Theory of Computation

Midterm Exam, Fall 2010

1. (15 pts) Which of the following languages are regular? If the language is regular, present a finite automaton or regular expression. If not, give a proof using the pumping lemma.

   (a) The set of all 0-1 strings in which the total number of zeros to the right of each 1 is even.
   Ans. Regular. $0^*(1^*00)^*1^*$

   (b) The set of all 0-1 strings that contain more 1s than 0s.
   Ans. Not regular. Use pumping lemma on string 0^m1^{m+1}, where m is the pumping constant.

   (c) The set of all 0-1 strings of the form 0^m1^n where m is odd and n is even.
   Ans. Regular. $0(00)^*(11)^*$.

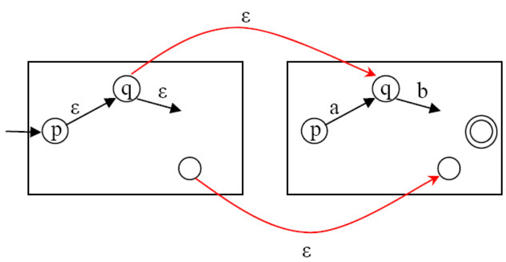
   (d) The set of all 0-1 strings in which the number of occurrences of "000" and of "111" are the same. (Note that the string "1110000111" contains two occurrences of each.)
   Ans. Not regular. Use pumping lemma.

2. (10 pts) Use the pumping lemma to show in detail that the language $\{a^{n^3} \mid n \geq 1\}$ is not regular.
Ans. Use the pumping lemma, along with the fact that $n^3 < n^3 + k < (n+1)^3$, for every $k \leq n$.

3. (10 pts) Prove that $L = \{a^m b^n c^k \mid m, n, k \geq 0, m \neq n \text{ or } m \neq k \text{ or } n \neq k\}$ is not regular. (Hint: Use closure properties of regular languages along with the pumping lemma.)
Ans. Consider the language $\bar{L} \cap a^*b^*c^*$. The rest is easy.

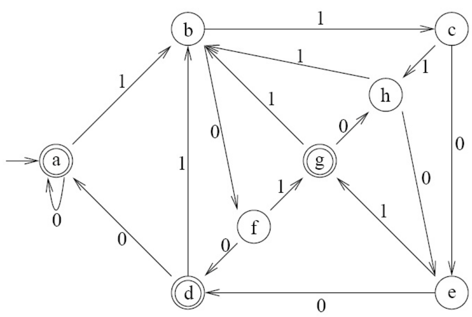
4. (15 pts) Answer the following two questions:

   (a) (6 pts) For regular languages $R \subseteq \Sigma^*$, prove that $TAIL(R) = \{y \mid \exists x \in \Sigma^*, xy \in R\}$ is regular language.
   Ans. Use the following construction.



   (b) (9 pts) Let r, s, r_I, s_I be regular expressions for $R, S, TAIL(R)$, and $TAIL(S)$ respectively. Using only these regular expressions and the operations +, concatenation, and *, give regular expressions for the following languages: (1) $TAIL(R \cup S)$; (2) $TAIL(RS)$; (3) $TAIL(R^*)$. No explanations are needed.
   Ans. (1) $TAIL(R \cup S) = r_I + s_I$; (2) $TAIL(RS) = s_I + r_I s$; (3) $TAIL(R^*) = r_I r^*$

5. (15 pts) Consider the following DFA.



(a) (9 pts) Find an equivalent DFA with the fewest number of states. Show your work in sufficient detail.

Solution: Below each new state is labeled by the set of states it is the result of merging.

	0	1
$\rightarrow *\{a,d\}$	$\{a,d\}$	$\{b,c,h\}$
$\{b,c,h\}$	$\{e,f\}$	$\{b,c,h\}$
$\{e,f\}$	$\{a,d\}$	$\{g\}$
$*\{g\}$	$\{b,c,h\}$	$\{b,c,h\}$

(b) (6 pts) For each pair of states $\{p,q\}$ in your minimized DFA, give a word w which distinguishes p and q.

Solution: The table supplied below contains all the strings necessary.

	$\{a,d\}$	$\{b,c,h\}$	$\{e,f\}$
$\{b,c,h\}$	ϵ		
$\{e,f\}$	ϵ	0	
$\{g\}$	0	ϵ	ϵ

6. (15 pts) For each of the following languages over $\Sigma = \{a,b\}$, write a context-free grammar for it.

   (a) $L_1 = \{a^n b^m \mid \frac{m}{2} \leq n \leq m\}$.
   Ans. $S \rightarrow aSb \mid aSbb \mid \epsilon$

   (b) $L_2 = \{ww^R \mid |w| \geq 1\}$.
   ANs. $S \rightarrow aSa \mid bSb \mid aa \mid bb$

   (c) $L_3 = \{a^{i+j}b^j \mid i \geq j \geq 0\}$.
   ANs. $S \rightarrow aaSb \mid aS \mid \epsilon$

7. (20 pts) True or False? Score=$Max\{0, Right - \frac{1}{2}Wrong\}$. No explanations needed.

   (a) ○ A PDA with two stacks can accept the language $\{0^n1^n2^n \mid n \geq 0\}$.

   (b) ○ For any language L, there are infinitely many different grammars G such that $L(G) = L$.

   (c) × If L is a CFL and R is a regular language, then $R - L$ is a CFL.

   (d) × If some word w in $L(G)$ has two different derivations, then G is ambiguous.

   (e) ○ If L is not context-free, then L^R is not context free either (where R is the reversal operator).

   (f) ○ $L = \{0^n1^m0^m : n + m = 3 \ mod \ 5\}$ is context-free but not regular.

   (g) ○ If L is context-free and R and S are regular, then *MAJORITY(L,R,S)*=$\{w \mid w$ is in at least two of $R, L, S\}$ is also context-free.
   Ans. Modify the PDA M_L that accepts L to simultaneously simulate R and S. Accept if at least two accept.

   (h) × $\{a^ib^jc^k \mid 0 < i < j < k\}$ is context-free.

   (i) × Let Σ be a finite alphabet, and let $h : \Sigma^* \rightarrow \Sigma^*$ be a homomorphism. For any language L, define $h^*(L)$ to be $h^*(L) = L \cup h(L) \cup h(h(L)) \cup ....$ If L is regular, then $h^*(L)$ is also regular.
   Ans. $h^*(L)$ is not necessarily regular, even if L is regular. Let $L = \{0\}$ and define $h(0) = 00$. Then $h^*(L) = \{0^{2^n} | n \geq 0\}$.

   (j) ○ $L = \{0^n1^m0^n \mid n < 12 < m\}$ is regular.