Theory of Computation

Spring 2025, Homework # 4

Due: May 27, 2025

1. (20 pts) Give an example of an undecidable (i.e., non-recursive) subset of 1^*. Prove that your answer is correct. (Hint: Consider the list of all TMs $M_1, M_2, ..., M_k, ...$ Construct a language $L \subseteq 1^*$, and use the diagonalization method to show L to be non-recursive.)

2. (20 pts) Is the language $L = \{\langle M, w\rangle |\ M$ at some point in time moves left while computing $w\}$ recursive? Why? Give a convincing argument.

3. (20 pts) Let $L = \{\langle M_1, M_2\rangle \mid M_1, M_2$ are TMs such that for some input x, both M_1 and M_2 halt on $x\}$. Prove that L is r.e. but not recursive.

4. (20 pts) Let $T = \{\langle M\rangle \mid M$ is a TM that accepts w^R whenever it accepts $w\}$. Show that T is not decidable. Do not use Rice's theorem. (Here w^R is the reversal of w.)

5. (20 pts) Suppose there are four languages A, B, C, and D. Each of the languages may or may not be recursively enumerable. However, we know the following about them: $A \leq_m B$, $B \leq_m C$, and $D \leq_m C$. Below are four statements. Indicate whether each one is

   (a) CERTAIN to be true, regardless of what languages A through D are.

   (b) MAYBE true, depending on what A through D are.

   (c) NEVER true, regardless of what A through D are.

   Justify your answers.

   (1) A is recursively enumerable but not recursive, and C is recursive.

   (2) A is not recursive, and D is not recursively enumerable.

   (3) If C is recursive, then the complement of D is recursive.

   (4) If C is recursively enumerable, then $B \cap D$ is recursively enumerable.