Section 5.4

23. Hit the relation
$$v_1 + \cdots + v_k \in W \tag{1}$$
by T to obtain
$$\lambda_1 v_1 + \cdots + \lambda_k v_k \in W. \tag{2}$$
Use (1) and (2) to obtain a shorter sum which belongs to W, and then use induction.

24. First we have
$$V = E_{\lambda_1} \oplus \cdots \oplus E_{\lambda_r}$$
and we want to show that
$$W = (W \cap E_{\lambda_1}) \oplus \cdots \oplus (W \cap E_{\lambda_r}).$$
(Why is this enough?)

    The inclusion "$\supset$" is clear. For "$\subset$" take any $w \in W$. Then $w = v_1 + \cdots + v_r$ for some $v_i \in E_{\lambda_i}$ (why?). By Exercise 23, each v_i is also in W.

25. To show that $U(E_\lambda) \subset E_\lambda$, one hits $U(E_\lambda)$ by T and obtains
$$T\big(U(E_\lambda)\big) = U\big(T(E_\lambda)\big) = U(\lambda \cdot E_\lambda) = \lambda \cdot U(E_\lambda).$$
(Make sure you know the reason for each equality.) Thus $U(E_\lambda) \subset E_\lambda$ (why?).

38. For "$\Rightarrow$", use Exercise 23.

42. The matrix A is in fact diagonalizable with eigenvalues 0 and n. Your job: find $n-1$ linearly independent eigenvectors with eigenvalue 0, and one eigenvector with eigenvalue n.

Section 2.6

13. (b) Construct a basis β of V which contains a basis of W and the vector x. Then take the dual basis of β.

    (c) This exercise shows that $(S^\circ)^\circ$ is the smallest subspace of V containing S. In particular if W is a subspace of V, then $(W^\circ)^\circ = W$.

17. If we have shown that "W is T-invariant $\Rightarrow$ W° is T^t-invariant", the other direction can be obtained by taking dual space again (using $(W^\circ)^\circ = W$ and $(T^t)^t = T$).