How to Construct Meromorphic Functions with Prescribed Principal Parts?

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Abstract: In order to study the geometry of an algebraic variety X, the space $H^0(X, mK_X + D)$ of m-th tensor of meromorphic forms of top degree whose poles are no worse than an effective divisor D, play the most important roles. The basic tool to estimate the size of $H^0(X, mK_X + D)$ is the Riemann-Roch theorem. For the simplest case, namely X being a compact Riemann surface of genus g, the Riemann-Roch theorem tells us that $\dim_{\mathbb{C}} H^0(X, K_X) = g$. Historically, however, Riemann got the Riemann-Roch theorem for compact Riemann surfaces after being able to construct explicitly g independent holomorphic 1-forms. In this talk we will revisit an approach due to Riemann to construct meromorphic functions on a compact Riemann surface with prescribed principal parts. This very explicit method, which seems to be missing in most of modern textbooks on complex analysis, Riemann surfaces, and algebraic geometry, was then used to produce holomorphic 1-forms and to prove the Riemann-Roch theorem. It is interesting (and also very challenging) to find appropriate higher dimensional generalizations of this approach.

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