

Experiments Games with Mixed Strategy Equilibrium

(混合策略均衡實驗)

Joseph Tao-yi Wang (王道一)
EE-BGT, Lecture 5

Some Comments Regarding 20-Minute Presentations

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Comments Regarding 20-Minute Presentations

- ▶ **Rule 1:** Don't use more than **20** slides
 - ▶ It takes on average 1 minute to go over 1 slide
- ▶ **Rule 2:** Don't use font sizes below **28**
 - ▶ Try looking at your slides from far behind
 - ▶ Font sizes < 28 means:
 - You DON'T want people to see it
- ▶ **Rule 3:** This is a **teaser-trailer** of the movie
 - ▶ Show the experiment + a snapshot of the results
- ▶ Write down a script and **NOT** memorize it

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Games with MSE 有混合策略均衡的賽局

- ▶ Zero-Sum Games (零和賽局)
 - ▶ Rock-Scissor-Paper (剪刀石頭布)
 - ▶ Sports (PK, tennis serves, basketball drives, etc.)
 - 足球罰踢、網球發球、籃球切入或投籃
 - ▶ Military attack (軍事行動如登陸諾曼地或加萊)
- ▶ Deter Undesired Behavior (嚇阻投機/不希望發生的行為)
 - ▶ Searches of passengers after 9/11 (機場安檢、海關抓走私)
 - ▶ Randomizing across exam questions (老師隨機出題)
- ▶ But, there are interesting folk theories about these games... (但總有一些有趣的「理論」)

玩家公開猜拳遊戲必勝絕招：先出剪刀 中央社 2007-12-19

▶ 媒體報導，大多數人都知道，在猜拳遊戲中，石頭贏剪刀，剪刀贏布，布勝拳頭，但很少有人知道，如何贏得這個相當普遍的遊戲。現在死忠玩家透露了必殺秘技：先出剪刀。 L0

▶ 英國「每日郵報」報導，研究顯示在這種快速擺出手部姿勢的猜拳遊戲中，石頭是三種猜拳手勢中玩家最喜歡出的一種。如果你的對手預期你會出石頭，他們就會選擇出布來贏過你，
L1 因此你要出剪刀才能贏，因為剪刀贏布。

L2

玩家公開猜拳遊戲必勝絕招：先出剪刀 中央社 2007-12-19

- ▶ 報導說，這套剪刀策略讓拍賣商佳士得前年成功贏得一千萬英鎊的生意。一名有錢的日本藝術品收藏家，無法決定要讓哪家拍賣公司來拍賣自己收藏的印象派畫作，於是 he 要求佳士得與蘇富比兩家公司猜拳決定。
- ▶ 佳士得向員工討教猜拳策略，最後在一名主管十一歲的女兒的建議下決定出剪刀。這名女孩現在還在讀書，經常玩猜拳，她推論「所有人都以為你會出石頭」。這代表蘇富比會出布，想要打敗石頭，因此佳士得應該選擇出剪刀。
- ▶ 一如預期，蘇富比最後出布，輸給了佳士得的剪刀，拱手將生意讓給對方。

Mixed-Strategy Equilibrium in RPS

- ▶ How do you play Rock-paper-scissors (RPS)?
 - 如果你來玩剪刀石頭布，你會出什麼？
- ▶ What is the MSE here? (剪刀石頭布賽局的均衡為何?)
- ▶ Mix with probabilities $(1/3, 1/3, 1/3)$ (三者隨機)
- ▶ Would you really play **this MSE** in RPS?
 - ▶ News article suggests a level-k model...
 - ▶ (你真的會按均衡策略來玩嗎？新聞故事所反映的多層次思考模型預測為何？想知道更多請看課本第五章)
- ▶ Janken/RPS Robot with 100% winning rate:
 - ▶ v1: <http://www.youtube.com/watch?v=3nxjjztQKtY>
 - ▶ v2: <https://www.youtube.com/watch?v=ZVNnoOcohaU>

Advantages of Games with MSE (此種賽局的優點)

- ▶ Typically have unique equilibrium (有唯一均衡)
 - ▶ All games discussed have unique equilibrium
- ▶ Constant sum: No room for social preference
 - ▶ Not possible to help others without hurting self (總報酬為常數下通常無社會偏好，因為幫助別人一定傷到自己)
- ▶ Maximin leads to Nash in zero sum (避兇就是均衡)
 - ▶ Maximin is a simple rule: (對方就是要害我如何趨吉避兇)
 - ▶ "I want to maximize the worse case scenario..."
- ▶ A good place to test theory! (這是驗證理論的好地方)

Maximin in Matching Pennies (黑白猜下避兇)

	H	T
H	1	-1
T	-1	1

- ▶ Rowena thinks: (列子認為)
- ▶ Play H: If Colin wants to scxxw me, worse case = -1
- ▶ Play T: Worse case = -1
- ▶ $(1/2, 1/2)$: Worse case 0*
(出正面最慘是對手選反面，出反面最慘是對手選正面，一半一半至少不賺不賠)
- ▶ Same for Colin (行家所見略同)

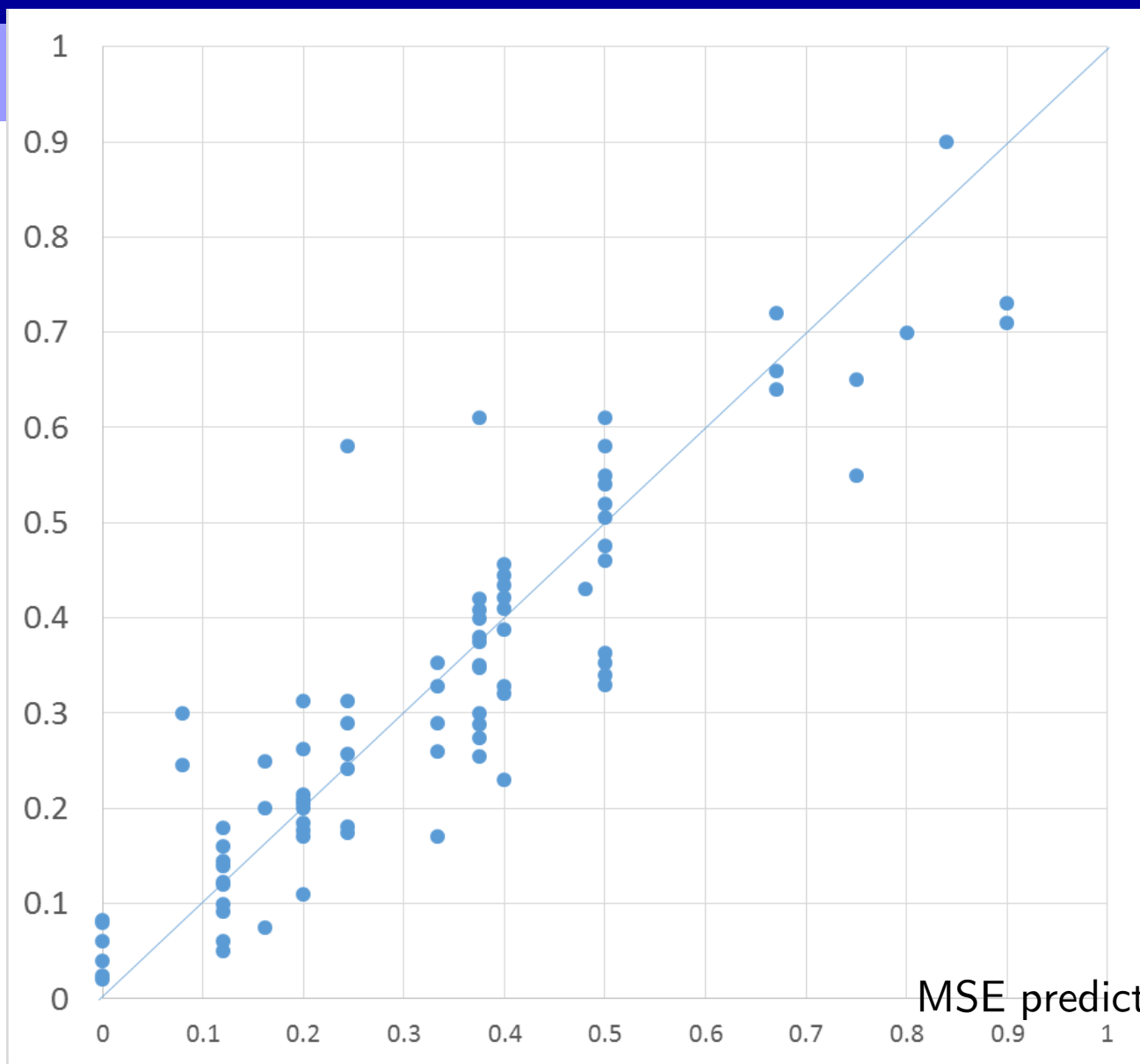
*We assume preferences satisfy axioms for EU... (假設偏好滿足期望效用公理)

▶ This is the MSE!
(這正好是此賽局的混合策略均衡!)

Challenges of Games with MSE (對理論的挑戰)

- ▶ Epistemic Foundation (認知基礎: 須清楚知道對手的策略)
 - ▶ Requires precise knowledge of opponent strategy
- ▶ Learning Dynamics may not work (動態學習不見得好)
 - ▶ Gradient processes spiral away (梯度逼近會螺旋脫離均衡)
 - ▶ No incentive to mix properly at MSE (均衡時亂做沒差)
- ▶ Randomization can be unnatural
 - ▶ Especially in repeated play (重複做的話, 隨機亂選不太自然)
- ▶ Purification (純化: 個體可做不同單純策略, 整體看起來「混合」即可)
 - ▶ MSE can occur at population level, not individually

Actual Data
(實驗資料)



Source
(資料來源):
Behavioral
Game Theory
Ch 3

Joker Game: O'Neill (1987) (出鬼牌賽局)

- ▶ Earlier studies: Play between MSE & random
 - ▶ But had computerized opponents and/or low incentives, so hard to interpret the results...
(早期實驗結果介於MSE和亂選之間，但通常對手是電腦且不見得有誘因)
- ▶ First "Modern" Studies: O'Neill (1987)
- ▶ **Good Design Trick:** (很棒的實驗設計技巧!)
 - ▶ Risk aversion plays no role when there are only two possible outcomes
(當實驗結果只有兩種可能時，風險偏好不會影響受試者的決定)

Joker Game: O'Neill (1987) (出鬼牌賽局)

	1	2	3	J	MSE	Actual	QRE
1	-5	5	5	-5	0.2	0.221	0.213
2	5	-5	5	-5	0.2	0.215	0.213
3	5	5	-5	-5	0.2	0.203	0.213
J	-5	-5	-5	5	0.4	0.362	0.360
MSE	0.2	0.2	0.2	0.4			
Actual	0.226	0.179	0.169	0.426			
QRE	0.191	0.191	0.191	0.427			

▶ 實際的出牌頻率跟MSE預測很接近

▶ QRE的預測更接近，但無法解釋「不平衡」

▶ Actual frequencies are quite close to MSE

▶ QRE better, but cannot get "imbalances"

Quantal Response Equilibrium (手滑反應均衡)

- ▶ QRE - McKelvey and Palfrey (1995)
- ▶ **Better response**, not best response (更適/非最適)
- ▶ Logit payoff response function: (常用logit報酬反應函數)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

Quantal Response Equilibrium (QRE)

- ▶ $\lambda = 0$: Noise (do not respond to payoffs)
 - ▶ (對報酬無反應)
- ▶ $\lambda = \infty$: Nash (perfectly respond to payoffs)
 - ▶ (完全反應)

$$P(s_i) = \frac{e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_i, s_{-i}) \right]}}{\sum_{s_k} e^{\lambda \cdot \left[\sum_{s_{-i}} P(s_{-i}) u_i(s_k, s_{-i}) \right]}}$$

Response to O'Neill (1987) (後續討論)

- ▶ Brown and Rosenthal (1990) criticize O'Neill:

- ▶ Overly support MSE (太過支持混合策略均衡)

- ▶ Aggregate tests not good enough (只有總體檢定不夠)

- ▶ They run (temporal dependence):

$$J_{t+1} = a_0 + a_1 J_t + a_2 J_{t-1} \quad (\text{應該檢定跨期相關性})$$

$$b_0 J_{t+1}^* + b_1 J_t^* + b_2 J_{t-1}^*$$

$$c_1 J_t J_t^* + c_2 J_{t-1} J_{t-1}^* + \epsilon$$

J_t = Own Choice; J_t^* = Other's Choice;

- ▶ MSE implies only a_0 is not zero (均衡: 只有 a_0 不是 0)

Brown & Rosenthal (1990) Results

Effect	Coefficient	% Players s.t. $p < 0.05$
Guessing	b_0	8%
Previous opp. choices	b_1, b_2	30%
Previous outcomes	c_1, c_2	38%
Previous choices & outcomes	b_1, b_2, c_1, c_2	44%
Previous own choices	a_1, a_2	48%
All effects		62%

Source: Table 3.4, BGT.

Response to O'Neill (1987) (後續討論)

- ▶ **Run: 2 JJJJ 1 2 33** (連發太短)
- ▶ **Too Short runs:** play J twice too rarely (鮮有連續J)
- ▶ **Subjects react to what they see/do** (對歷史有反應)
 - ▶ But most cannot use temporal dependence to guess opponent current action
 - ▶ (無法用跨期相關性猜中對方這次行動)
- ▶ **Equilibrium-in-beliefs** somewhat supported
 - ▶ (證據支持信念上的均衡)
- ▶ Each player may deviate from MSE (每人各自可能偏離)
- ▶ But these deviations cannot be detected (卻未被破解)

Response to O'Neill (1987) (後續討論)

- ▶ **Purification interpretation of MSE** (純化的MSE)
 - ▶ Equilibrium in beliefs, not in mixtures (信念非策略)
- ▶ **Other similar studies** (相關延伸研究)
 - ▶ Rapoport and Boebel (1992) [BGT, Table 3.5]
 - ▶ Mookerjee and Sopher (1997)
 - ▶ [BGT, Table 3.6-3.7]
 - ▶ Tang (1996abc, 2001)
 - ▶ [BGT, Table 3.8]
 - ▶ Binmore, Swierzbinski and Proulx (2001)
 - ▶ [BGT, Table 3.9]

Response to O'Neill (1987) (後續討論)

▶ Stylized Facts: (整體實驗結果)

1. Actual frequencies not far from MSE
 - ▶ (出牌頻衡很接近MSE)
2. Deviations small but significant
 - ▶ (跟MSE差距小但統計上顯著)
3. Temporal dependence at individual level
 - ▶ (個人有跨期相關性)

▶ Can a theory explain these?

- ▶ (有何理論可以解釋這些實驗結果?)

Response to O'Neill (1987) (後續討論)

- ▶ Ask subjects generate random sequences (產生數列)
- ▶ Sequences resemble the underlying statistical process **more closely than** what short random sequences actually do (產生的比真正隨機還要更「隨機」)
 - ▶ Too balanced (太平衡)
 - ▶ Too few runs (連發太少)
 - ▶ Longest run is too short (最長的連發太短)
- ▶ Children do not learn this misconception until after 5th grade (小孩子在五年級之前沒有這個問題)
 - ▶ A learned mistake (這是一個後天學會的錯誤)

Game Play (賽局實驗) vs. Production (產生數列)

- ▶ Rapoport and Budescu (1992, 1994, 1997)
 - ▶ Compare sequences from a production task to strategies in a constant-sum game (R&B, 1992)
 - 比較產生的數列和零和賽局實驗中的數列
- ▶ **Condition D:** Matching pennies 150 times 1-by-1
 - ▶ 150次逐次黑白猜
- ▶ **Condition S:** Give sequence of 150 plays at once
 - ▶ 一次給150回合黑白猜的決定
- ▶ **Condition R:** Produce the outcome of tossing an unbiased coin 150 times
 - ▶ 產生數列——丟銅板150次的結果

Game Play (賽局實驗) vs. Production (產生數列)

- ▶ iid rejected for 40% (D), 65% (S), 80% (R) of the subjects in the three conditions
 - ▶ 三種分別有40%, 65% 和80%的受試者拒絕 iid 假設
 - ▶ **Game play** reduces deviations from randomness
 - ▶ 真的去玩會讓受試者比較隨機(降低偏離情形)
- ▶ Are subjects better motivated?
 - ▶ 這是因為受試者有更好的誘因,
- ▶ Or, are their working memory interfered and randomize "memory-lessly"?
 - ▶ 還是因為他們的腦部運作(工作記憶)受到干擾, 以致於「忘記過去, 努力面前」?

3-action Matching Pennies

	1	2	3
1	2	-1	-1
2	-1	2	-1
3	-1	-1	2

MSE
1/3
1/3
1/3

MSE	1/3	1/3	1/3
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- ▶ Rapoport and Budescu (1994)

Runs in 3-action Matching Pennies R&B '94

Pattern	Game (%)	Production (%)	iid (%)
xx	26.9%	27.2%	33.3%
xyy	19.6%	21.0%	22.2%
xyx	19.6%	20.9%	22.2%
xxx	7.3%	6.3%	11.1%
xxxx	2.0%	1.8%	3.7%
xxxy	5.3%	4.5%	7.4%
xyxx	5.6%	4.5%	7.4%
xyyx	5.8%	3.5%	7.4%
xyxx	5.6%	3.7%	7.4%

Other Play in 3-action Matching Pennies

Pattern	Game (%)	Production (%)	iid (%)
xy	73.1%	72.8%	66.7%
xyx	23.7%	16.0%	22.2%
xyz	29.7%	35.9%	22.2%
yxzx	9.6%	7.8%	7.4%
xyxz	9.9%	7.9%	7.4%
xyzx	12.1%	17.3%	7.4%

Source: Table 3.10, BGT.

A Limited Memory Model (有限記憶模型)

- ▶ Subjects only remember last m elements
- ▶ Chose the $(m+1)$ st to balance the number of H and T choices in the last $(m+1)$ flips
 - ▶ 受試者只記得最後 m 回合，第 $(m+1)$ 回合做決定來平衡正反面在 $(m+1)$ 次中出現的次數(如果 m 很小就會正反變換太頻繁)
- ▶ If m is small, alternate choices too frequently
- ▶ Experimental Data: (Should all be 0.5 if iid)
 - ▶ $P(H|H)=0.42$, $P(H|HH)=0.32$, $P(H|HHH)=0.21$
- ▶ Requires $m=7$ to generate this (Magic 7?)
 - ▶ 實驗結果：如果 iid 的話應該都是 0.5，但需要 $m=7$ 才能解釋實驗結果

Explicit Randomization (使用亂數產生器)

- ▶ Observe the randomization subjects want to play
 - ▶ 觀察人們會為亂數產生器設定何種機率來做
 - ▶ Bloomfield (1994), Ochs (1995b), Shachat (2002)
- ▶ Explicit Randomization: (使用亂數產生器)
 1. Allocate 100 choices to either strategies
 2. Choices are shuffled and computer selects one
 - ▶ 總共100張牌/選擇，決定兩邊各放幾張讓電腦隨機打一張出來...
- ▶ Deviations cannot be due to cognitive limit!
 - ▶ 如果還偏離均衡，就不是因為不能產生亂數！
- ▶ Result: Deviations from MSE small but significant
- ▶ About 10% purists (偏離MSE很小但顯著。10%「單純的人」)

Explicit Randomization (使用亂數產生器)

- ▶ Ex: Ochs (1995b) - Matching Pennies (黑白猜)
 - ▶ Row player payoff of (H, H): $1 \rightarrow 9 \rightarrow 4$ (改列子報酬)
- ▶ MSE: Column MSE changes; row is same...
 - ▶ 行家的MSE會改變；列子的反而不會變
- ▶ Allocate 10 plays of H or T (分配十個選擇給正或反)
 - ▶ Becomes a 10-play sequence (變成「做十次的數列」)
- ▶ Note: Random draw without replacement
 - ▶ This is not exactly randomization of MSE...
 - ▶ 註：這是隨機抽取不放回，不是真的MSE...

Matching Pennies (Baseline)

	H	T
H	1, 0	0, 1
T	0, 1	1, 0

- ▶ MSE:
 - ▶ R: (0.500, 0.500)
 - ▶ C: (0.500, 0.500)
- ▶ Actual Frequency: (實際頻率)
 - ▶ R: (0.500, 0.500)
 - ▶ C: (0.480, 0.520)
- ▶ QRE:
 - ▶ R: (0.500, 0.500)
 - ▶ C: (0.500, 0.500)

Matching Pennies (Game 2)

	H	T
H	9, 0	0, 1
T	0, 1	1, 0

- ▶ MSE:
 - ▶ R: (0.500, 0.500)
 - ▶ C: (0.100, 0.900)
- ▶ Actual Frequency: (實際頻率)
 - ▶ R: (0.600, 0.400)
 - ▶ C: (0.300, 0.700)
- ▶ QRE:
 - ▶ R: (0.649, 0.351)
 - ▶ C: (0.254, 0.746)

Matching Pennies (Game 3)

	H	T
H	4, 0	0, 1
T	0, 1	1, 0

- ▶ MSE:
 - ▶ R: (0.500, 0.500)
 - ▶ C: (0.200, 0.800)
- ▶ Actual Frequency: (實際頻率)
 - ▶ R: (0.540, 0.460)
 - ▶ C: (0.340, 0.660)
- ▶ QRE:
 - ▶ R: (0.619, 0.381)
 - ▶ C: (0.331, 0.669)

MSE in Field Context (實際現場的MSE)

- ▶ Rapoport and Almadoss (2000)
- ▶ Patent races games (競相專利賽局)
 - ▶ Two firms with endowment e (兩家廠商, 各有財產)
 - ▶ Invest $1, 2, \dots, e$ (integer)
 - ▶ Win r if invest the most
- ▶ **Unique MSE:** Invest e with prob. $1 - e/r$,
invest others with prob. $1/r$ (not obvious)

Patent Race Results (競相專利賽局實驗結果)

(Table 3.14)	Game L: $e = 5, r = 8$		Game H: $e = 5, r = 20$	
	Investment	MSE	Actual	MSE
0	12.5%	16.9%	5.0%	14.1%
1	12.5%	11.6%	5.0%	5.5%
2	12.5%	8.8%	5.0%	5.3%
3	12.5%	11.8%	5.0%	5.3%
4	12.5%	9.0%	5.0%	6.9%
5	37.5%	41.8%	75.0%	62.8%

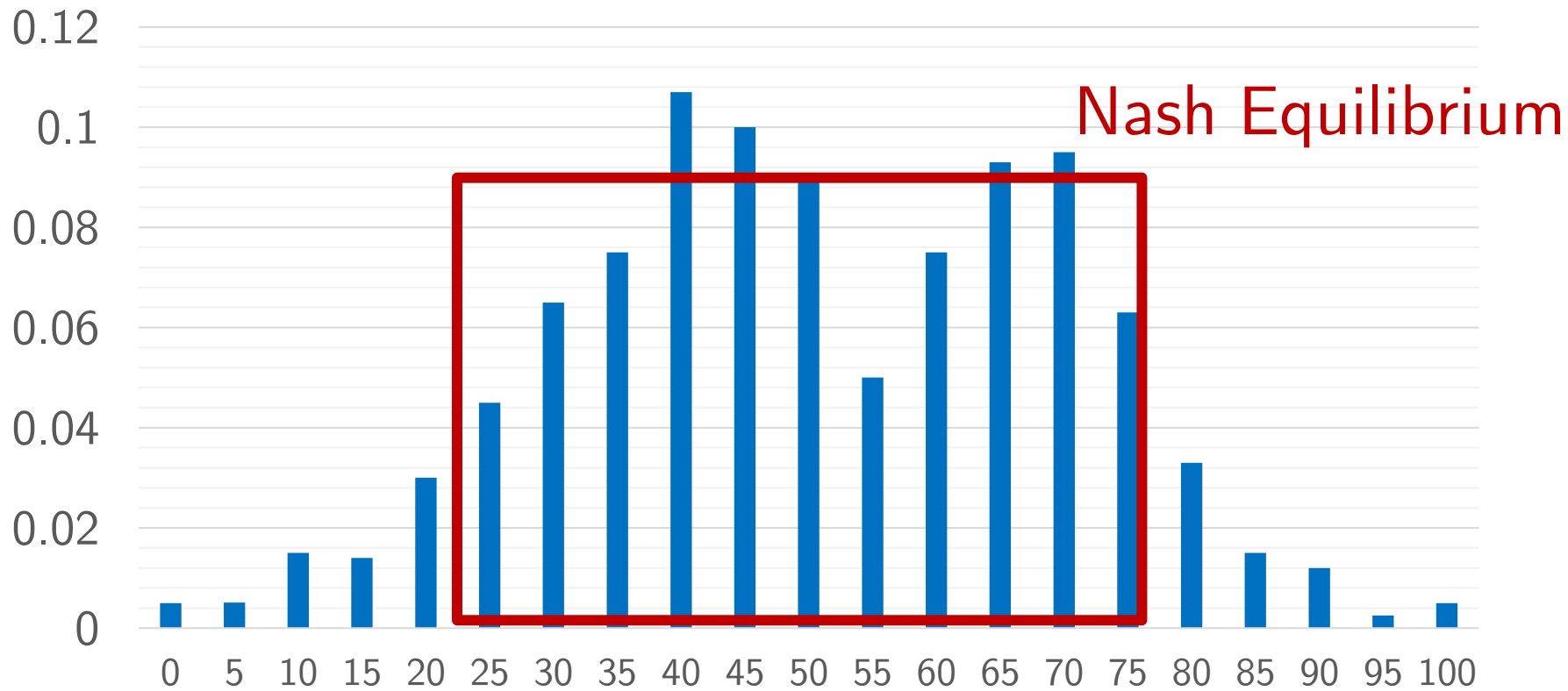
MSE in Field Context (實際現場的MSE)

- ▶ **3 Firm Hotelling:** Collins and Sherstyuk (2000)
 - ▶ 2-Firm: Brown-Kruse, Cronshaw & Schenk (1993)
 - ▶ 4-Firm: Huck, Muller and Vreind (2002)
- ▶ **Location Games (3 Firm Hotelling Model)**
 - ▶ Three firms simultaneously choose $[0,100]$
 - ▶ Consumers go to nearest firm
 - ▶ Profits proportional to units sold
- ▶ **Unique MSE:** Randomize uniformly $[25,75]$

MSE in Field Context (實際現場的MSE)

Frequency

Source: Figure 3.2, BGT; Based on Colins and Sherstyuk (2000).



Distribution of location choices

Two Field Studies

- ▶ **Walker and Wooders (2001)**
 - ▶ serve decisions (L or R) of tennis players in 10 Grand Slam matches
- ▶ **Result:**
 - ▶ Win rates across two different directions are not statistically different ($p < 0.10$ for only 2/40)
 - ▶ Players still exhibit some over-alteration in serve choices through temporal dependence ($p < 0.10$ for 8/40) [weaker than lab subjects]

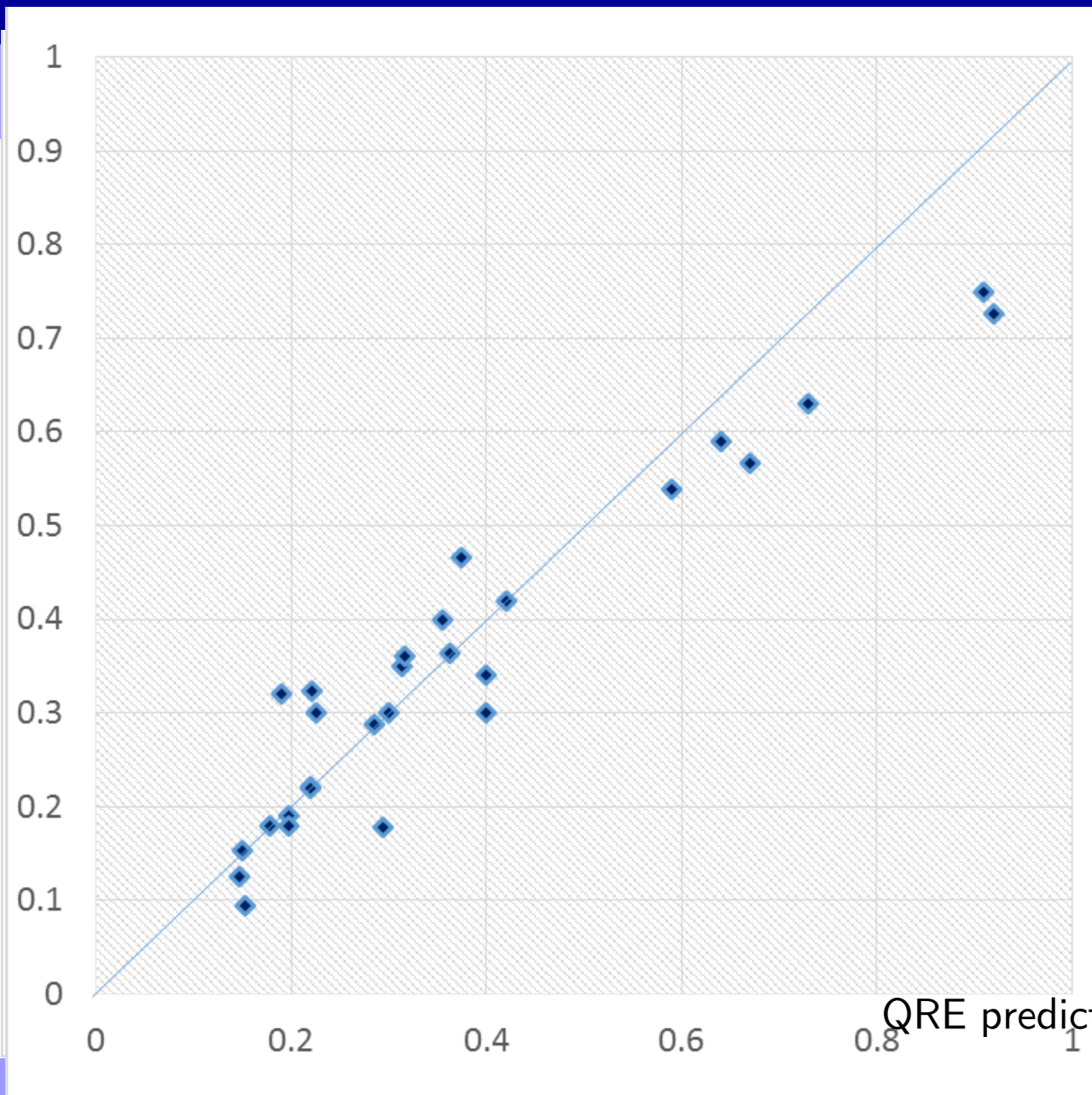
Two Field Studies

- ▶ **Palacios-Huerta (2001): soccer penalty kicks**
 - ▶ Code both kicker and goalie's choices
 - ▶ No selection bias (look at all games)
- ▶ Win rates are equal; no serial dependence
 - ▶ Not surprising since penalty kicks are few and are often done by different players
- ▶ Recent: Huang, Hsu, and Tang (AER 2007)
 - ▶ Chen-Ying Huang (here at NTU)

Conclusion

- ▶ **Take-home Message:**
- ▶ Aggregate frequencies of play are close to MSE but the deviations are statistically significant.
- ▶ **QRE seems to fit behaviors well.**
- ▶ Temporal dependence frequently observed

Actual Data
(實驗資料)



Source
(資料來源):
Behavioral
Game Theory
Ch 3

ions(均衡預測)

QRE predictions(均衡預測)

Conclusion

- ▶ With explicit randomization, the existence of purists hint on **equilibrium in beliefs**
 - ▶ Players cannot guess what opponents are doing
 - ▶ Beliefs about opponents are correct on average
 - ▶ But, they may not be randomizing themselves
- ▶ **Field-Lab-Theory:** Ostling, Wang, Chou and Camerer (2011), "[Testing Game Theory in the Field: Evidence from Swedish Poisson LUPI Lottery Games](#)," *American Economic Journal: Microeconomics*, 3(3), 1-33.