Empirical Process Theory: ULLN, UCLT and Rate of Convergence of LSE

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Abstract: In this talk, I would like to give an introductory-level lecture about an idea in statistics. There are no specific prerequisites for audience.

The framework of **Empirical Process Theory** enable the classic **Law of Large Number (LLN)** and **Central Limit Theorem (CLT)** to have their functional version **Uniform Law of Large Number (ULLN)** and **Uniform Central Limit Theorem (UCLT)**. It also gives the rate of convergence of **Least Square Estimator (LSE)** of different regression function, which is widely used in statistical inference.

Roughly, LLN and CLT tell us the limit and rate of convergence of random variable (or, function) of certain type while ULLN and UCLT give us the limit and rate of convergence of measure (or, functional) of certain type.

Through this talk, I wish I could achieve the following goal:

1. Derive Empirical Process (EP) from Empirical CDF (ECDF).
2. Understanding ULLN and Glivenko-Cantelli Class.
3. Understanding UCLT and Donsker Class.
4. Understanding entropy of set of function and VC-class.
5. See Rate of Convergence of LSE of different regression function.

All object will be defined in the talk, and there are no specific prerequisites for audience. Everyone is welcomed to participate.