Assignment 2

Due on November 26, 2009

1. ([1, Ex 9.13]) Prove the statement: Whenever a homomorphism $f : K \to K'$ of double complexes induces H_d-isomorphism, it also induces H_D-isomorphism.

2. Let S be a compact, orientable, connected surface with g holes in $\mathbb{R}^3$. Let $p_1, \cdots, p_r$ be r distinct points on S ($r > 0$) and $M = S \setminus \{p_1, \cdots, p_r\}$. Compute the de Rham cohomology groups $H(M)$ and $H_c(M)$.

3. Verify that for any vector bundle E over M, there exists a *natural* trivial sub line bundle in $E^\vee \otimes E$, where $E^\vee$ is the dual vector bundle of E. (The sections of this line bundle are constant scalars in $E^\vee \otimes E = \mathrm{Hom}_M(E, E)$.)

4. Find the de Rham cohomology groups of the manifolds $GL(1, \mathbb{R})$ and $GL(2, \mathbb{R})$. (Hint: For the latter, you may use the decomposition in [1, Ex 6.6.(a)].)

5. ([1, Ex 6.10]) Compute $\mathrm{Vect}_k(S^1)$ (= the isomorphism classes of rank k real vector bundles over the unit circle S^1). (Hint: You may use the decomposition in [1, Ex 6.6.(a)] and the fact that $SO(n)$ is connected.)

References

[1] R. Bott abd L.W. Tu, *Differential forms in algebraic topology.* Graduate Texts in Mathematics, 82. Springer-Verlag, New York-Berlin, 1982.