Theory of Computation

Spring 2024, Homework #1

Due: March 26, 2024

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1. (20 pts) Use the Myhill-Nerode Theorem to prove that $L = \{a^i b^j c^k \mid i, j, k \geq 0$ and if $i = 1$ then $j = k\ \}$ is not regular. Recall that the pumping lemma fails to show L to be non-regular.

2. (40 pts) Which of the following statements are correct? If you think a statement is correct, give a proof. If you think it is incorrect, give a counterexample.

   (a) If A is a regular language over Σ and $\{a, b\} \subseteq \Sigma$, then the language $L = \{c^n \mid \exists w \in A, \#_a(w) + \#_b(w) = n\}$ is always regular. Here $\#_a(w)$ and $\#_b(w)$ denote the numbers of occurrences of a and b in w, respectively.

   (b) Given two regular languages L_1 and L_2, the language $L = \{x0y \mid x \in L_1, y \in L_2, |x| = |y|\}$ is always regular.

   (c) If L is a regular language over $\Sigma = \{a, b\}$, then $L' = \{x \mid ax \in L$ or $xb \in L\}$ is always regular.

   (d) The language $L = \{a^m b^n \mid m + n$ is a prime number $\}$ is not regular.

3. (40 pts) An Infinite Input Finite Automaton (IIFA) is a tuple $M = (Q, \Sigma, \delta, q_0, F)$, as was defined for nondeterministic finite automata where $\delta : Q \times \Sigma \to 2^Q$, except that M now operates on infinite strings of symbols $s = s_0 s_1 \ldots$ over Σ (i.e., $\forall i \geq 0, s_i \in \Sigma$). A run of M on s is an infinite sequence of states $r = r_0, r_1, \ldots$, where $r_0 = q_0$ and $r_{i+1} \in \delta(r_i, s_i)$, for all $i \geq 0$. We define $inf(r)$ to be the set of states that occurs infinitely many times along r. A run r is accepting if $inf(r) \cap F \neq \emptyset$, i.e., some accepting state is visited infinitely often. Automaton M accepts string s if there is an accepting run r of M on s. The language of M, denoted $L(M)$, is the set of infinite strings accepted by M. Answer the following questions:

   (a) (10 pts) Show that if A and B are IIFAs, then there is an IIFA C such that $L(C) = L(A) \cup L(B)$. (Hint: Given $A = (Q_A, \Sigma, \delta_A, q_{A0}, F_A)$ and $B = (Q_B, \Sigma, \delta_B, q_{B0}, F_B)$, construct $C = (Q_C, \Sigma, \delta_C, q_{C0}, F_C)$ from A and B.)

   (b) (10 pts) Show that if A and B are IIFAs, then there is an IIFA C such that $L(C) = L(A) \cap L(B)$. (Note that letting $F_C = F_A \times F_B$ in the product automaton $A \times B$ does not work as an accepting run for $L(C)$ does not have to visite states of F_A and F_B simultaneously infinitely many times.)

   (c) Consider an IIFA M whose language is $L(M) = (0 + 1)^* 00^\omega$, where 0^ω stands for "000..." (i.e., "0" repeats infinitely many times).
   [(i)](5 pts) Design an nondeterministic IIFA to accept L. (15 pts) Show that nondeterministic IIFAs are more powerful than deterministic IIFAs by proving that $L(M)$ cannot be accepted by any deterministic IIFA.