Dominance-Solvable Games (優勢可解賽局實驗)

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EE-BGT, Lecture 7

Dominance

- Strategy A dominates strategy B (B dominated by A)
  - Strategy A gives you better payoffs than Strategy B regardless of opponent strategy
- Dominance Solvable
  - A game that can be solved by iteratively eliminating dominated strategy (IEDS)
- Do people obey dominance?
- Will you bet on others obeying dominance?

Dominance

- Do people obey dominance?
  - Looking both sides to cross a 1-way street
  - "If you can see this, I can't see you."
  - Guess above 67 in the p-Beauty Contest (with $p = 2/3$)
- Behavior in Dominant-Solvable Games measures
  - Extent of Iterative Elimination of Dominated Strategies (IEDS)
  - Belief about others (Theory of Mind)
  - Degree of others' strategic sophistication

Belief About Dominance

- Will you bet on others obeying dominance? In...
- Diplomatic Decisions:
  - Knowing how leaders behave before impose tariffs/call a bluff
- Designing Incentive Contracts: (Prendergast, 1999)
  - Workers respond to incentives rationally, but...
  - Companies unwilling to bet on it/do not use optimal contracts
- Voting Theory vs. Practice: (Alverez and Nagler, 2002)
  - Predictions of Strategic Voting vs. Voters surprisingly sincere

Belief About Dominance

- SOPH: Knowing other's steps of reasoning
  - Good Advice: Do not guess 0 in the p-beauty contest game!
  - Why? Goal is to "Reason one step ahead, but no further!"
- Why limited steps of iterative thinking?
  - There is a huge difference (in cognitive status) between:

1. Do you obey dominance?
2. Will you bet on others obeying dominance?
   - And, going to 3+ levels of iterated reasoning is nearly impossible:
3. Will you believe that others think you obey dominance?

Belief of Iterated Dominance

1. Obey Dominance, (=One Level of Iterated Dominance)
   - Do you obey dominance? Do others obey dominance?
2. Believe that others obey dominance,
   - Will you bet on others obeying dominance?
   - Will others bet on you obeying dominance?
3. Believe that others believe you will obey dominance,
   - Will you believe that others think you obey dominance?
   - Will others believe that you think they obey dominance?

Belief of Iterated Dominance

4. Believe that others believe you believe they obey dominance,
   - Will you believe others believe you think they obey dominance?
   - Will others believe you believe they think you obey dominance?
5. Believe that others believe that you believe that they believe you obey dominance,
   - Will you believe others believe you believe they think you obey dominance?
   - Will others believe you believe they believe you think they obey dominance?

- etc.

Empirical Upper Bound on Steps of Reasoning

- Established by Experimental Results (since 1995) under:
  - Definition: Obey Dominance = One Step of Iterated Dominance
  - Qualification: Players' utility depends only on own payoffs
- Nearly all use one step of iterated dominance
- At least 10% have two levels of iterated dominance
  - Another 10% or more have three levels of iterated dominance
  - Yet another 10+% have four levels of iterated dominance
- Median steps of iterated dominance = 2 (Oversimplified?!)

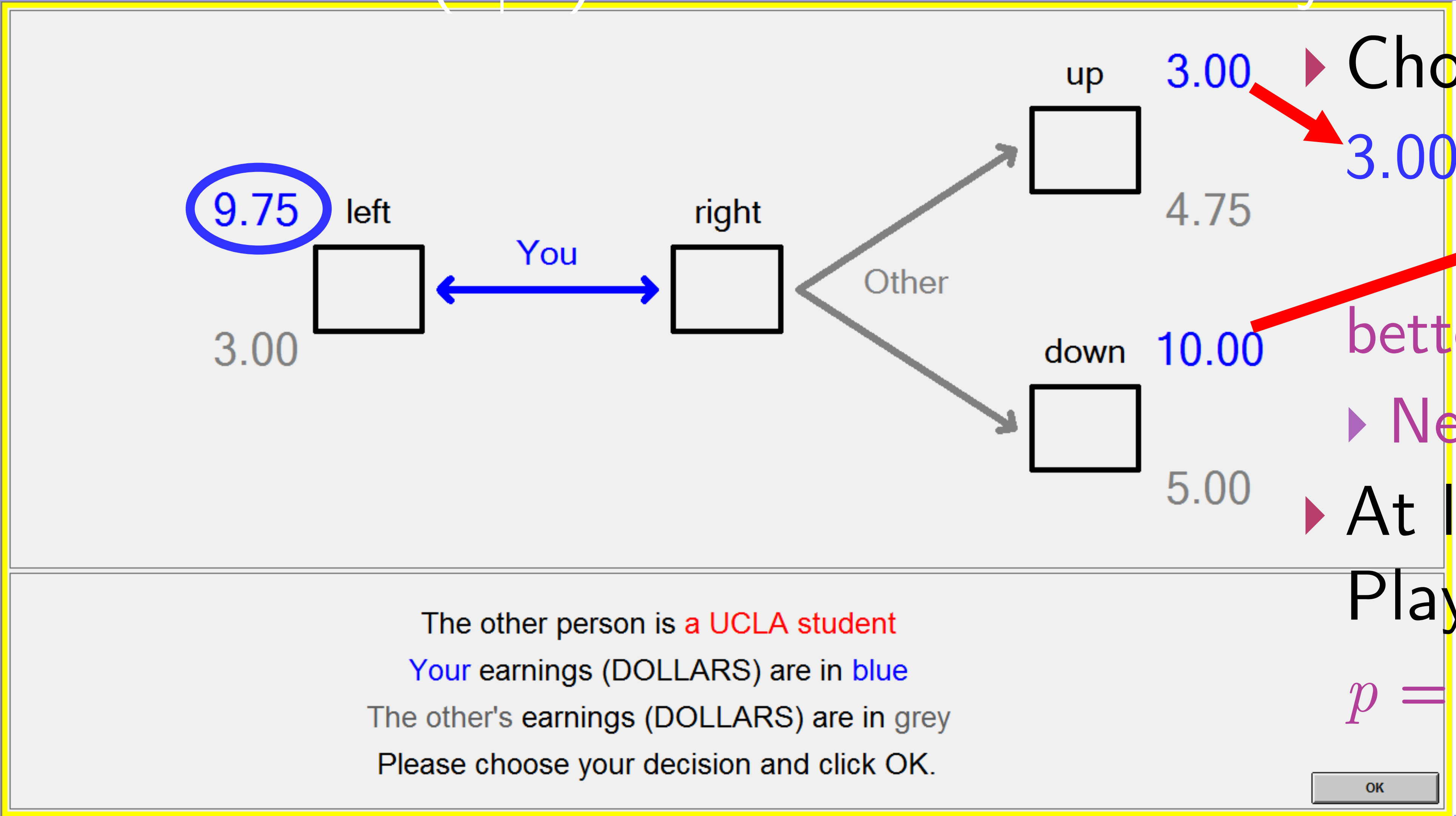
Outline

- A Simple Test: Beard and Beil (MS 1994)
  - Goeree and Holt (AER 2001), Schotter, Weigelt and Wilson (GEB 1994)
- Centipede: McKelvey and Palfrey (ECMA 1992)
- Mechanism Design:
  - Sefton and Yavas (GEB 1996)
- Dirty Face:
  - Weber (EE 2001)

A Simple Test: Beard and Beil (MS 1994)

Iterated Dominance Game		
Player 1 Move	Player 2 Move	
	l (up)	r (down)
L	9.75, 3	
R	3, 4.75	10, 5

Threshold P(r|R) for Risk Neutral Player 1 to Choose R



- Choose R yields:
  $3.00(1\text{-}p) + 10.00p \geq 9.75$
  better than Choose L
  - Need $p \geq 6.75/7$
- At least 96.4% of Player 2 choose D:
  $p = \text{P}(r|\text{R}) \geq 96.4\%$

A Simple Test: Beard and Beil (MS 1994)

Treatment	Payoffs from			Frequency		N	Threshold P(r | R)
	(L, l)	(R, l)	(R, r)	L	r|R		
1 (baseline)	(9.75,3)	(3, 4.75)	(10, 5)	66%	83%	35	96.4%
2 (less risk)	(9, 3)	(3, 4.75)	(10, 5)	65%	100%	31	85.7%
3 (even less risk)	(7, 3)	(3, 4.75)	(10, 5)	20%	100%	25	57.1%
4 (more assurance)	(9.75,3)	(3, 3)	(10, 5)	47%	100%	32	96.4%
5(more resentment)	(9.75,6)	(3, 4.75)	(10, 5)	86%	100%	21	96.4%
6(less risk, more reciprocity)	(9.75,5)	(5, 9.75)	(10,10)	31%	100%	26	95%
7 (1/6 payoff)	(58.5,18)	(18,28.5)	(60,30)	67%	100%	30	96.4%

A Simple Test: Beard and Beil (MS 1994)

- Player 2 mostly do obey dominance
- Player 1 is inclined to believe this
  - Though they can be convinced if incentives are strong for the other side to comply
- Follow-up studies show similar results:
  - Goeree and Holt (AER 2001)
  - Schotter, Weigelt and Wilson (GEB 1994)

Follow-up #1: Goeree and Holt (AER 2001)

Condition	N	Threshold $P(r \mid R)$	Payoffs (L, l)	Payoffs (R, l)	Payoffs (R, r)	Frequency L	Frequency $r \mid R$
Baseline 1	25	33.3%	(70, 60)	(60, 10)	(90, 50)	12%	100%
Lower Assurance	25	33.3%	(70, 60)	(60, 48)	(90, 50)	32%	53%
Baseline 2	15	85.7%	(80, 50)	(20, 10)	(90, 70)	13%	100%
Low Assurance	25	85.7%	(80, 50)	(20, 68)	(90, 70)	52%	75%
Very Low Assurance	25	85.7%	(400,250)	(100,348)	(450,350)	80%	80%

#2: Schotter, Weigelt and Wilson (GEB 1994)

Normal Form	Player 2		Game 1M
Player 1	l	r	Frequency
L	4, 4	4, 4	(57%)
R	0, 1	6, 3	(43%)
Frequency	(20%)	(80%)	

- In Game 1M:
  - Player 2 obey (weak) dominance
    - Actually 80% choose r
  - Player 1 unwilling to bet on it
    - But only 43% choose R

#2: Schotter, Weigelt and Wilson (GEB 1994)

Normal Form	Player 2		Game 1M
Player 1	l	r	Frequency
L	4, 4	4, 4	(57%)
R	0, 1	6, 3	(43%)
Frequency	(20%)	(80%)	

- Player 2 obey (weak) dominance
  - 80% choose r
- Player 1 unsure
  - 43% choose R

Sequential Form			Game 1S
L	4, 4		(8%)
	l	r	
R	0, 1	6, 3	(92%)
Frequency	(2%)	(98%)	

- Player 2 obey dominance (in subgame)
  - 98% choose r
- Player 1 expects this
  - 92% choose R

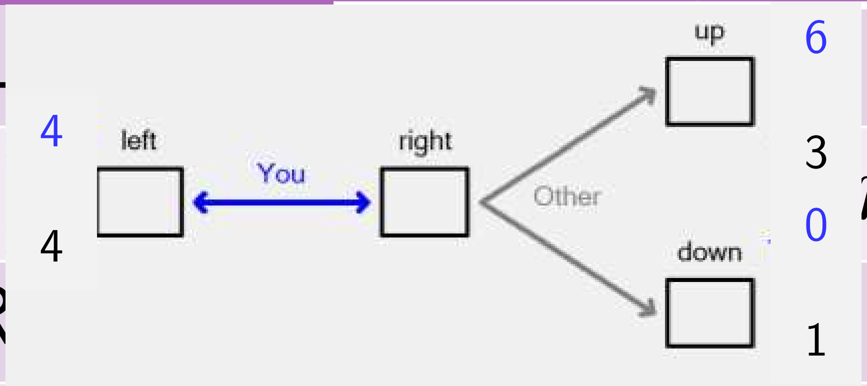
#2: Schotter et al. (1994) - Tree Presentation Effect

- Player 2 obeys dominance (choose r) in both Game 1M and 1S:
  - 98% in Game 1S, and 80% in Game 1M
- But Player 1 willing to bet on this (choose R) only in Game 1S:
  - 92% in Game 1S, but only 43% in Game 1M

Tree Presentation!!

- Game 1H like 1S:
- Player 2 obey dominance
  - 88% choose r
- Player 1 expects this
  - 86% choose R

Hybrid: Tree Played Simultaneously			Game 1H
L			L (14%)
		r	
R		3	R (86%)
Frequency	l (12%)	r (88%)	



Normal Form	Player 2			Game 3M
Player 1	t	m	b	Frequency
T	4, 4	4, 4	4, 4	(82%)
M	0, 1	6, 3	0, 0	(16%)
B	0, 1	0, 0	3, 6	(2%)
Frequency	(70%)	(26%)	(4%)	

94)

- 3M: (M, m) selected by 3 steps of iterated dominance
  - Player 1 almost never violates dominance
    - Only 2% choose B (dominated)
  - Few Player 2 anticipate this and play 2nd order dominance
    - Only 26% choose m (weakly dominant)

Few beyond 1-step ID!

#2: Schotter, Weigelt and Wilson (GEB 1994)

- Game 3S: 1-step ID + Forward Induction selects (M, m)
  - t dominated by MSE of BoS (⅔M + ⅓B, ⅓m + ⅔b): 1<2
  - Rejecting (4, 4) implies expecting (6, 3) by FI and playing M
- Before move, player 2 sees action of player 1
  - Player 2 only hypothesizes it in Game 3M
  - And player 1 knows this!

Sequential Form | Game 3S

T	4, 4	t			
		0, 1		m	b
			M	6, 3	0, 0
			B	0, 0	3, 6

Normal Form	Player 2			Game 3M
Player 1	t	m	b	Frequency
T	4, 4	4, 4	4, 4	(82%)
M	0, 1	6, 3	0, 0	(16%)
B	0, 1	0, 0	3, 6	(2%)
Frequency	(70%)	(26%)	(4%)	

94)

Similarly to 3M: Few beyond FI

- Player 1 plays FI
  - 100% choose M
- Player 2 unsure/disagrees
  - 69% choose b
- Player 1 expects this
  - 70% choose T

				Sequential Form		Game 3S
T	4, 4	t				(70%)
		0, 1		m	b	
			M	6, 3	0, 0	(100%)
			B	0, 0	3, 6	(0%)
Frequency		(13%)		(31%)	(69%)	

#2: Schotter, Weigelt and Wilson (GEB 1994)

- Conclusion of Schotter et al. (GEB 1994):
- Limited evidence of iteration of dominance (beyond 1-step), or SPE, forward induction
  - Can more experience fix this?
- No for forward induction in 8 periods...
  - Brandts and Holt (1995)
- But, Yes for 3-step iteration in 160 periods
  - Rapoport and Amaldoss (1997): Patent Race

Centipede Game: 4-Move SPNE

- McKelvey and Palfrey (Econometrica 1992)

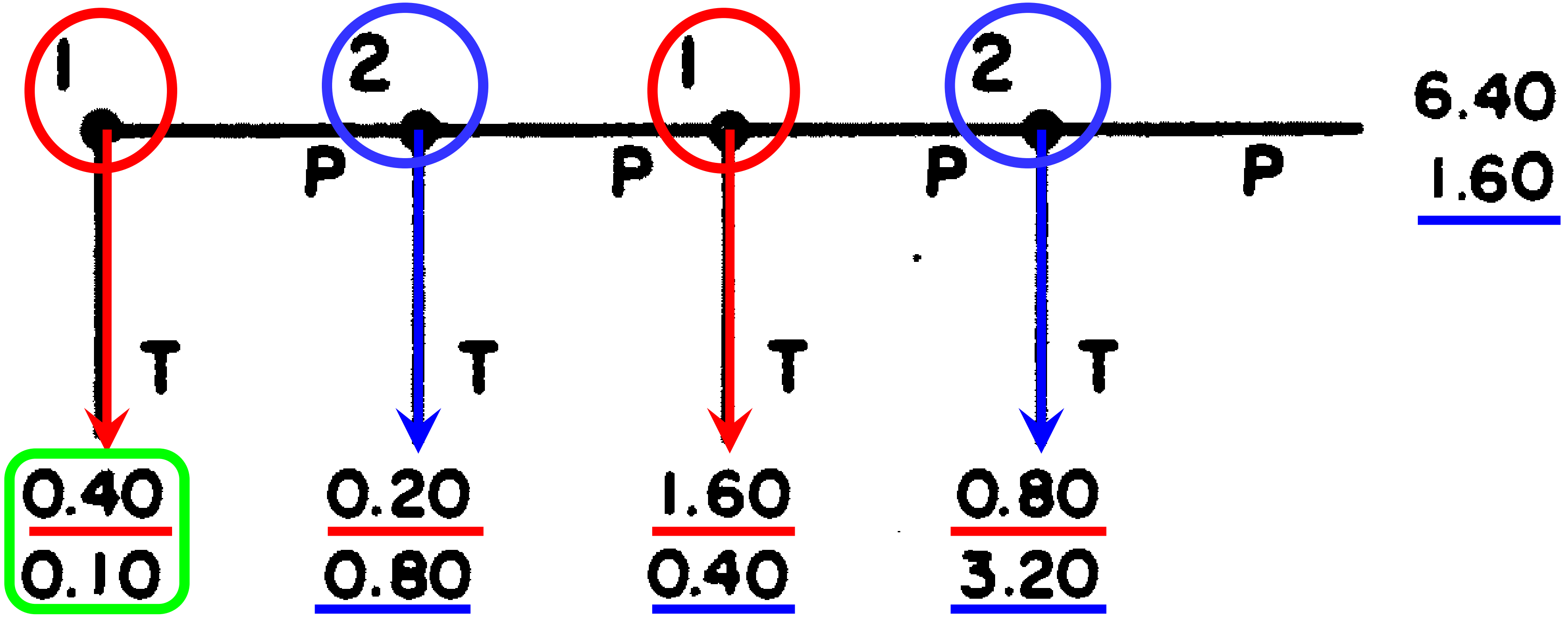


FIGURE 1.—The four move centipede game.

Centipede Game: 6-Move SPNE

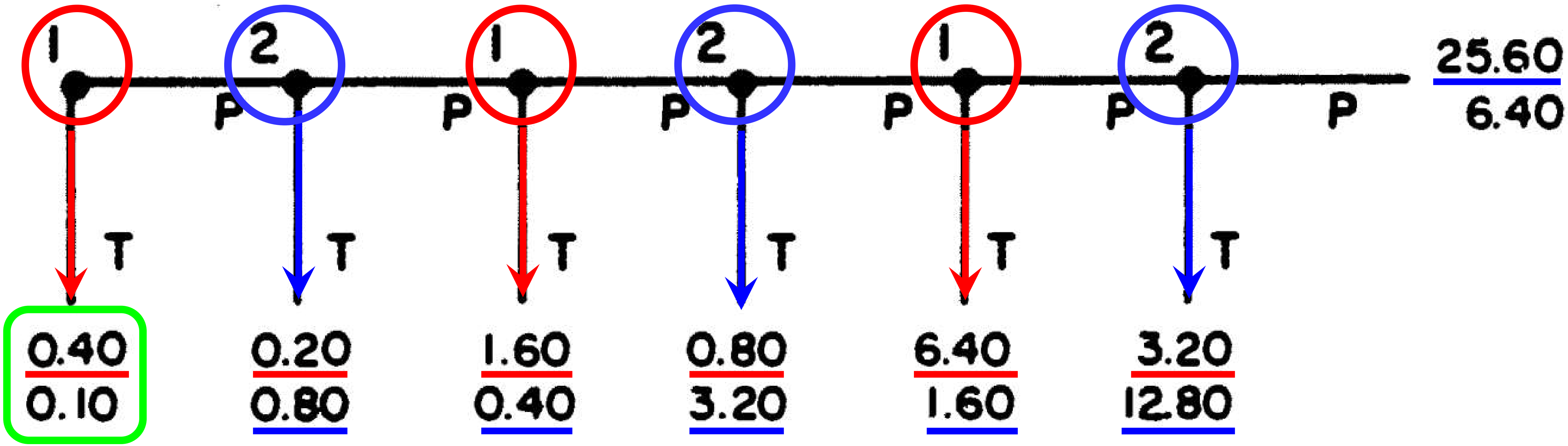


FIGURE 2.—The six move centipede game.

Centipede Game: Outcome

TABLE IIA

PROPORTION OF OBSERVATIONS AT EACH TERMINAL NODE

	Session		N	f_1	f_2	f_3	f_4	f_5	f_6	f_7
Four Move	1	(PCC)	100	.06	.26	.44	.20	.04		
	2	(PCC)	81	.10	.38	.40	.11	.01		
	3	(CIT)	100	.06	.43	.28	.14	.09		
	Total	1–3	281	.071	.356	.370	.153	.049		
High Payoff	4	(High-CIT)	100	.150	.370	.320	.110	.050		
Six Move	5	(CIT)	100	.02	.09	.39	.28	.20	.01	.01
	6	(PCC)	81	.00	.02	.04	.46	.35	.11	.02
	7	(PCC)	100	.00	.07	.14	.43	.23	.12	.01
	Total	5–7	281	.007	.064	.199	.384	.253	.078	.014

Centipede Game: Pr(Take)

TABLE IIB

IMPLIED TAKE PROBABILITIES FOR THE CENTIPEDE GAME

	Session	p_1	p_2	p_3	p_4	p_5	p_6
Four Move	1 (PCC)	.06 (100)	.28 (94)	.65 (68)	.83 (24)		
	2 (PCC)	.10 (81)	.42 (73)	.76 (42)	.90 (10)		
	3 (CIT)	.06 (100)	.46 (94)	.55 (51)	.61 (23)		
	Total 1–3	.07 (281)	.38 (261)	.65 (161)	.75 (57)		
High Payoff	4 (CIT)	.15 (100)	.44 (85)	.67 (48)	.69 (16)		
Six Move	5 (CIT)	.02 (100)	.09 (98)	.44 (89)	.56 (50)	.91 (22)	.50 (2)
	6 (PCC)	.00 (81)	.02 (81)	.04 (79)	.49 (76)	.72 (39)	.82 (11)
	7 (PCC)	.00 (100)	.07 (100)	.15 (93)	.54 (79)	.64 (36)	.92 (13)
	Total 5–7	.01 (281)	.06 (279)	.21 (261)	.53 (205)	.73 (97)	.85 (26)

Centipede Game

TABLE IIIB

IMPLIED TAKE PROBABILITIES
COMPARISON OF EARLY VERSUS LATE PLAYS IN THE LOW PAYOFF CENTIPEDE GAMES

Treatment	Game	p_1	p_2	p_3	p_4	p_5	p_6
Four Move	1–5	.06 (145)	.32 (136)	.57 (92)	.75 (40)		
	6–10	.08 (136)	.49 (125)	.75 (69)	.82 (17)		
Four Move	1–5	.00 (145)	.06 (145)	.18 (137)	.43 (112)	.75 (64)	.81 (16)
	6–10	.01 (136)	.07 (134)	.25 (124)	.65 (93)	.70 (33)	.90 (10)

Centipede Game: Mimic Model

- What theory can explain this?
- Altruistic Types (1-$q = 7\%$): Prefer to Pass
- Selfish Types (q):
  - Mimic altruistic types up to a point (to gain)
- Unraveling: error rate shrinks over time

Centipede Game: Mimic Model

- Selfish guys sometimes pass (mimic altruist)
- Imitating an altruist might lure an opponent into passing at the next move
  - Raising one's final payoff in the game
- Equilibrium imitation rate depends directly on beliefs about the likelihood $(1 - q)$ of a randomly selected player being an altruist
  - The more likely players believe there are altruists, the more imitation there is

Mimic: Predictions for Normal Types

1. On the last move, Player 2 TAKE for any q

2. If $1 - q > 1/7$, both Player 1 and 2 PASS
   - Except on the last move Player 2 always TAKE

3. If $0 < 1 - q < 1/7 \rightarrow$ Mixed Strategy Equilibrium

4. If $1 - q = 0$ both Player 1 & Player 2 TAKE

Mimic: Predictions for Normal Types

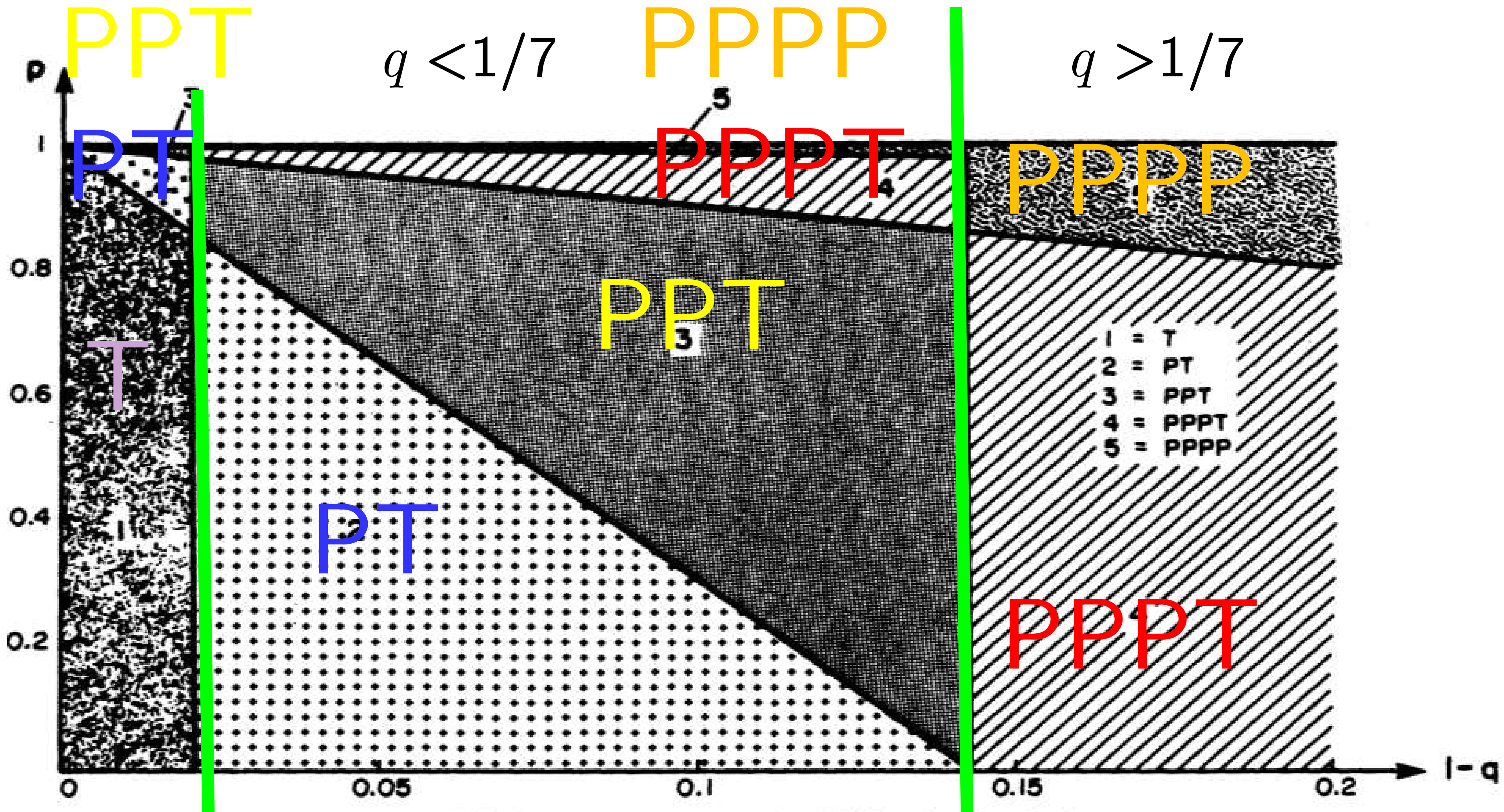
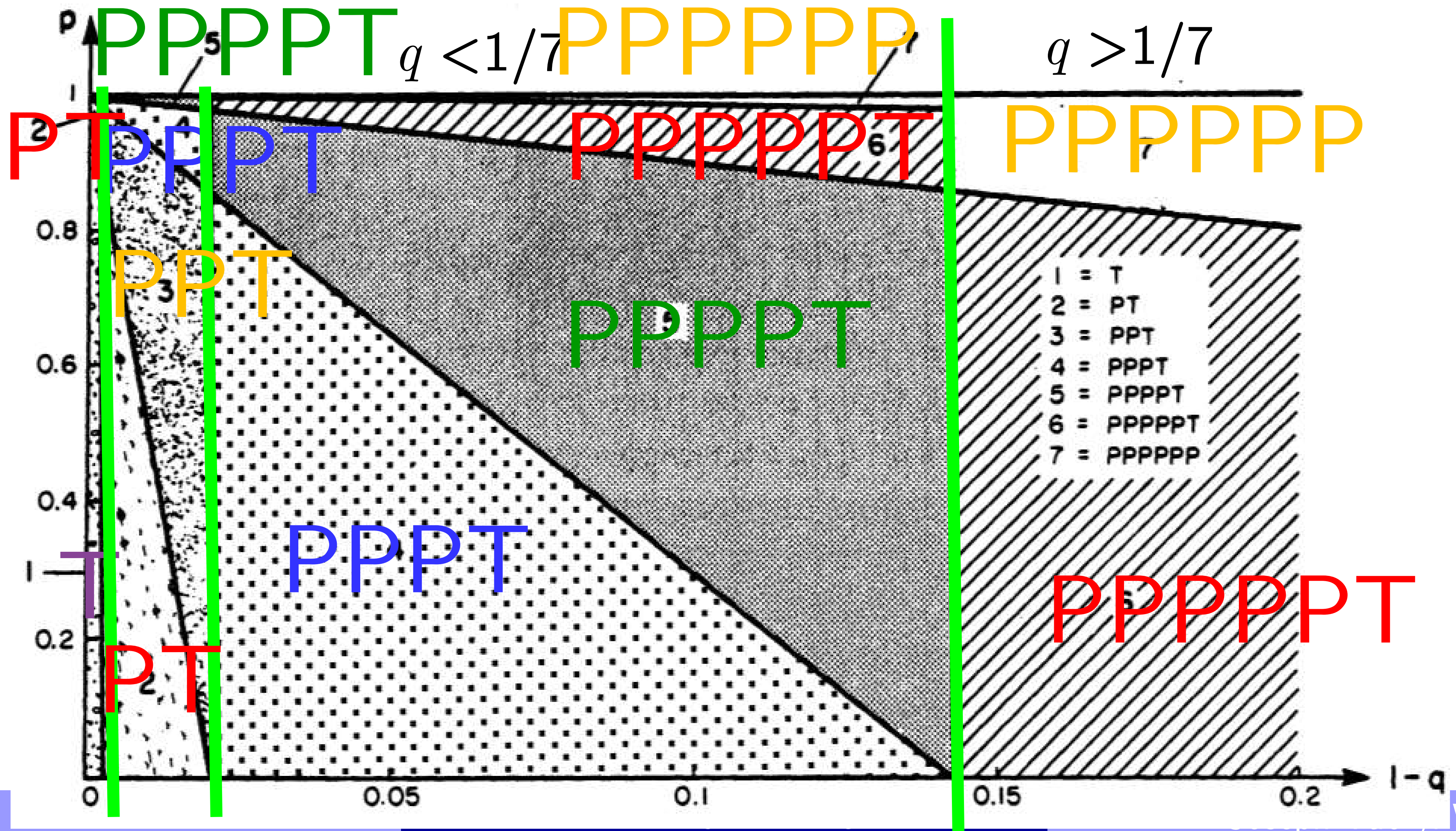


FIGURE 3.—Equilibrium outcome probabilities for basic four move game.

Mimic Model Equilibrium Outcome



Centipede: Mimic Model Add Noisy Play

- We model noisy play in the following way.
- In game t, at node s, if p^* is the equilibrium probability of TAKE
- Assume player actually chooses TAKE with probability $(1\text{-}\varepsilon_t)p^*$, and makes a random move with probability

$$\epsilon_t = \epsilon e^{-\delta(t-1)}$$

  - Explains further deviation from mimic model

Centipede: Follow-up Studies

- Fey, McKelvey and Palfrey (IJGT 1996)
  - Use constant-sum to kill social preferences
  - Take 50% at 1st, 80% at 2nd
- Nagel and Tang (JMathPsych 1998)
  - Don't know other's choice if you took first; take half way
- Rapoport et al. (GEB 2003)
  - 3-person & high stakes: Many take immediately
  - CH can explain this (but not QRE) – see theory

Mechanism Design

- Pure coordination game with \$1.20 & \$0.60
- How can you implement a Pareto-inferior equilibrium in a pure coordination games?
- Abreu and Matsushima (ECMA 1992)
  - Slice the game into T periods
  - F: Fine paid by first subject to deviate
  - Will not deviate if $F > \$1.20/T$
  - Can set $T = 1$, $F = \$1.20$; more credible if T large

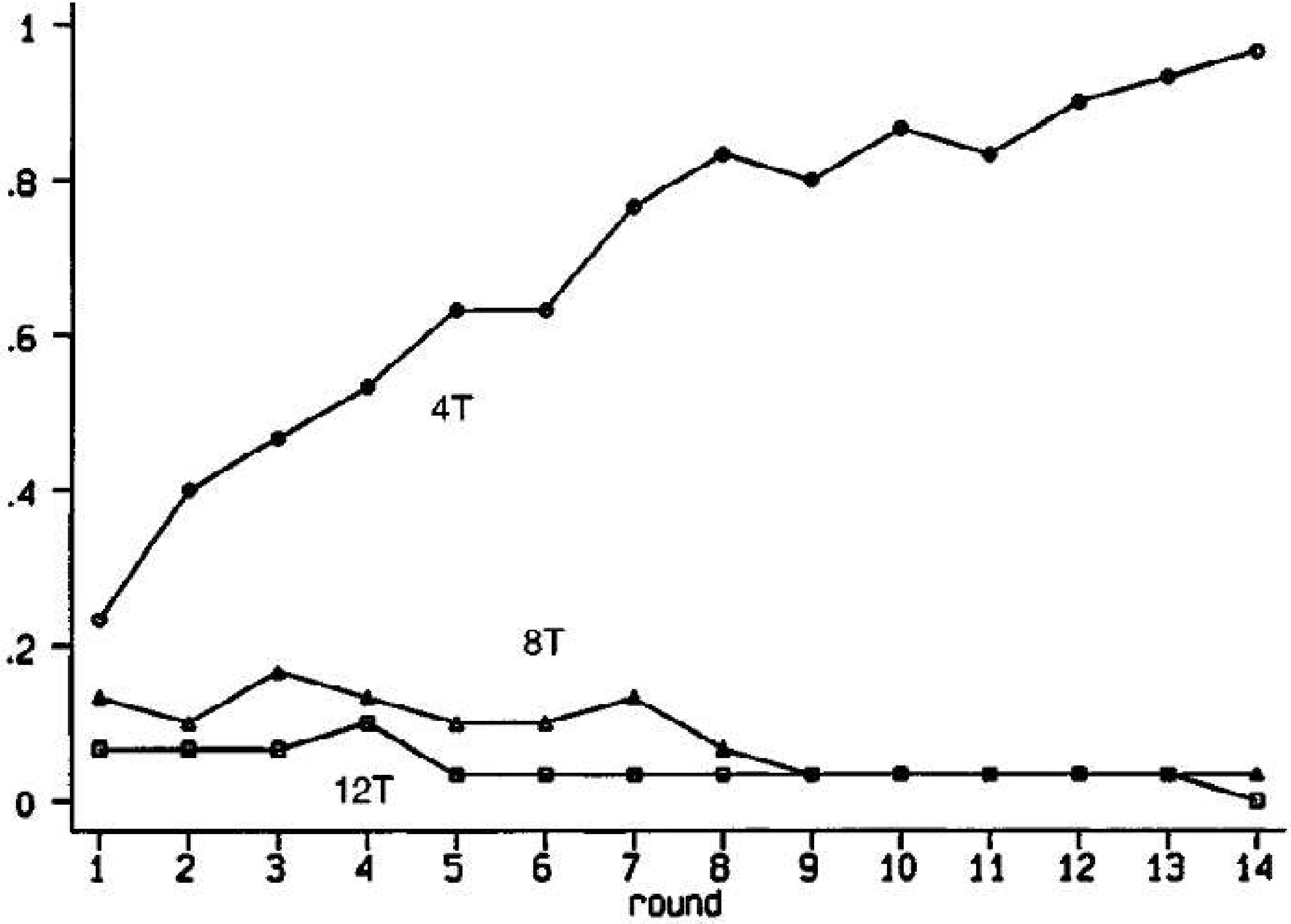
Mechanism Design

- Glazer and Rosenthal (ECMA 1992)
  - Comment: AM mechanism requires more steps of iterated deletion of dominated strategies
- Abreu and Matsushima (ECMA 1992)
  - Respond: "[Our] gut instinct is that our mechanism will not fare poorly in terms of the essential feature of its construction, that is, the significant multiplicative effect of fines.'"
- This invites an experiment!

Mechanism Design

- Sefton and Yavas (GEB 1996)
- F =$0.225
- T =4, 8, or 12
  - Theory: Play inferior NE at T =8, 12, not T =4
- Results: Opposite, and diverge...
- Why? Choose only 1 switch-point in middle
  - Goal: switch soon, but 1 period after opponent

Mechanism Design



Mechanism Design

- Glazer and Perry (GEB 1996)
  - Implemental can work in sequential game via backward induction
- Katok, Sefton and Yavas (JET 2002)
  - Does not work either
- Can any approximately rational explanation get this result?
  - Maybe "Limited steps of IEDS + Learning"?

Dirty Face Game

- Three ladies, A, B, C, in a railway carriage all have dirty faces and are all laughing.
- It sudden flashes on A:
- Why doesn't B realize C is laughing at her?
- Heavens! *I* must be laughable.
  - Littlewood (1953), *A Mathematician's Miscellany*
- Requires A to think that B is rational enough to draw inference from C

Dirty Face Game: Weber (Exp Econ 2001)

- Independent Types: X or O
  - $\Pr(X) = 0.8$, $\Pr(O) = 0.2$ (X is like "dirty face")
- Commonly told: At least one player is type X.
  - $P(XX) = 0.64 \rightarrow 2/3$, $P(XO) = 0.32 \rightarrow 1/3$
- Observe other's type
- Choose Up/Down (figure out one is type X)
  - If nobody chooses Down, reveal other's choice and play again

Dirty Face Game: Weber (Exp Econ 01')

		Type	
		X	O
Probability		0.8	0.2
Action	Up	$0	$0
	Down	$1	-$5

Dirty Face Game

- Case XO: Players play (Up, Down) since
- Type X player thinks:
  - I know that "at least one person is type X"
  - I see the other person is type O
- So, I must be type X → Chooses Down
- Type O player thinks:
  - I know that "at least one person is type X"
  - I see the other person is type X: No inference → Chooses Up

Dirty Face Game

- Case XX - First round:
  - At least one is type X, but the other guy is type X
- No inference → Both choose Up
- Case XX - Second round:
- Seeing UU in first
  - The other is not sure about his type
  - He must see me being type X
- I must be Type X → Both choose Down

Dirty Face Game

		Trial 1		Trial 2	
		XO	XX	XO	XX
Round 1	UU	0	7*	1	7*
	DU	3*	3	4*	1
	DD	0	0	0	0
Round 2 (after UU)	UU	-	1	-	2
	DU	-	5	-	2
	DD	-	1*	-	3*
	Other	-	-	1	-

Dirty Face Game

- Results: 87% rational in XO, but only 53% in 2nd round of XX
- Significance:
- Choices reveal limited reasoning, not pure cooperativeness
  - More iteration is better here...
- Upper bound of iterative reasoning
  - Even Caltech students cannot do 2 steps!

Conclusion

- Do you obey dominance?
- Would you count on others obeying dominance?
  - Little evidence beyond 1-step iterative dominance
- Limit of Strategic Thinking: At most 2-3 steps
- Compare with Theories of Initial Responses
  - Level-k: Stahl-Wilson95, CGCB01, CGC06
  - Cognitive Hierarchy: CHC04