Quiz III

2013 / 5 / 17

Problem 1 *Prove that if T is a self adjoint operator on a finite dimensional inner product space V, then every eigenvalue of T is real.*

Problem 2 *Let T be a linear operator on a finite dimensional inner product space V. Prove the following:*

(a) $N(T^*T) = N(T)$. *Deduce that* $rank(T^*T) = rank(T)$.

(b) $rank(T) = rank(T^*)$

(c) For any $n \times n$ *matrix* A, $rank(A^*A) = rank(AA^*) = rank(A)$

Problem 3 *Let $T : V \to W$ be a linear transformation, where V and W are finite dimensional inner product spaces. Prove that $(R(T^*))^{\perp} = N(T)$.*