Vector bundles associated with Higgs bundles with applications to the Bogomolov inequality of semistable Higgs bundles

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Abstract: A Higgs bundle on a smooth variety X is a pair (E, φ) of a vector bundle and a homomorphism $\varphi : E \to \Omega^1 \otimes E$. If a subsheaf F of E satisfies $\varphi(F) \subset \Omega^1 \otimes F$, it is called a Higgs subsheaf. When X is projective with an ample divisor H, we can define the semistability of a Higgs bundle (E, φ) in the same manner as in the case of usual vector bundles. In the lecture, we show that a Higgs bundle (E, φ) gives rise to a vector bundle $\widetilde{E}$ defined over a variety Y together with a surjective morphism $f : Y \to X$ such that (1) $\widetilde{E}$ has the same rank as E; (2) Higgs subsheaves F of E corresponds to a subsheaf of $\widetilde{E}$; and (3) Semistability of E translates to a weak semistability of $\widetilde{E}$. From the above correspondence between (E, φ) and $\widetilde{E}$, we deduce several results such as: (1) Semistability is preserved by pull-backs and tensor products; (2) The 1st and the 2nd Chern classes of a semistable Higgs bundle satisfy the Bogomolov inequality. The Bogomolov inequality was first proved by Simpson by constructing harmonic metrics on Higgs bundles, which depends on hard analysis. Our approach gives a much more elementary alternative proof.

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