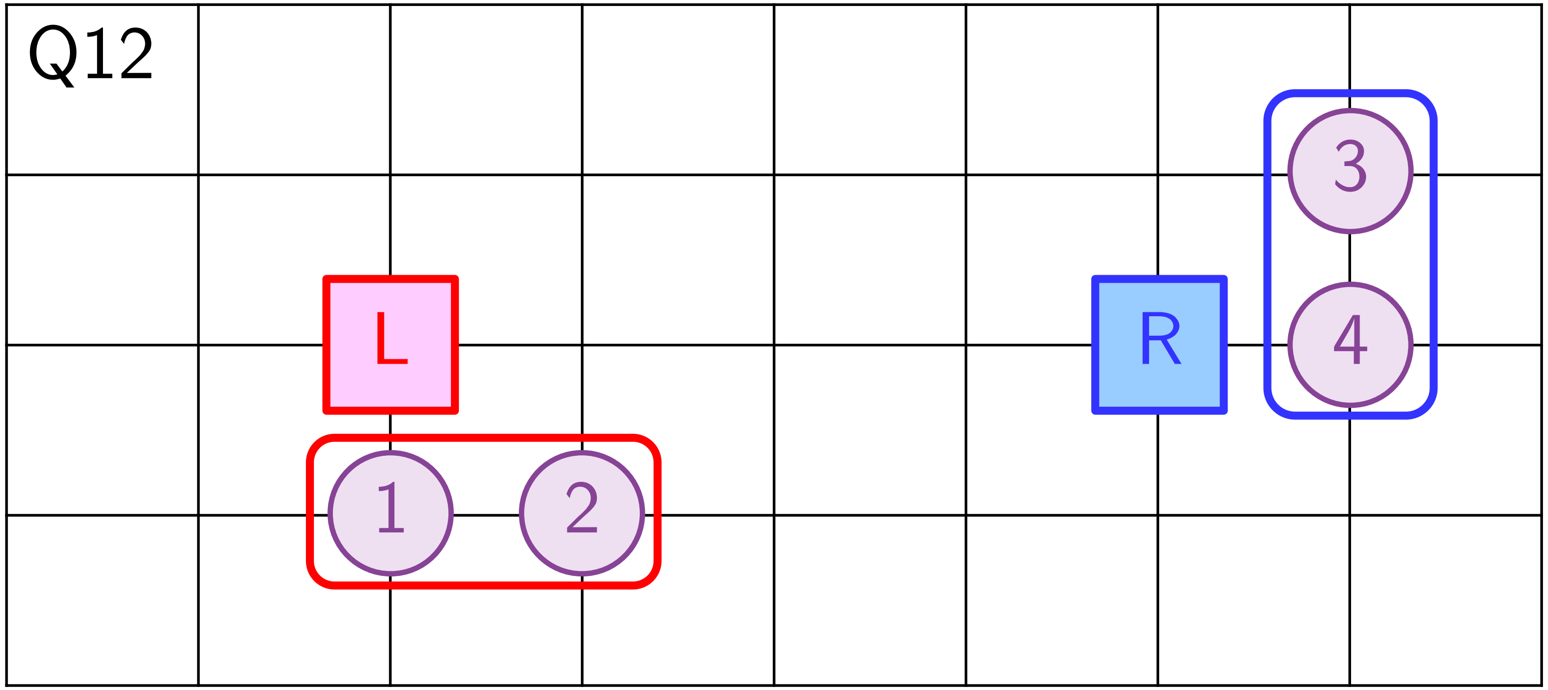
Assignment Game and Visual Selection

- Assign circles to L or R;
- Earn $$ if all circles match partner assignment
- Focal Principle 1: Closeness (C)



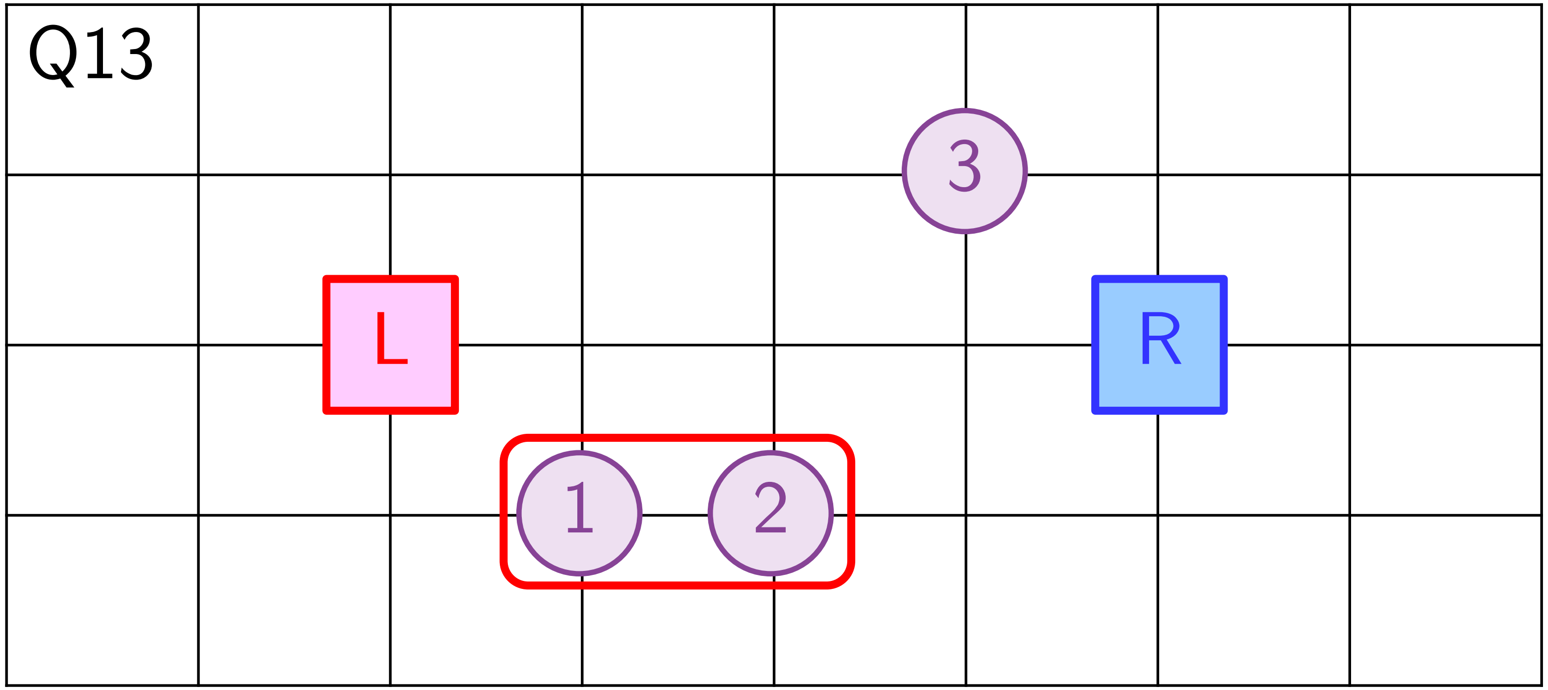
Assignment Game and Visual Selection

- Assign circles to L or R
- Earn $$ if all circles match partner assignment
- Focal Principle 2: Equality (E)



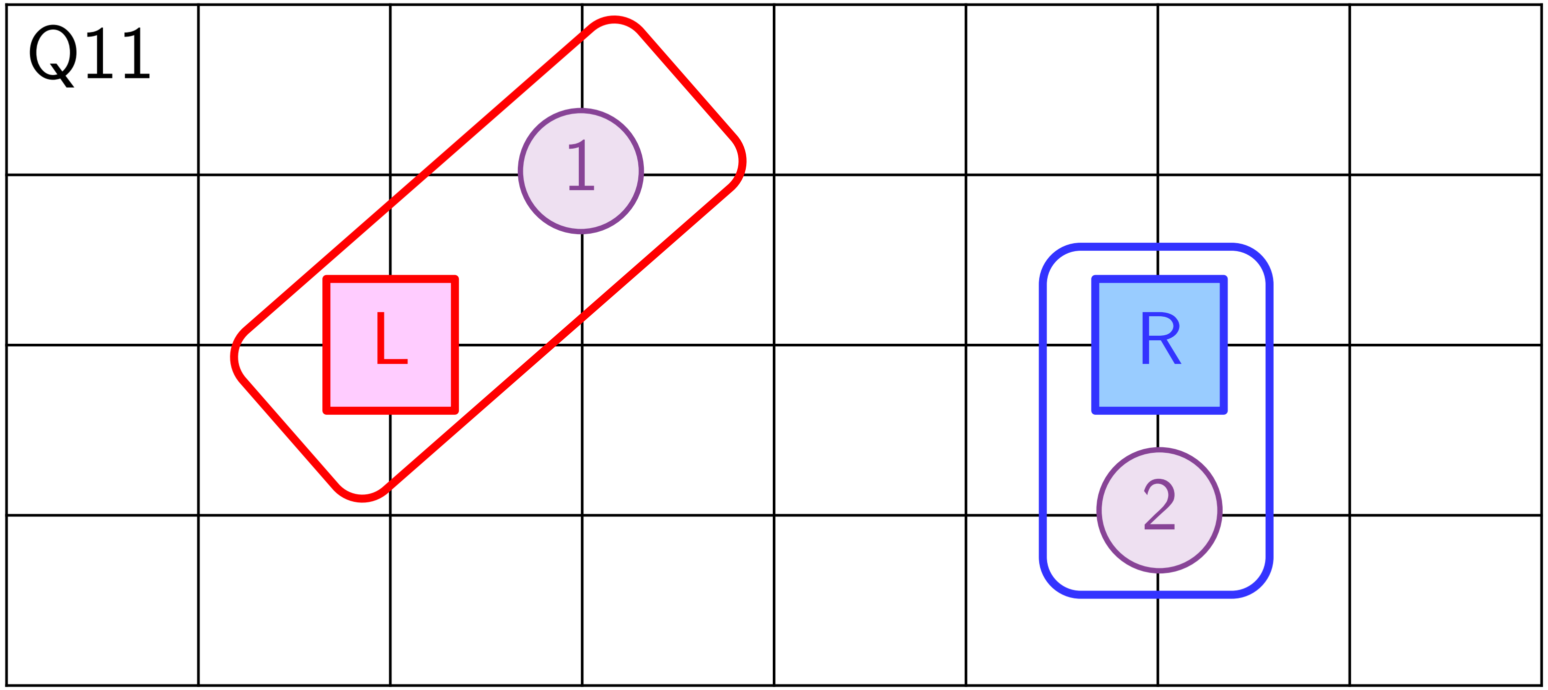
Assignment Game and Visual Selection

- Assign circles to L or R
- Earn $$ if all circles match partner assignment
- Focal Principle 3: Accession (A)



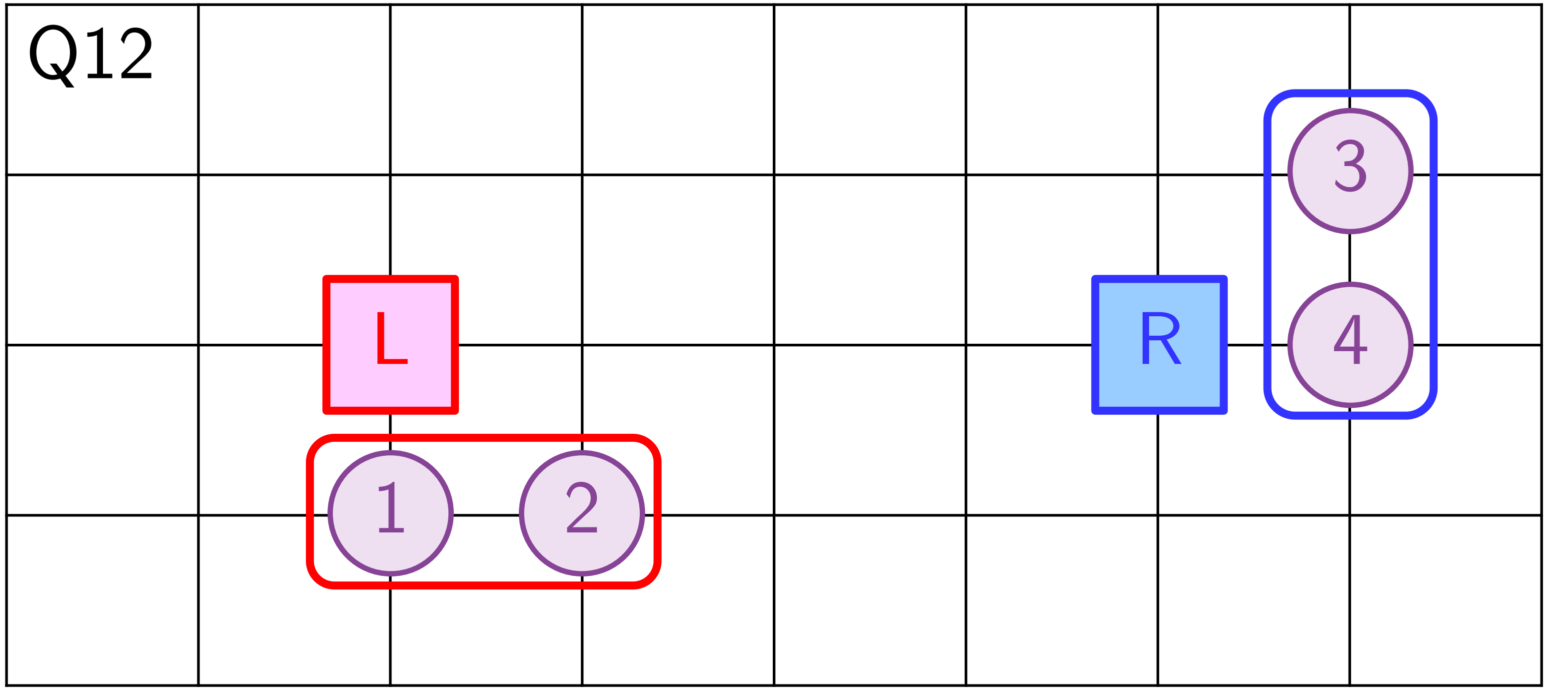
Assignment Game and Visual Selection

- How would you assign the circles?
- What about this? (C = A = E)
  - In fact, 74% chose this!



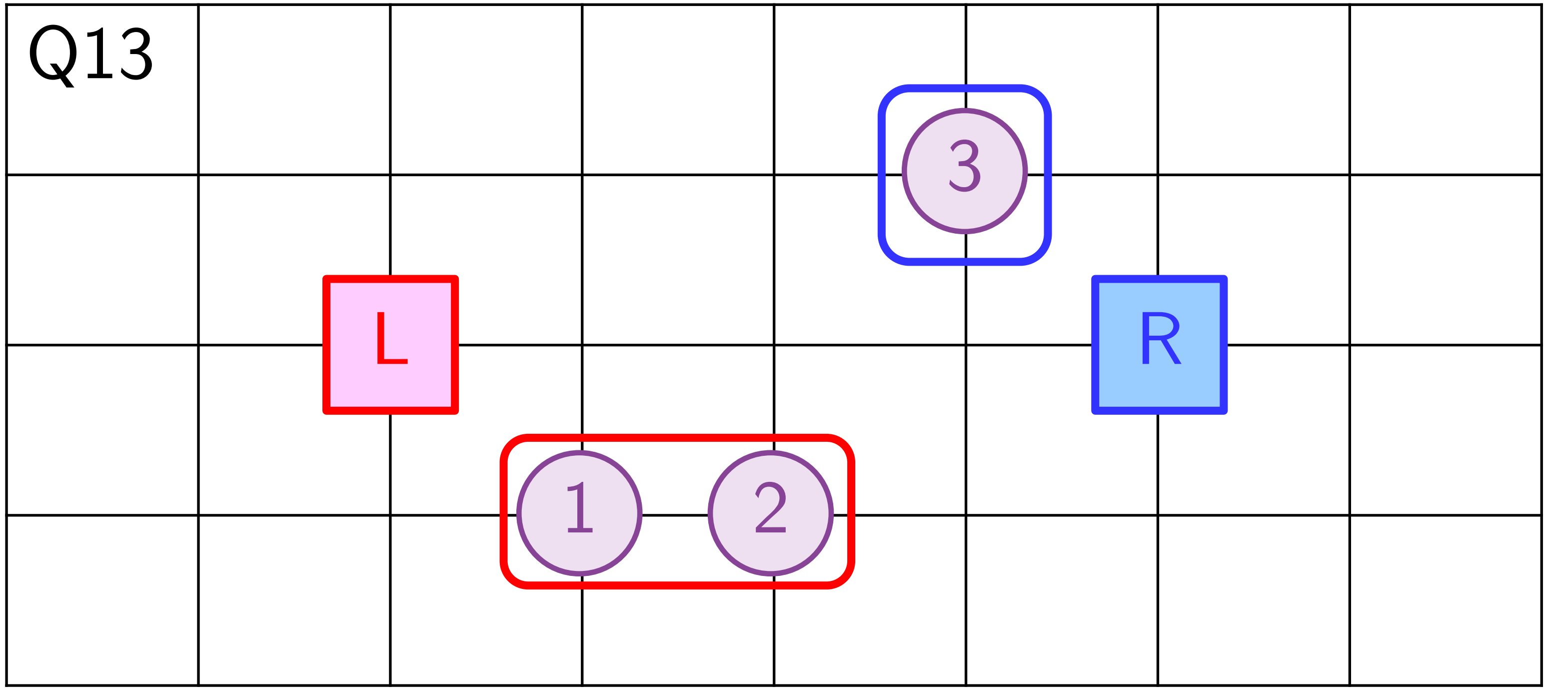
Assignment Game and Visual Selection

- How would you assign the circles?
- What about this? (C = A = E)
  - In fact, 68% chose this!



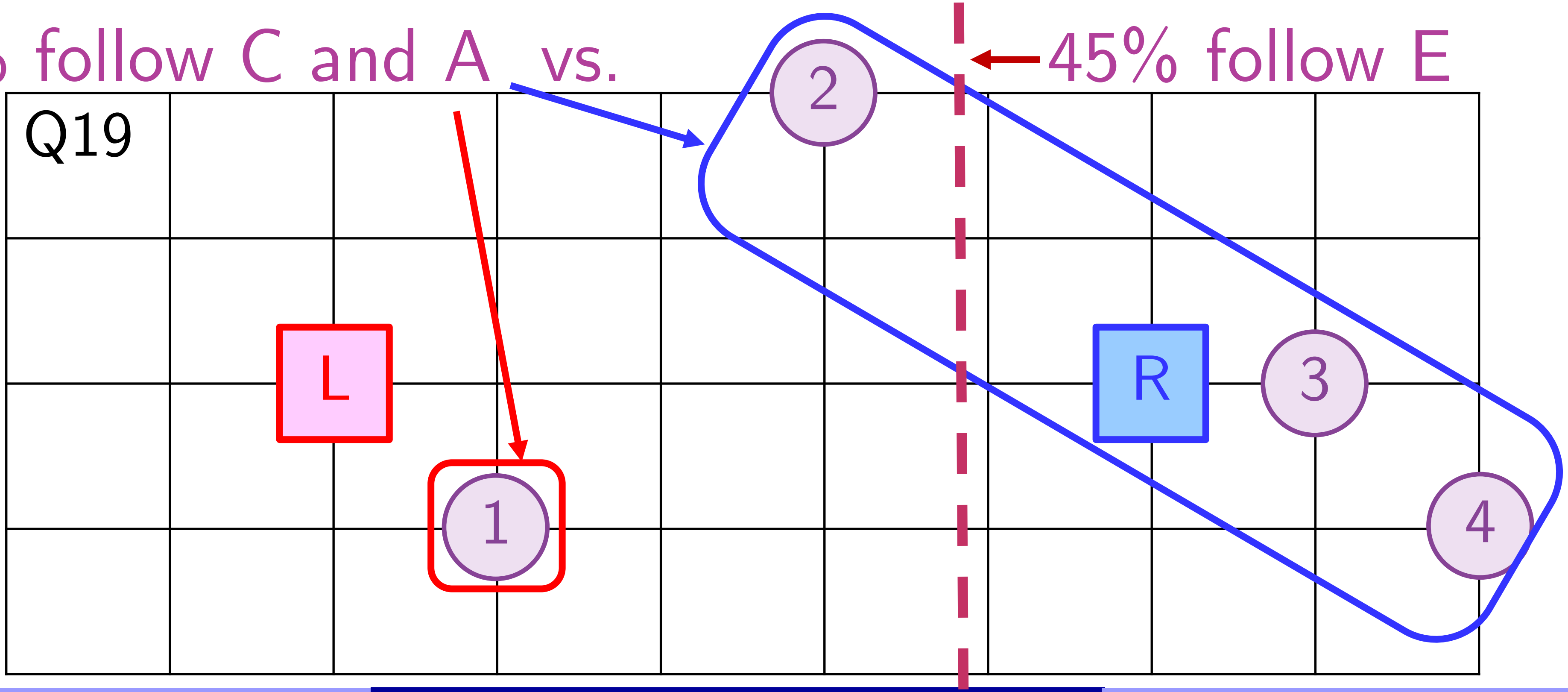
Assignment Game and Visual Selection

  - How would you assign the circles?
- What about this? (Accession!)
  - In fact, 70% chose this! (What does C/E say?)



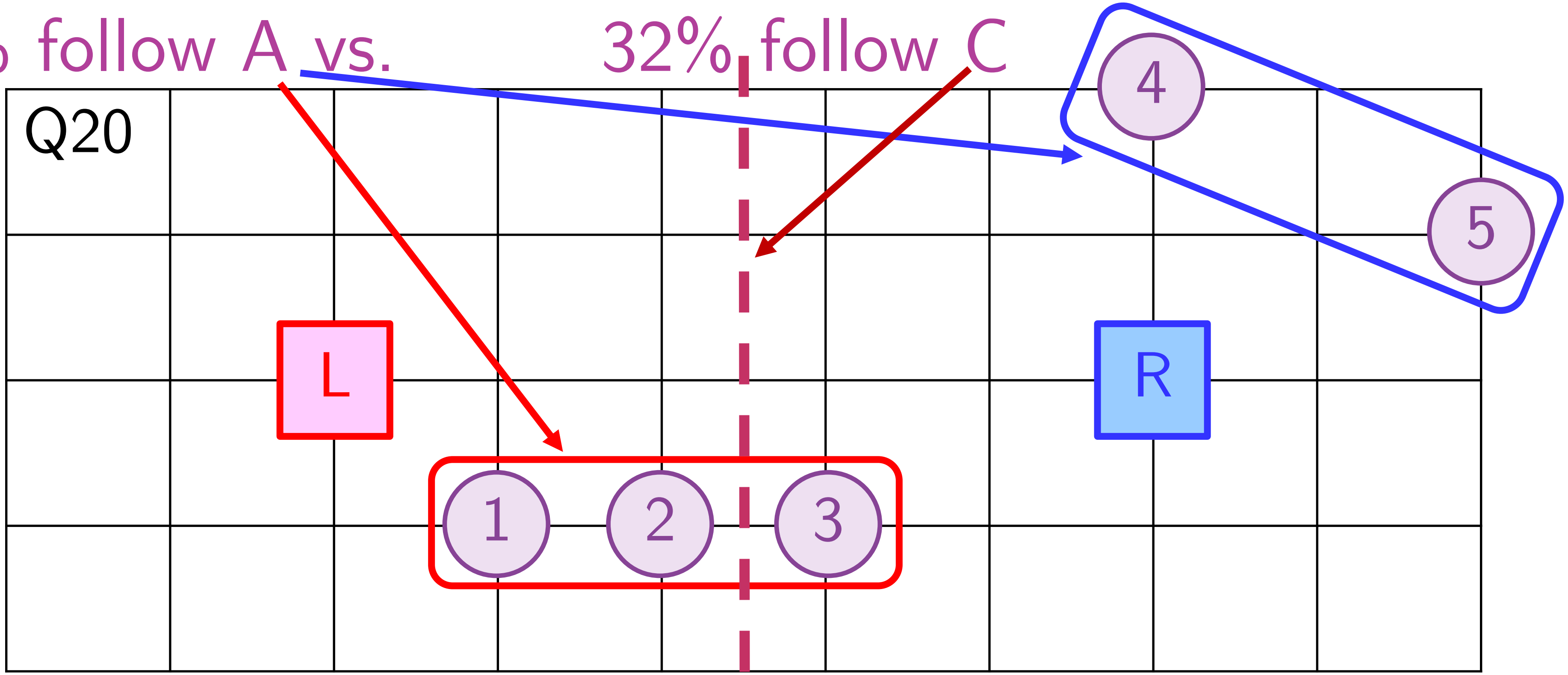
Assignment Game: Closeness and Accession vs. Equality

- What does Closeness/Accession say?
- What does Equality say about this?
  - 29% follow C and A vs. 45% follow E



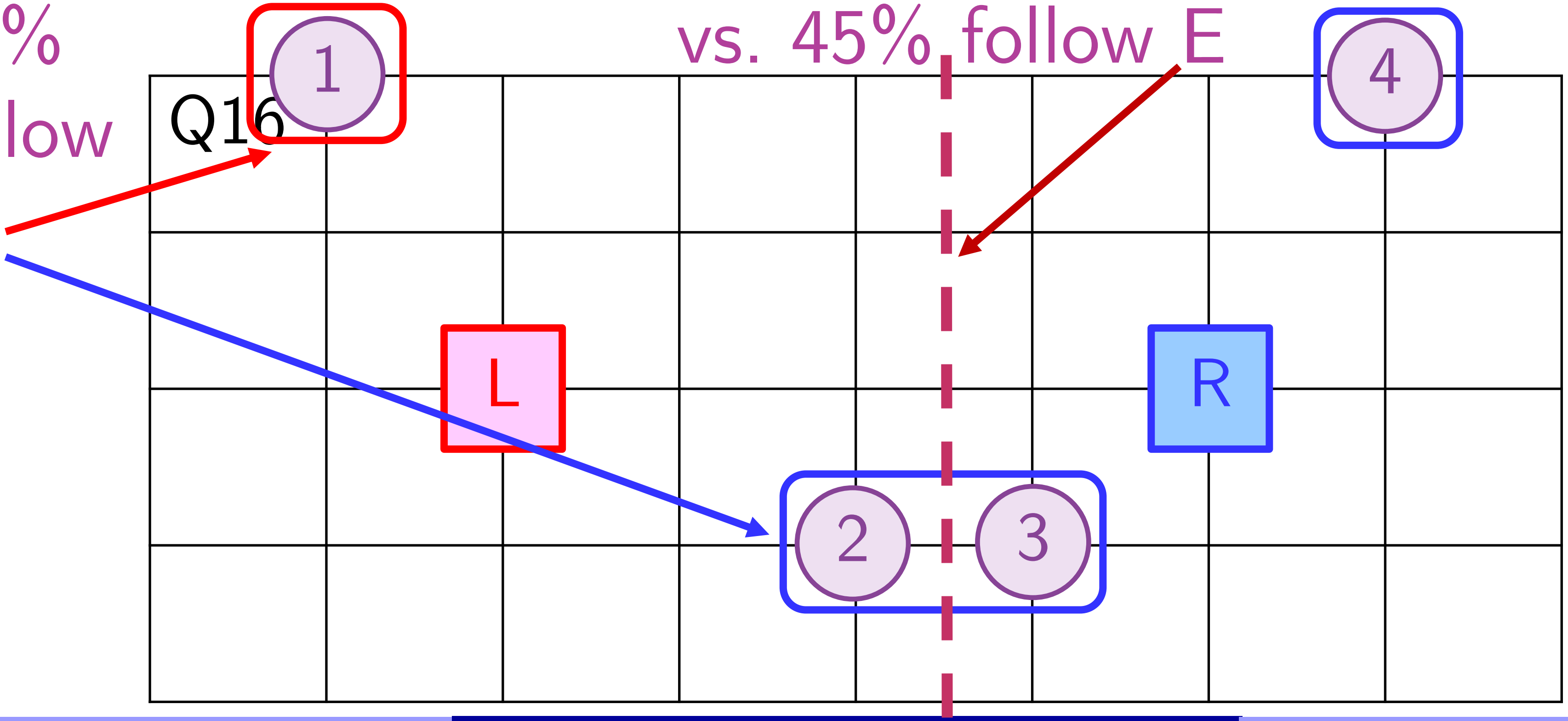
Assignment Game: Accession vs. Closeness

- What does Accession say about this?
- What does Closeness say about this?
  - 43% follow A vs. 32% follow C



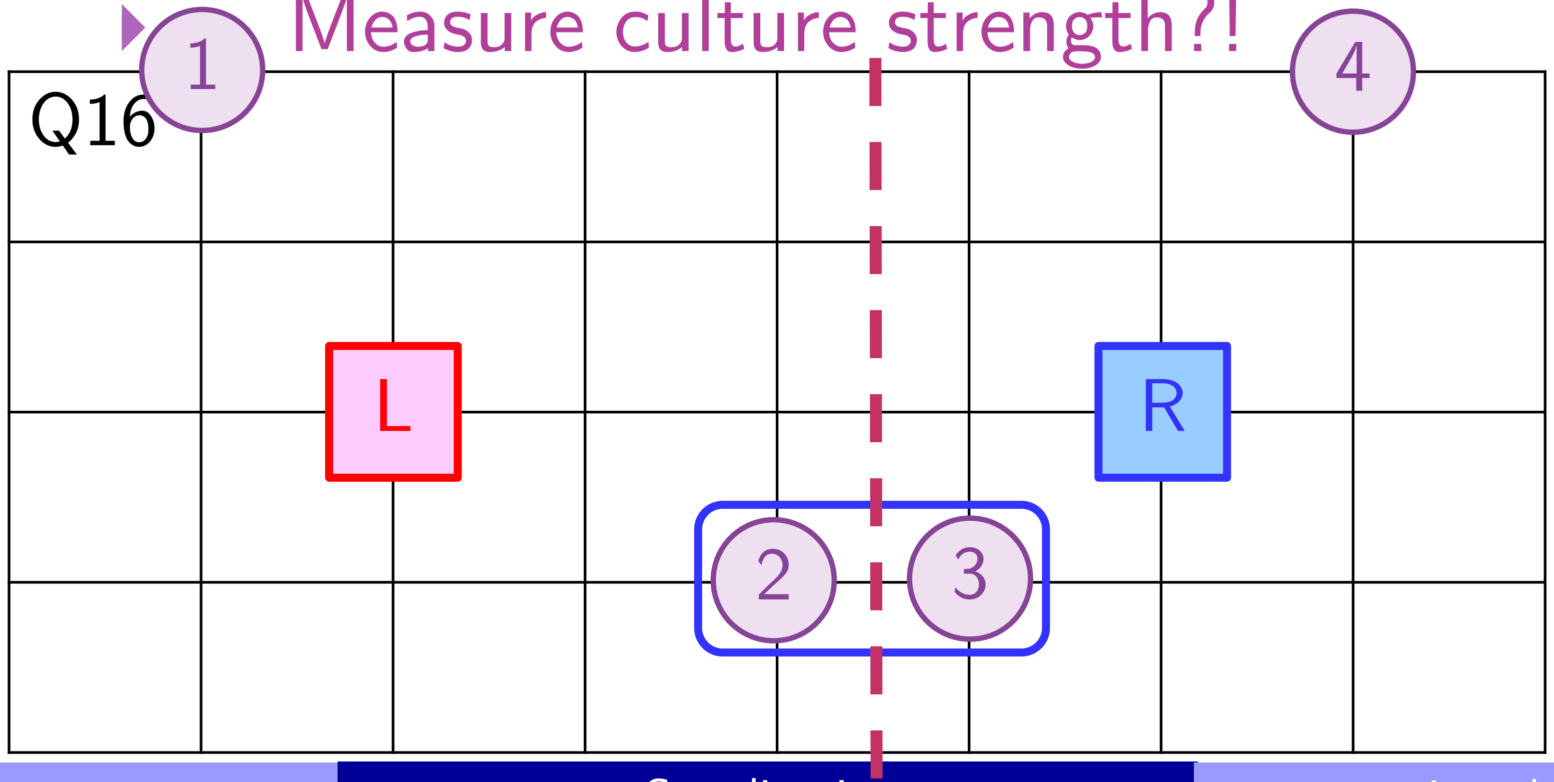
Assignment Game: Accession vs. Equality

- What does Accession say about this?
- What does Equality say about this? ☺
  - 29% follow A vs. 45% follow E



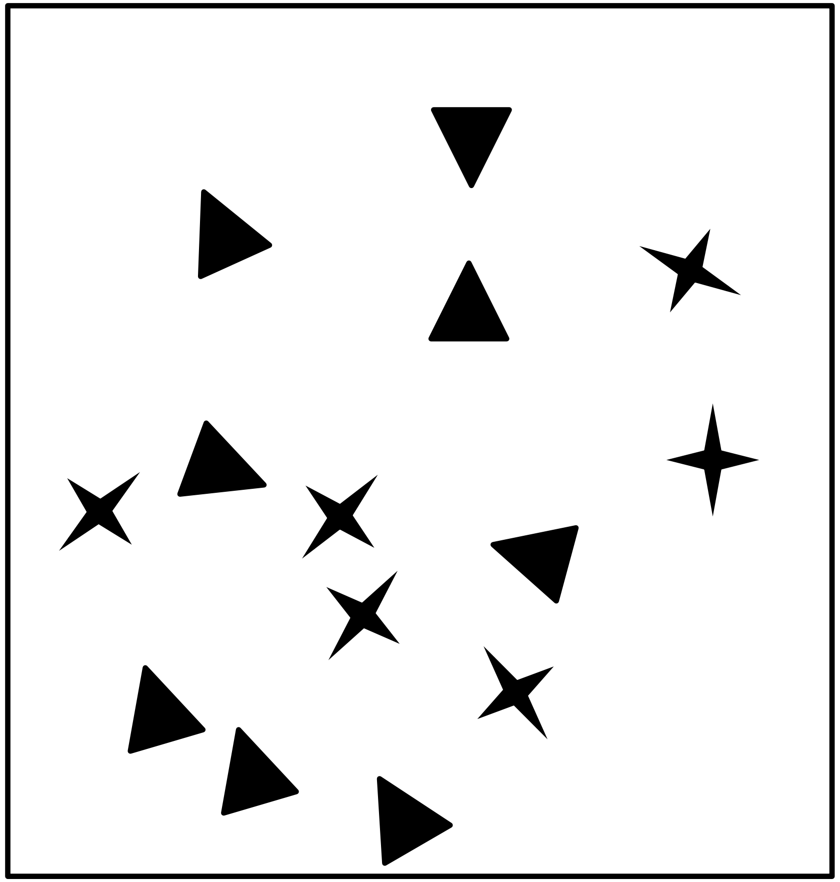
Equality > Accession > Closeness

- First Focal Principle: Equality ☺
- Then Accession (if Equality satisfied/silent)
  - Measure culture strength?!

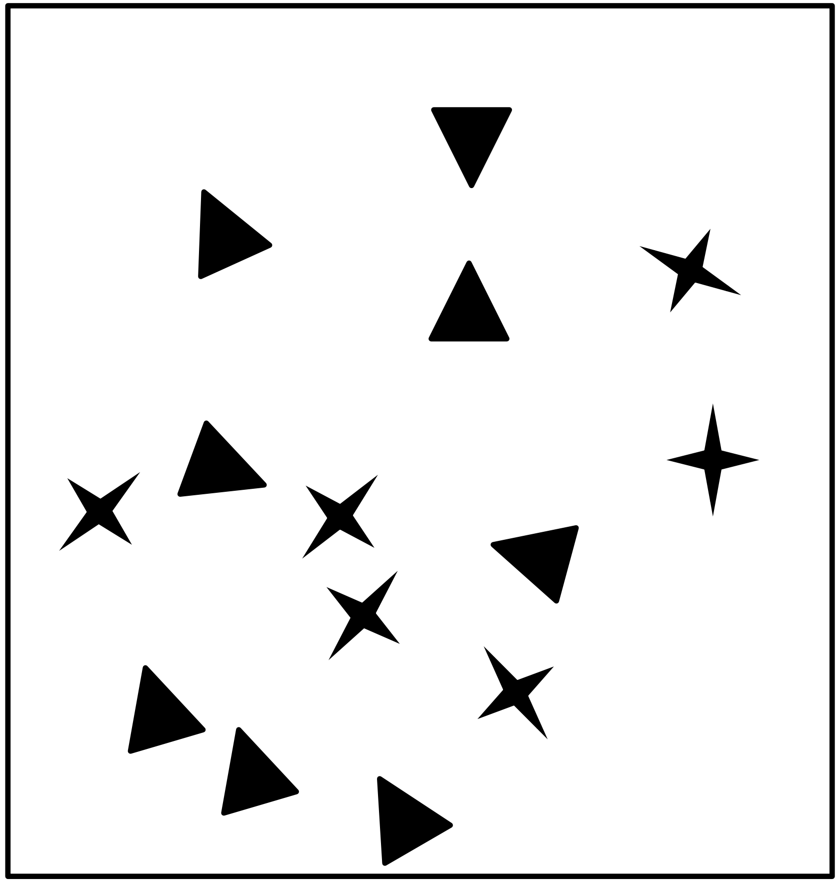


Unpacking Focality

- Bacharach and Bernasconi (GEB 1997)
- Visual matching game
  - Pick one from picture:
- Test rarity preferences
  - 6 vs. 8
- Are Rare item chosen more frequently
  - As Rarity increases?
  - 6/8, 2/3, 6/18, 1/15



Unpacking Focality: Test Rarity

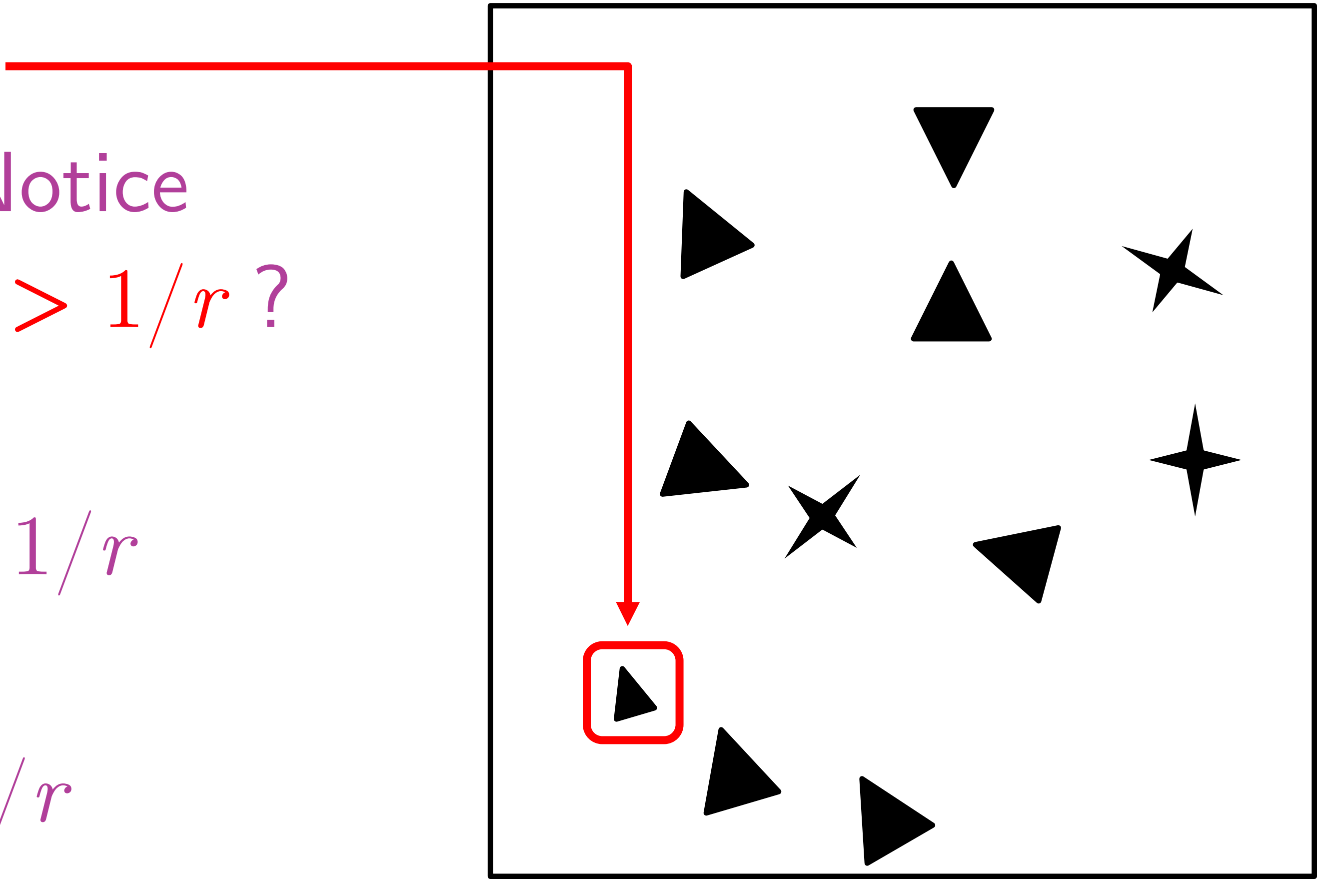


- Yes!
- As Rarity increases,
  - Frequency of rare choice increases!

	# of Rare/Frequent Items			
	6/8	2/3	6/18	1/15
Rare Item	65%	76%	77%	94%
Frequent Item	35%	24%	23%	6%

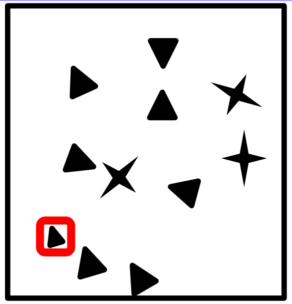
Unpacking Focality: Test Trade-offs

- Rarity ($r = 3$ vs. $n = 8$) against
- Oddity (size or color)
  - $p(\mathrm{F})=$ Probability of Notice
  - Choose Oddity if $p(\mathrm{F}) > 1/r$?
- Obvious Treatments:
  - $p(\mathrm{F})$=0.94 >> 1/3 $= 1/r$
- Subtle Treatments:
  - $p(\mathrm{F})$=0.40 > 1/3 $= 1/r$



Unpacking Focality: Test Trade-offs

- Violate $p(\mathrm{F}) > 1/\mathrm{r}$ **Proportion to Difference!**
  - Mostly chose Obvious vs. Less than half chose Subtle



$r = \#$ of Rare	Obvious Oddity ($1/r$)				Subtle Oddity ($1/r$)				
	1/2	1/3	1/4	1/5	1/2	1/3	1/4	1/5	1/6
$p(\mathrm{F})$	0.95	0.91	0.95	0.93	0.55	0.40	0.62	0.25	0.25
Difference: r - $p(\mathrm{F})$	0.45	0.58	0.7	0.73	0.05	0.07	0.37	0.05	0.09
Rare	14%	19%	9%	7%	77%	55%	45%	69%	55%
Oddity	83%	79%	91%	88%	23%	31%	45%	19%	20%
Other	2%	2%	0%	5%	0%	14%	10%	12%	25%

Unpacking Focality

- Munro (wp 1999)
- Field study of coordination
- Narrow bike lanes in Japan
  - No center line
- Two bikes coming from opposite directions
  - Both ride close to middle
- How they avoid colliding?
  - Both move Left!

Asymmetric Players: Battle of Sexes

- 100 lottery tickets =
  - 10% chance to win \$1/\$2
- Pure NE: (1,2) and (2,1)
  - Players prefer equilibrium where they play strategy 2
- Mixed NE:
  - (1/4, 3/4) each
- Which would you pick?

	1	2
1	0, 0	200, 600
2	200, 600	0, 0

Asymmetric Players: Battle of Sexes

- Cooper, DeJong, Forsythe and Ross (AER 1990)
- BOS: Baseline (MSE mismatch 62.5%)
- BOS-300: Row player has outside option 300
  - Forward Induction predicts (2,1)
- BOS-100: Row player has outside option 100
  - Forward Induction doesn't apply
- Compare BOS-100 and BOS-300
  - Shows if "any outside option" works...

Battle of Sexes (Last 11 Periods)

Game	Outside	(1,2)	(2,1)	Other	# Obs
BOS	-	37 (22%)	31 (19%)	97 (59%)	165
BOS-300	33	0 (0%)	119 (90%)	13 (10%)	165
BOS-100	3	5 (3%)	102 (63%)	55 (34%)	165
BOS-1W					165
BOS-2W					165
BOS-SEQ					165

Asymmetric Players: Battle of Sexes

  - Cooper, DeJong, Forsythe and Ross (AER 1990)
- BOS-1W: 1 way communication by Row
- BOS-2W: 2 way communication by Both
- BOS-SEQ: Both know that Row went first, but Column doesn't know what Row did
  - Information set same as simultaneous move
  - Would a sequential move act as an coordination device?

Battle of Sexes (Last 11 Periods)

Game	Outside	(1,2)	(2,1)	Other	# Obs
BOS	-	37(22%)	31 (19%)	97(59%)	165
BOS-300	33	0 (0%)	119 (90%)	13(10%)	165
BOS-100	3	5 (3%)	102 (63%)	55(34%)	165
BOS-1W	-	1 (1%)	158(96%)	6 (4%)	165
BOS-2W	-	49(30%)	47(28%)	69(42%)	165
BOS-SEQ	-	6 (4%)	103(62%)	56(34%)	165

Where Does Meaning Come From?

  - Communication can help us coordinate
- But how did the common language for communication emerge in the first place?
- Put people in a situation of no meaning and see how they create it!

- Blume, DeJong, Kim and Sprinkle (AER 1998)
  - See also BDKS (GEB 2001) which is better!

Evolution of Meaning: Game 1 (Baseline)

- Game 1: Blume et al. (AER 1998)
- Sender has private type T1 or T2
- Sends message "*" or "#" to receiver
- Receiver chooses A or B (to coordinate type)
- Game 1NH: See only history of own match

	A	B
T1	0, 0	7, 7
T2	7, 7	0, 0

Evolution of Meaning: Game 2

- Game 2:
- Receiver can choose C (safe action) that gives (4,4) regardless of T1/T2
  - Theory: Pooling or Separating Equilibrium

	A	B	C
T1	0, 0	7, 7	4, 4
T2	7, 7	0, 0	4, 4

Evolution of Meaning

- Blume, DeJong, Kim and Sprinkle (AER 1998)
- Game 1: Baseline as above
- Game 1NH: See only history of own match
- Game 2: Receiver can choose C (safe action) that gives (4,4) regardless of T1/T2
  - Theory: Pooling or Separating Equilibrium

Percentage Consistent with Separating Equilibrium

Game \ Period	1	5	10	15	20
1st Session: Game 1	48%	65%	74%	89%	95%
2nd Session					
Game 1	49%	72%	61%	89%	100%
Game 1NH	55%	55%	28%	55%	72%
Game 2					
Separating	44%	88%	88%	88%	94%
Pooling	39%	5%	0%	5%	5%

Evolution of Meaning: Game 3

- Game 3: Coordinate payoffs become (2,7)
  - So sender wants to disguise types to force receiver to choose C (safe action)
  - Allowed to send 2 or 3 messages…

	A	B	C
T1	0, 0	2, 7	4, 4
T2	2, 7	0, 0	4, 4

Evolution of Meaning (Blume et al. AER 1998)

- Game 1: Baseline as above
- Game 1NH: See only history of own match
- Game 2: Receiver can choose C (safe action) that gives (4,4) regardless of T1/T2
  - Theory: Pooling or Separating Equilibrium
- Game 3: Coordinate payoffs become (2,7)
  - Sender wants to disguise type so receiver picks C (safe action)
  - Allowed to send 2 or 3 messages...

Results of Game 3: 2 vs. 3 messages

# of Messages-Equil. Played	1-10	11-20	21-30	31-40	41-50	51-60
2nd Session: 2-Separating	43%	53%	38%	39%		
2-Pooling	33%	34%	41%	43%		
3-Separating	43%	38%	33%	24%		
3-Pooling	33%	37%	42%	60%		
1st Session: 2-Separating	39%	27%	23%	24%	24%	23%
2-Pooling	39%	48%	51%	60%	63%	61%
3-Separating	23%	22%	23%	25%	22%	24%
3-Pooling	55%	61%	58%	56%	57%	61%

Example of Asymmetric Payoffs

- Market Entry Game
  - n players decide to enter market with capacity c
  - Payoffs declines as number of entrants increase
  - " < 0 " if number $> c$ ($=$ market capacity)

- Sundali, Rapoport and Seal (OBHDP 1995)
  - Number of Entrants: Predicted vs. Actual

Market Entry Game: Results Close to Equilibrium

Capacity	1	3	5	7	9	11	13	15	17	19
Predicted Number of Entrants										
MSE	0	2.1	4.2	6.3	8.4	10.5	12.6	14.7	16.8	18.9
Actual Number of Entrants										
All Data	1.0	3.7	5.1	7.4	8.7	11.2	12.1	14.1	16.5	18.2
1st Block	1.3	5.7	9.7	6.7	3.7	14.0	11.3	11.3	16.0	18.0

- Kahneman (1988): "To a psychologist, it looks like magic."
  - See BI-SAW paper by Chen et al. (2012)...

Games with Asymmetric Equilibria

- Stag Hunt
  - Cooper, DeJong, Forsythe & Ross (AER 1990)
- 100 lottery tickets =
  - 10% chance to win \$1/\$2
- Pure NE:
  - (1,1) and (2,2)
  - Mixed NE?
- Which would you pick?

	1	2
1	800, 800	800, 0
2	0 , 800	1000, 1000

Games with Asymmetric Equilibria

- Cooper, DeJong, Forsythe and Ross (AER 1990)
- CG: Baseline Stag Hunt
- CG-900: Row has outside option 900 each
  - Forward Induction predicts (2,2)
- CG-700: Row has outside option 700 each
  - Forward Induction doesn't apply
- CG-1W: 1-way communication by Row
- CG-2W: 2-way communication by both

Stage Hunt (Last 11 Periods)

Game	Outside	(1,1)	(2,2)	Other	# Obs
CG	-	160(97%)	0(0%)	5(3%)	165
CG-900	65	2(2%)	77(77%)	21(21%)	165
CG-700	20	119(82%)	0(0%)	26(18%)	165
CG-1W	-	26(16%)	88(53%)	51(31%)	165
CG-2W	-	0(0%)	150(91%)	15(9%)	165

- (1,1) Payoff = 800 vs. (2,2) Payoff = 1000

Weak-Link Game (aka Minimum Effort Game)

- Van Huyck, Battalio and Beil (AER 1990)
- Each of you belong to a team of n players
- Each of you can choose effort $X_i = $ 1-7
- Earnings depend on
  - Your own effort X_i, and
  - The smallest effort $\min\{X_j\}$ of your team
- Payoff = 60 + 20 * $\min\{X_j\}$ – 10 * X_i

Cost of Effort X_i

Team Project Payoff

Weak-Link Game: Van Huyck et al. (AER 1990)

- Payoff = 60 + 10 * $\min\{X_j\}$ – 10 * $(X_i - \min\{X_j\})$

Team Minimum → $\min\{X_j\}$

Deviation from Min → $\min\{X_j\}$

- Payoff sensitive to weakest link in production chain:

1. Cobb-Douglas Production Function (Leontief)
2. All have to arrive for restaurant to seat your group
3. Each has to do their job for whole project to fly
   - Law firms, accounting firms, investment banks, etc.
4. Prepare an airplane for departure

Weak-Link Game: Van Huyck et al. (AER 1990)

$m = \min\{X_j\}$

Team Minimum

- Payoff $= 60$
  $+ 10 * m$
  $- 10 * (X_i - m)$

Deviation from Min

Your X_i	Smallest X_j in the Team						
	7	6	5	4	3	2	1
7	130	110	90	70	50	30	10
6	-	120	100	80	60	40	20
5	-	-	110	90	70	50	30
4	-	-	-	100	80	60	40
3	-	-	-	-	90	70	50
2	-	-	-	-	-	80	60
1	-	-	-	-	-	-	70

Weak-Link Game: Van Huyck et al. (AER 1990)

- What is your choice when...
  - Group size = 2?
  - Group size = 3?
  - Group size = 20?
- Can some kind of communication help coordinate everyone's effort?
- Let's conduct a classroom experiment first...

Classroom Experiment: 害群之馬

最弱環節賽局
(Weak-Link Game)

Weak-Link Game (最弱環節賽局)

- Each DM chooses effort X_i =1-4
  - Spade = 4, Heart = 3, Diamond = 2, Club = 1
- DM (Decision Maker) = a team of two
  - 每組每回合都會有四張撲克牌，分別為黑桃(4)、紅心(3)、方塊(2)、梅花(1)
    - 主持人會跟每組收一張牌
  - 交出來的花色代表你們花多少時間排練
    - 你們的努力程度：黑桃 = 4小時、紅心 = 3小時、方塊 = 2小時、梅花 = 1小時
  - 各組要討論屆時交出哪一張牌...

Payoff Calculation (記分方式)

- Payoff = $3 * \min\{X_j\}$ – $1 * X_i$ ← Cost of Effort X

Team Project Payoff

- $\min\{X_j\}$ =「花最少時間排練那一組的排練時數」，
- 每一小時的排練大家都會得到3分
- 各組自己每花一小時排練，就少1分

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Payoff Calculation (記分方式)

1. How much would you earn if all DM choose $X_i = 4$?

- 8!
- 如果所有各組都花四小時排練，這樣各組會拿幾分？
- 8分！

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Payoff Calculation (記分方式)

2. How much would you earn if you choose X_i =3 while others choose $X_j = 4$?

- 6 (< 8)
- Not worth it!
- 如果別組都花四小時排練，但你們這組只花三小時排練，這樣你們會拿幾分？這麼做值得嗎？
- 6分！小於8分所以不值得！

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Payoff Calculation (記分方式)

3. How much would you earn if you choose $X_i = 2$ while some other DM choose $X_i = 1$?

- 1 (< 2)
  - If you also choose $X_i = 1$!
- 如果有某一組只花一小時排練，你們這組如果花兩小時排練，值得嗎？
- 不值得，因只得1分，但如果也花一小時就會跟他們一樣得到2分!

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Weak-Link Game (最弱環節賽局)

- Please decide now and we will see the results...

6. Are you satisfied with the results? How can you encourage cooperation next time?
   - 你對結果滿意嗎？如果你希望大家都更好，該怎麼鼓勵大家合作？
   - 讓我們再來做一次...

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Weak-Link Game (最弱環節賽局)

- In reality, people would see each other's effort and increase effort gradually
- Let's try again by committing hour-by-hour!
  - 現實中你們彼此多半清楚大家的排練情況，而且時數可以逐步加碼。這次我們採一小時、一小時逐步加碼方式進行

Your X_i (本組時數)	$\min\{X_j\}$ (最低那組時數)			
	4	3	2	1
4	8	5	2	-1
3	-	6	3	0
2	-	-	4	1
1	-	-	-	2

Back to Van Huyck et al. (AER 1990)...

$m = \min\{X_j\}$

Team Minimum

- Payoff $= 60$
  $+ 10 * m$
  $- 10 * (X_i - m)$

Deviation from Min

Your X_i	Smallest X_j in the Team						
	7	6	5	4	3	2	1
7	130	110	90	70	50	30	10
6	-	120	100	80	60	40	20
5	-	-	110	90	70	50	30
4	-	-	-	100	80	60	40
3	-	-	-	-	90	70	50
2	-	-	-	-	-	80	60
1	-	-	-	-	-	-	70

Weak-Link Game: Large Group (Extensions)

- 7 Large Group ($n = 14$-16) sessions (Table 7.25)
  - X_i starts at 4-7, but quickly drop to 1-2!

Choice Frequencies in 7 Large Group Sessions

X_i	Round									(group size $n =$ 14-16)
	1	2	3	4	5	6	7	8	9	10
7	33	13	9	4	4	4	6	3	3	8
6	10	11	7	-	1	2	-	-	-	-
5	34	24	10	12	2	2	24	1	-	1
4	17	23	24	18	15	5	3	3	2	2
3	5	18	25	25	17	9	8	3	4	2
2	5	13	17	23	31	35	39	27	26	17
1	2	5	15	25	37	50	47	70	72	77

(2 modes in red/pink)
Table 7.25 of Camerer (BGT 2003)

Weak-Link Game: Large Group (Extensions)

- 7 Large Group ($n = 14$-16) sessions (Table 7.25)
  - X_i starts at 4-7, but quickly drop to 1-2!
- Extensions in Van Huyck et al. (AER 1990):
  - No penalty above min: 83% choose 7 in round 1
  - See effort distribution: Accelerate race to bottom
- 1 Small Group (n=2) Session (Table 7.26)
  - X_i starts at 1 or 7, but quickly converges to 7!
  - If choose $X_i = 7$ first, will wait a couple rounds for partner to follow...

Choice Frequencies in Small Group Session

X_i	Round			(group size $n = 2$)			
	1	2	3	4	5	6	7
7	9	13	13	17	19	19	21
6	0	1	4	2	1	1	0
5	4	1	1	1	0	0	0
4	0	1	2	0	1	1	0
3	1	2	1	1	0	0	0
2	1	2	0	0	0	0	1
1	8	4	3	3	3	3	2

(2 modes in red/pink)
Table 7.26,
Camerer (BGT 2003)

Weak-Link Game: Small Group Extension

  - Van Huyck et al. (AER 1990) also did
- Small Group (n=2) + Random Matching:
  - Start high (4-7), but drop to 1!
- Small group size not enough
  - Need stability/mutual adjustment of fixed pairing!
- Clark and Sefton (wp 1999)
  - Replicate random-matching results in stag hunt
  - Still unpublished: Difficult to publish replications?
- Group Size Meta-Study (Table 7.27)

Round 1 Group Minima

Group size n	Distribution of $\min\{X_j\}$							Total Obs.
	1	2	3	4	5	6	7	
2	43%	7%	7%	7%	29%	-	7%	14
3	25%	5%	35%	15%	5%	-	15%	20
6	73%	16%	11%	-	-	-	-	19
9	-	100%	-	-	-	-	-	2
12	100%	-	-	-	-	-	-	2
14-16	28%	28%	14%	28%	-	-	-	7

(Median underlined; 2 modes in red/pink) Middle Panel of Table 7.27, Camerer (BGT 2003)

Round 5 Group Minima

Group size n	Distribution of $\min\{X_j\}$							Total Obs.
	1	2	3	4	5	6	7	
2	14%	-	-	-	-	-	86%	14
3	30%	15%	20%	15%	-	-	20%	20
6	80%	10%	10%	-	-	-	-	19
9	100%	-	-	-	-	-	-	2
12	-	-	-	-	-	-	-	-
14-16	100%	-	-	-	-	-	-	7

(Median underlined; 2 modes in red/pink) Bottom Panel of Table 7.27, Camerer (BGT 2003)

Weak-Link Game: Group Size Meta-Study

- Large Group size ($n \geq 6$):
  - 1st period $\min\{X_j\} \leq 4$ vs. 5th period $\min\{X_j\}$ mostly 1
- Small Group size ($n =$ 2-3):
  - 1st period $\min\{X_j\}$ only partly in 5-7
  - 5th period $\min\{X_j\}$ mostly (86%) reaches 7 if n=2
- But 1st period median $X_i =$ 4-5 for all n!
  - Why? Maybe subjects think they play against representative opponent (and clone for large n)

Round 1 Choices (Median Underlined)

Group size n	Distribution of X_i							Total Obs.
	1	2	3	4	5	6	7	
2	28%	3%	3%	7%	21%	-	36%	28
3	8%	5%	8%	17%	7%	2%	41%	60
6	18%	7%	13%	16%	7%	7%	39%	114
9	0%	11%	28%	39%	5%	-	17%	18
12	25%	4%	13%	8%	16%	4%	29%	24
14-16	2%	5%	5%	17%	32%	9%	31%	104

(Median underlined; 2 modes in red/pink) Top Panel of Table 7.27, Camerer (BGT 2003)

Weak-Link Game: Local Interaction

- Berninghaus, Erhart and Keser (GEB 2002)
  - 3-person weak-link game
- What does Game Theory say?
  - Inefficient Nash: Each earn 80 if (X, X, X)
  - Efficient Nash: Each earn 90 if (Y, Y, Y)

		Other Player Choices		
		Both X	One X, One Y	Both Y
Row Player	X	80	60	60
	Y	10	10	90

Weak-Link Game: Local Interaction

- Baseline: Play 20 rounds with the same 2 opponents
  - See opponent choices (but not who made what)
- Local Interaction: 8 subjects form a circle to play the 2 neighbors next to you
  - Contagion: Can spread Equilibrium around circle

		Other Player Choices		
		Both X	One X, One Y	Both Y
Row Player	X	80	60	60
	Y	10	10	90

Weak-Link Game: Local Interaction

- Baseline (Fixed): 75% initially play Y
  - 7 of 8 groups converge to all-Y equilibrium
- Local Interaction: half initially play Y
  - Drop to None play Y in round 20
  - Because 64% play X if one neighbor played X

		Other Player Choices		
		Both X	One X, One Y	Both Y
Row Player	X	80	60	60
	Y	10	10	90

Weak-Link Game: Mergers

- Camerer and Knez (SMJ 1994):
  - Two groups each play 3-person weak-link game
  - Then merge into one 6-person group
- Two Possible Predictions:
  - Mergers Fail: Large group size reduces efficiency
  - Mergers Restart: Coordinate on good equilibrium
- Results: Mergers Fail! (Table 7.29)
  - Group Minima mostly 1 in Round 1 and 5
    - Regardless knowing other group minimum or not

Group Minima Before/After Mergers

Know Other Group Minimum				Don't Know Other Minimum			
Before Merger		After Merger		Before Merger		After Merger	
Round	5	1	5	Round	5	1	5
Session 1	(1,2)→	(1,2)→1	1	Session 1	(2,4)→	(1,2)→1	1
Session 2	(1,4)→	(1,1)→1	1	Session 2	(7,3)→	(7,1)→1	1
Session 3	(1,1)→	(1,2)→1	1	Session 3	(3,2)→	(3,1)→1	2
Session 4	(4,1)→	(4,1)→1	1	Session 4	(7,3)→	(7,3)→3	3
Session 5	(1,7)→	(1,7)→1	1	Session 5	(7,3)→	(7,2)→2	1

(.,.) show min of 3-person group   min of 6-person group   Table 7.29, Camerer (BGT 2003)

Weak-Link Game: Bonus

- Camerer and Knez (SMJ 1994): 2nd Treatment
  - Announce a bonus of $0.20/$0.50 if all choose 7
  - Additional bonus + announcement (beyond implicit gains if all choose 7)
- Results: 90% choose 7 in next period
  - Compared to 85% choose 1-2 last period
- Confirms Knez and Simester (JLE 2001)
  - Why group-level bonuses work so well

Weak-Link Game: Leadership

- Weber, Camerer, Rottenstreich and Knez (OS 2001)
- Play in large (n=8-10) or small (n=2) group
  - Each choose $s_i = 0, 1, 2, 3$;
  - Payoff = \$2.50 + \$1.25 x $[\min s_i - 1] - s_i$ − 0.25 x $1_{\{\min s_i = 0\}}$
  - Payoff = \$2.50 − s_i if $\min s_i = 1$
  - Payoff = \$3.75 − s_i if $\min s_i = 2$
  - Payoff = \$5.00 − s_i if $\min s_i = 3$
  - Payoff = \$1.00 − s_i if $\min s_i = 0$

Weak-Link Game: Leadership

- Weber, Camerer, Rottenstreich and Knez (OS 2001)
- Play in large (n=8-10) or small (n=2) group
  - Each choose $s_i = 0, 1, 2, 3$;
  - Payoff = \$2.50 + \$1.25 x $[\min s_i - 1] - s_i$ − 0.25 x $1_{\{\min s_i = 0\}}$
- After 2 rounds, randomly select a leader
  - Makes short speech to encourage more effort
  - Then, rate leader before/after 5 more rounds
- Attribute success to leadership personalities?

Weak-Link Game: Leadership

Effort Level	Large (n=8-10)					Small (n=2)			
	0	1	2	3		0	1	2	3
Round 1-2	25%	24%	20%	32%		5%	24%	26%	45%
Leadership	Rating (before)			5.88		Rating (before)			5.80
Round 3-8	47%	4%	-	49%		6%	6%	6%	83%
Leadership	Rating (after)			4.53		Rating (after)			6.17

Table 7.30, Camerer (BGT 2003)

- Confirm Nisbett and Ross (bk 1991)
  - Attribute too much cause of success/failure to leadership personalities

Median-Action Game: Van Huyck, Battalio and Beil (QJE 1991)

- In a team of $n = 9$, you choose effort $X_i = 1\text{-}7$
- Earnings depend on your own effort, and
  - The median effort M of your team
- Payoff $= 70 + 10 \times (M - 1) - 5 \times (X_i - M)^2$

  Team Project Payoff: $10 \times (M - 1)$

  Cost of Non-Conformity: $5 \times (X_i - M)^2$

  - Situations where players prefer to conform
  - Example: Prefer to not work too hard or too little
- Maximin $X_i = 3$ vs. Payoff-dominant $X_i = 7$

Median-Action Game: Van Huyck et al. (QJE1991)

Team Median

- Payoff (¢) $= 70$ $+ 10 \times (M - 1)$ $- 5 \times (X_i - M)^2$

Deviation from M

Your X_i	Median Value of X_j in the team						
	7	6	5	4	3	2	1
7	130	115	90	55	10	-45	-110
6	125	120	105	80	45	0	-55
5	110	115	110	95	70	35	-10
4	85	100	105	100	85	60	25
3	50	75	90	95	90	75	50
2	5	40	65	80	85	80	65
1	-50	-5	30	55	70	75	70

Median-Action Game Results

X_i	Round								(6 groups; 54 subjects)	
	1	2	3	4	5	6	7	8	9	10
7	8	2	2	-	-	1	1	-	-	-
6	4	6	6	6	3	3	4	1	3	1
5	15	15	22	19	22	20	20	24[1]	23[1]	26[2]
4	19	26	22	29[1]	27[1]	30[2]	30[2]	28[2]	28[3]	27[3]
3	8	3	2	-	-	-	-	1	-	-
2	-	1	-	-	1	-	-	-	-	-
1	-	1	-	-	-	-	-	-	-	-

Dispersion

Lock-in: same group medians

(2 modes in red/pink)[1-3] of groups in equilibrium

Table 7.32, Camerer (BGT 2003)

Median-Action Game (γ): Original

Team Median

- Payoff (¢)
  $= 70$
  $+ 10 \times (M - 1)$
  $- 5 \times (X_i - M)^2$

Deviation from M

Your X_i	Median Value of X_j in the team						
	7	6	5	4	3	2	1
7	130	115	90	55	10	-45	-110
6	125	120	105	80	45	0	-55
5	110	115	110	95	70	35	-10
4	85	100	105	100	85	60	25
3	50	75	90	95	90	75	50
2	5	40	65	80	85	80	65
1	-50	-5	30	55	70	75	70

Median-Action Game (ω): non-BR $\pi = 0$

Team Median

- Payoff (¢) $= 70$ $+ 10 \times (M - 1)$ $- ~~5 \times (X_i - M)^2~~$

Deviation from M

- Maximin no longer $X_i = 3$

Your X_i	Median Value of X_j in the team						
	7	6	5	4	3	2	1
7	130	0	0	0	0	0	0
6	0	120	0	0	0	0	0
5	0	0	110	0	0	0	0
4	0	0	0	100	0	0	0
3	0	0	0	0	90	0	0
2	0	0	0	0	0	80	0
1	0	0	0	0	0	0	70

Median-Action Game Results: Round 1

	Game (γ)		Game (ω)		Game (ϕ)	
X_i	Principle	Round 1	Principle	Round 1	Principle	Round 1
7	Payoff-Dom.	15%	Payoff-Dom.	52%	-	8%
6	-	7%	-	4%	-	11%
5	-	28%	-	33%	-	33%
4	-	35%	-	11%	Maximin	41%
3	Maximin	15%	-	-	-	8%
2	-	-	-	-	-	-
1	-	-	-	-	-	-

(2 modes in red/pink); Table 7.33, Camerer (BGT 2003)

Median-Action Game (γ): Original

Team Median

- Payoff (¢)
  $= 70$
  $+ 10 \times (M - 1)$
  $- 5 \times (X_i - M)^2$

Deviation from M

Your X_i	Median Value of X_j in the team						
	7	6	5	4	3	2	1
7	130	115	90	55	10	-45	-110
6	125	120	105	80	45	0	-55
5	110	115	110	95	70	35	-10
4	85	100	105	100	85	60	25
3	50	75	90	95	90	75	50
2	5	40	65	80	85	80	65
1	-50	-5	30	55	70	75	70

Median-Action Game (ϕ)

- Payoff (¢)
  $= 70$
  $+$ ~~$10 \times (M - 1)$~~
  $- 5 \times (X_i - M)^2$

  Deviation from M

Your X_i	Median Value of X_j in the team						
	7	6	5	4	3	2	1
7	70	65	50	25	-10	-55	-110
6	65	70	65	50	25	-10	-55
5	50	65	70	65	50	25	-10
4	25	50	65	70	65	50	25
3	-10	25	50	65	70	65	50
2	-55	-10	25	50	65	70	65
1	-110	-55	-10	25	50	65	70

Median-Action Game Results: Round 1

	Game (γ)		Game (ω)		Game (ϕ)	
X_i	Principle	Round 1	Principle	Round 1	Principle	Round 1
7	Payoff-Dom.	15%	Payoff-Dom.	52%	-	8%
6	-	7%	-	4%	-	11%
5	In-between	28%	-	33%	-	33%
4		35%	-	11%	Maximin	41%
3	Maximin	15%			-	8%
2	-	-			-	-
1	-	-	-	-	-	-

Follow Single Principles

(2 modes in red/pink); Table 7.33, Camerer (BGT 2003)