Quiz II

Name: ______________________ ID: ______________________

1. Determine whether the following are inner products. You do NOT need to explain the reasons.
   (a) $\langle \begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} c \\ d \end{pmatrix} \rangle = ad + bc$ on $\mathbb{R}^2$.
   (b) $\langle A, B \rangle = \operatorname{tr}(A^t B)$ on $M_{n\times n}(\mathbb{C})$.
   (c) $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ on $C([-1, 1])$.

2. Let V be a finite dimensional inner product space, and suppose $T : V \to V$ is a linear transformation that preserves the inner product, i.e. $\langle T(x), T(y)\rangle = \langle x, y\rangle$ for all x, y. Show that T is an isomorphism.

3. Let $V = M_{2\times 2}(\mathbb{R})$ with $\langle A, B\rangle = \operatorname{tr}(A^t B)$ and let

$$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\}.$$

   Find an orthonormal basis β of V by applying the Gram-Schmidt process to S.