Recent Progress on the Search of 3D Euler Singularities

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Abstract: Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This is closely related to the Clay Millennium Problem on 3D Navier-Stokes Equations. We first review some recent theoretical and computational studies of the 3D Euler equations which show that there is a subtle dynamic depletion of nonlinear vortex stretching due to local geometric regularity of vortex filaments. Our study suggests that the convection term could have a nonlinear stabilizing effect for certain flow geometry. We then present strong numerical evidence that the 3D Euler equations develop finite time singularities. To resolve the nearly singular solution, we develop specially designed adaptive (moving) meshes with a maximum effective resolution of order 10^12. A careful local analysis also suggests that the blowing-up solution is highly anisotropic and is not of Leray type. However, the solution develops a self-similar structure near the point of the singularity in the radial and axial directions as the singularity time is approached. A 1D model is proposed to study the mechanism of the finite time singularity. Recently we prove rigorously that this 1D model develops finite time singularity.

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