Combinatorics and Geometry of KP Solitons and Applications to Tsunami

Yuji Kodama*

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Abstract: Let $\mathrm{Gr}(N, M)$ be the real Grassmann variety defined by the set of all N-dimensional subspaces of $\mathbb{R}^M$. Each point on $\mathrm{Gr}(N, M)$ can be represented by an $N \times M$ matrix A of rank N. If all the $N \times N$ minors of A are nonnegative, the set of all points associated with those matrices forms the totally nonnegative part of the Grassmannian, denoted by $\mathrm{Gr}(N, M)^+$. In this talk, I start to give a realization of $\mathrm{Gr}(N, M)^+$ in terms of the (regular) soliton solutions of the KP (Kadomtsev-Petviashvili) equation which is a two-dimensional extension of the KdV equation. The KP equation describes small amplitude and long waves on a surface of shallow water. I then construct a cellular decomposition of $\mathrm{Gr}(N, M)^+$ with the asymptotic form of the soliton solutions. This leads to a classification theorem of all solitons solutions of the KP equation, showing that each soliton solution is uniquely parametrized by a derangement of the symmetric group S_M. Each derangement defines a combinatorial object called the Le-diagram (a Young diagram with zeros in particular boxes). Then I show that the Le-diagram provides a complete classification of the "entire" spatial patterns of the soliton solutions coming from the $\mathrm{Gr}(N, M)^+$ for asymptotic values of the time. I will also present some movies of real experiments of shallow water waves which represent some of those solutions obtained in the classification problem. Finally I will discuss an application of those results to analyze the Tohoku-tsunami on March 2011. The talk is elementary, and shows interesting connections among combinatorics, geometry and integrable systems.

*The Ohio State University