Systems of nonlinear, dispersive evolution equations

Part I: The initial-value problem

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Part II: Solitary waves and stability theory

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Abstract: Considered here are coupled systems

$$\begin{cases} u_t + u_{xxx} + P(u,v)_x = 0, \\ v_t + v_{xxx} + Q(u,v)_x = 0, \end{cases} \tag{1}$$

of equations of Korteweg-de Vries type, where $u = u(x,t)$ and $v = v(x,t)$ are real-valued functions defined for $x \in \mathbb{R}, t \geq 0$, and $P(u,v)$ and $Q(u,v)$ are homogeneous quadratic polynomials of the form

$$P(u,v) = Au^2 + Buv + Cv^2$$

and

$$Q(u,v) = Du^2 + Euv + Fv^2,$$

with $A, B, \cdots, F$ being given real constants. It will be shown that under certain conditions on P and Q, the system is well-posed globally in time provided the initial data $(u(x,0), v(x,0)) = (u_0(x), v_0(x))$ lies in the space $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for some $s > -\frac{3}{4}$. More specifically, if the system of linear equations

$$\begin{cases} 2Ba + (E-2A)b - 4Dc = 0, \\ 4Ca + (2F-B)b - 2Ec = 0, \end{cases} \tag{2}$$

has a solution (a,b,c) such that $a^2 + c^2 > \frac{1}{4}b^2$, then (1) is globally well-posed as described above.

These systems possess solitary-wave solutions, and the lectures continue with an analysis of their existence, multiplicity and especially, their stability.