Assignment 2

Due on December 3, 2009

1. Show that the Lubin-Tate formal group (introduced in the course *Algebraic Number Theory II*) for $\mathbb{Q}_p$ can be taken to be the formal multiplicative group, defined by the formal power series $X + Y + XY$. Thus its p (= a uniformizer of $\mathbb{Q}_p$)-power torsions are elements of the form $\zeta - 1$, where $\zeta^{p^n} = 1$ for some $n \geq 1$, and they generate the maximal totally ramified abelian extension of $\mathbb{Q}_p$.

2. Let R be a complete discrete valuation ring with maximal ideal $\mathfrak{m}$; $X = (X_i)_{i=1}^n$. Define two subrings A, B of $R[[X]]$ respectively by

$$A = \left\{ \sum_{s=0}^{\infty} a_s P_s(X) \,|\, a_s \in \mathfrak{m}^s, P_s(X) \in R[X], \deg P_s(X) \leq c(s+1) \text{ for some } c > 0 \right\}$$

$$B = \left\{ \sum_{I \in (\mathbb{Z}_{\geq 0})^n} \alpha_I X^I \,|\, \exists\, \rho > 1 \text{ s.t. } \lim \|\alpha_I\| \rho^{|I|} = 0 \right\}.$$

Show that $A = B$.

3. ([2, §3.0]) Let k be a perfect field of positive characteristic; W = the ring of Witt vectors of k. Let $\mathbb{A}^1$ be the affine line over k. Show that the rank of the W-module $H^1_{cris}(\mathbb{A}^1/W)$ is infinity.

4. ([1, p.3.3]) Let k be a ring of characteristic p; $A = k[x_1, \cdots, x_6]/(x_1^p, \cdots, x_6^p, x_1x_2 + x_3x_4 + x_5x_6)$ and $I \subset A$ be the ideal generated by $x_1, \cdots, x_6$. Show that there is no PD-structure on I.

References

[1] P. Berthelot and A. Ogus, *Notes on crystalline cohomology.* Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo, 1978.

[2] L. Illusie, Report on crystalline cohomology. *Algebraic geometry (Proc. Sympos. Pure Math., Vol. 29, Humboldt State Univ., Arcata, Calif., 1974)*, pp. 459-478. Amer. Math. Soc., Providence, R.I., 1975.